Determination of similarity criteria in optimization tasks by means of neuro-fuzzy modelling

Abstract. A method is proposed for solving optimization problems with high complexity when searching for the function minimum by using methods and means of similarity theory and neuro-fuzzy modelling. The problem with nonlinear objective function and constraints is transformed into a task with a nonlinear objective function and linear constraints. In this task, the basic similarity criteria are presented in the form of membership functions. Means of similarity theory, in particular the criterion (1) and means of similarity theory and neuro-fuzzy modelling. The problem with nonlinear objective function and constraints is transformed into a task with a nonlinear objective function and linear constraints. In this task, the basic similarity criteria are presented in the form of membership functions. The corresponding dual tasks can be formulated like this [4, 5]: To maximise (3)

$\delta(x) = \prod_{i=1}^{m} \left( \sum_{j=1}^{n} x_{ij}^{a_{ij}} \right)^{\pi_{ij}}$

given orthogonally (4)

$\sum_{i=1}^{m} a_{ij} \pi_{ij} = 0, s = -1/n$

and normalization (5)

$\sum_{j=1}^{n} \pi_{ij} = 1$

where $\pi_{ij}$ is the similarity criterion and

$\lambda_i = \prod_{j=m+1}^{m+1} \pi_{ij}$

denote Lagrange (correlation) coefficients.

In criterion programming the orthogonality and normalization system of equations can be written as:

$\alpha \cdot \pi = b$

where $\alpha$ is the matrix of indicators

$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha_{n1} & \alpha_{n2} & \alpha_{n3} & \cdots & \alpha_{nn} \end{pmatrix}$

and $\pi$ is the vector of similarity criteria

$\pi = \begin{pmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{pmatrix}$
When \( \alpha \) is a square matrix, and it can be only when the total amount of criterion function members and restrictions per unit is more than the amount of variables the system of equations (6) easily solved by any known method. In any other cases the system of equations is not defined or has a lot of solutions.

In all other cases, for example when \( \alpha \) – rectangular matrix, the system of equations not defined or has lot solutions. In CM, the value \( s=m-n-1 \) is called the degree of complexity of the task. For such problems, orthogonal systems of equations (6) is written in the form:

\[
\pi = \begin{bmatrix}
\pi_1 \\
\pi_2 \\
\pi_3 \\
\vdots \\
\pi_m \\
\end{bmatrix},
\]

where \( m_1+1; \ m_1+2 \) etc. – the indices of the members of the system of equations, which correspond to the members of the constraint (2).

### Analysis of existing methods for challenging problems solution

In [1] it is shown that similarity criteria in the system of normalized orthogonally equations (4) - (5), are defined as:

\[
\pi_i = \beta_{i0} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j
\]

where \( \beta_{i0} \) is the normalisation vector, \( \beta_{ij} \) the nullity vector, \( \pi_j \) is the basic criteria of similarity and \( s=m-n-1 \) the degree of complexity of a criteria programming problem.

If we express similarity criteria through basic criteria and nullity vectors dual function will be written in such a way (8). Here \( \mathcal{N} \) - set which consists of indexes of members of \( k \)-restriction.

As we consider problems where function \( d(\pi) \) is convex then it is always possible to replace definition \( d(\pi)_{\text{max}} \) by definition of a stationary function point because sets of maximizing points of these functions coincide [1]. Let’s find algorithm of expression (8) in (9):

Take derivative of function (9) by basic criteria of similarity \( \pi_j \) (10). Having set it equal to zero and having found antilogarithm of it, we will receive a system of equations from which it is possible to receive maximum function conditions \( d(\pi_j) \). They will be the following (11) to (12).

### Determination of optimal criteria for similarity method of neuro-fuzzy modelling

Membership function, are similar to similarity criteria \( \pi \) which are a dimensionless system parameters correspondence. And in that case when they are defined by a method of integrated analogues, they also are weight coefficients of criterion function components (rated to unit) [5, 6].

Membership function and similarity criterion change from 0 to 1. Hence it is possible to draw analogy between membership function and similarity criterion [5, 7].

Similarity of membership function and similarity criterion allows to use membership function instead of similarity

The given approach offered in [1], can be applied only when \( t \) is small. When \( t \) is about 10 its use is in doubt as it is necessary to solve systems of the nonlinear equations of a high order.
The normalisation vector, the nullity vector, are the membership functions for basic similarity criteria and s = m - n - 1 the degree of complexity of a criteria programming problem.

If we express similarity criteria through membership functions, which after the substitution in (13) implements the procedure for determining optimum values of criteria of similarity.

Defining the membership function form the alternative approach was used. The best results were received using π function: m(x) = mS(x) mZ(x), Z-function in Matlab has a name pimf, order of its parameters: [a b c d], where [a d] is the carrier of fuzzy set; [b c] - core of fuzzy set; [a b] - smf-function parameters smf, [c d] - zmf-function parameters zmf.

Let membership function maximum for similarity criterion optimum meaning is defined as:

\[
\mu^{\pi_j}(x_j) = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} d_i}
\]

Using formula (14) it is possible to calculate approximate optimum meaning of optimum criterion.

For improved similarity criterion calculation:
1. Let’s construct membership function with a core calculated using (14).
2. Then normalise fuzzy set, dividing all membership functions into maximum value of membership function.
3. Fuzzy set \( \pi_j \) we transform into a set of \( \alpha \)-level to reduce area of possible values.
4. Specified value \( \mu^{\pi_j} \) is found using methods of linear programming in an interval between transition points. The offered approach has the following advantages:
   1. It does not depend on the degree of complexity of a problem.
   2. It is not necessary to solve equations of (10)–(12).
   3. Considering specific character of \( \mu^{\pi_j} \) it is possible to represent \( \pi_j \), with the help of linguistic variables, that is to involve experts.

If the algorithm is used to solve a specific technical problem with the presence of the sample of retrospective data, to find the basic criteria you can use a simplified approach, which is shown in Fig. 1.

The first algorithm is used when there is a fuzzy set of values of the basic criteria of similarity.
Conclusions

The proposed approach to determining similarity criteria in optimization problems with application of neuro-fuzzy modeling allows to widen the criteria-based method, relatively large-scale problems.

The algorithm of definition of criteria of similarity in this case can be constructed using standard computational procedures. To do this task with a nonlinear objective function and constraints is transformed into a task with a nonlinear objective function and linear constraints. In this task, the basic similarity criteria are presented in the form of membership functions. In turn dependent similarity criteria are defined through the base.

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REFERENCES