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Methods of Teaching English to Future Mathematicians in Ukraine: The Authors' Experience

The knowledge of foreign languages, especially the knowledge of English is very important for future mathematicians. It is highly beneficial for them to have profound level of the foreign language skills, understand and know certain mathematical terms and notions, be able to read mathematical signs and formulae. They should be competent enough to use linguistic skills in order to interpret and infer meaning both from oral professional communication and from specialized literature given in the foreign language. Besides, they should be able to participate in professional discussions, to generate ideas and to express their point of view in the English language. But the process of the language learning can become a challenge for students specializing in Mathematics due to some cognitive and psychological peculiarities inherent to them. The aim of the paper is to study the most efficient means and methods of the English language teaching to future mathematicians at Ukrainian universities. It also states the importance to apply up-to-date techniques in foreign languages teaching and to select relevant tasks to form deep understanding of the peculiar mathematical terminology and to develop main foreign language competences. On the basis of their own experience, the authors prove that appropriately selected teaching methods can have good results with students of mathematical specialties.

Keywords: Mathematics, mathematical academic language, mathematical terminology, mathematical expressions, theorems proofs, language learning, the English language competence, Logical / Mathematical intelligence, methods of teaching English, educational problems and their solution, linguistic skills, a supportive learning environment

Introduction

The language of Mathematics is international. It underlies all the branches of modern science making technological and scientific progress possible. The emphasis on international communication has brought the necessity to know English by people specializing at mathematics to a high level. English has become an international means of the information exchange. The importance of the language in the mathematics learning used to be underestimated for a long time. However, this thinking is slowly changing due to the globalization processes. There is a steady tendency towards the intensification of the language learning in modern educational establishments. The knowledge of English has become a must for every specialist and for future mathematicians as well.

In the article, the authors touch upon the theoretical grounds for the English language acquisition by Math students and reveal the psychological issues related to the language learning. They highlight the possible problems with which future mathematicians can encounter with when mastering the language and suggest the ways of their solution. The authors also build out the algorithm for the successful language learning by the students specializing at Mathematics.

K. Hakuta, Y.G. Butler and D. Witt (2000) state that achieving full proficiency in English includes far more than mere fluency in conversation; it means language learners should know English well enough to be academically competitive with native English-speaking peers. Academic language proficiency helps mathematics students achieve long-term success both at their studies and in future career.

That is why special attention must be paid to the study of English. Considering the importance it has for mastering the content area, the English language instruction should be based on sound theoretical principles related to how students learn a foreign language. Susana Dutro and Moran Carrol (2003) discuss the differences between the theories of the natural acquisition of English (Krashen and Terrell, 1983), which is the idea that language can be acquired in a natural way through meaningful interactions, similar to how we acquire our first language, and the direct instruction of English (McLaughlin, 1985).

Fillmore and Snow (2000) explain that certain conditions must be created for students to be successful in English learning. They state that language learners must interact directly and frequently with people who are expert speakers of English; however, if that condition is not met for any reason, then direct instruction in English is essential for language skills development. It is possible to maximize their academic opportunities by providing direct instruction for learning English that is embedded in a natural, meaningful context with many opportunities for practice.

In general, the demands towards the English language competence are really very high in the international arena. They are on the level of the advanced language competence. The surveys show that Math scientists are considered to be competent if they cope with the following communicative tasks given in English:

1. read and understand specialized literature within their sphere of knowledge;
2. are able to read mathematic symbols and characters in English;
3. read complex mathematical formulas;
4. find the necessary information in a foreign language text without help;
5. understand manuals, technical documentation, instructive materials, theoretical tips easily and adequately;
6. are competent enough to discuss mathematical problems and tasks, to prove the theorems, etc.;
7. are able to perform written communication on the adequate level of the language command;
8. are able to use the means facilitating foreign language understanding (dictionaries, glossaries, thesaurus, etc.);
9. are able to perform structured presentations in English;
10. are able to understand the main message which is delivered in English;
11. possess some sociocultural competences needed for communication with foreigners;
12. constantly improve the foreign language command.

These demands can set the real challenge for people specializing in Mathematics as many of them have difficulty in language learning.

We tend to think of Mathematics as the subject that does not require a strong command of language. In reality, however, mathematical reasoning and problem solving are closely linked to language and rely upon a firm understanding of basic math vocabulary (Dale, Cuevas, 1992). For many educators, the challenge of bringing language and math instruction together is not an easy task.

To begin with, it should be born in mind that future mathematicians have rather peculiar learning styles and their own inherent comprehension mechanisms. These differences arise from psychology. In 1983 American psychologist, Howard Gardner developed a theory of Multiple Intelligences, which can explain different learner styles. According to Gardner, there are eight different types of intelligences: Linguistic, Logical/Mathematical, Visual/Spatial, Musical, Bodily/Kinaesthetic, Interpersonal, Intrapersonal, Naturalistic.

The differences in the psychotypes of students cause the differences in the language acquisition processes thus creating different approaches towards language learning and teaching.

It has been proved that Logical/Mathematical intelligence is predominant among people who have chosen mathematical sciences as the basis for their profession. Gardner defines them as 'the questioners'. While Linguistic intelligence prevails among language lovers and polyglots who were called 'the word players' by Gardner (Arnold, Fonseca, 2004, p. 119–136).

The selection of the study methods is very essential while teaching languages to mathematicians. It is evident that abstract and rational type of thinking is often inherent for them. This fact must be taken into consideration by the foreign language teachers who work at the development of the language competences. On the subconscious level future mathematicians tend to algoritmise their actions; it is typical of them to subdivide the task into some minor elements that can be solved gradually step by step. Besides, they prefer new material to be presented logically, schematically and algorithmically.

To achieve good results, new information especially grammar structures should be explained from the logical point of view. It is highly beneficial to apply schemes and formulae. For instance, brief and accurate grammar formulae can prove to be much more effective than long explanations as people having predominant mathematical/logical intelligence tend to comprehend information in blocks.

Another aspect of instruction for English language learners is that teachers need to use strategies that give students access to the content in mathematics and help them learn the sophisticated vocabulary and language structures required in the mathematical academic settings.

Mathematical academic language is different from everyday speech and conversation. It is the language of numbers and formulas, theories and hypotheses, of academic discussions and of formal writing. Academic language proficiency in the sphere of mathematics requires students to use linguistic skills to interpret and infer meaning from oral and written language, discern precise meaning and information from text, relate ideas and information, recognize conventions of various genres, and use a variety of strategies for distinct purposes. Learning mathematical academic uses of language can become a lifelong endeavor for many language learners. Thus, it is highly beneficial to apply specific language learning technologies and strategies so that future mathematicians succeeded in foreign language acquisition. Here, the authors suggest a number of peculiar language learning techniques aimed at the formation of deep and established knowledge of English among students who specialize in mathematics.

The first thing the teacher of English should do is to get Math students acquainted with basic mathematical terms. It is important that mathematicians should be able to correlate them with appropriate notions given in their native language and certainly should be able to read them correctly in English. The aim of the English language teacher is to build definite parallels between English terms and notions and their counterparts in the mother tongue. Even the most basic mathematical characters can cause difficulties among the students at the process of the language learning. To avoid gaps at their understanding, it is highly recommended to teach students to read them correctly in English.

It is very useful to suggest doing "fill in the blank" tasks for developing students' language skills in an English context of mathematics with emphasis on reading and writing.

Task 1. *Let's "translate" the following mathematical definitions and theorems from the Math language into English*:

Math language	English
Example	
$(\forall \varepsilon > 0)(\exists \delta > 0): (\forall x \in B^0(a, \delta))(\lvert f(x) - l \rvert < \varepsilon)$	Let : A → B with $a \in A$ an interior point. The limit of f at a is l, if for every $\varepsilon > 0$, there exists a $\delta > 0$, such that $\lvert f(x) - l \rvert < \varepsilon$ for all $x \in B^0(a, \delta)$.
$f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$	…
$\int\limits_a^b f(x) = F(b) - F(a)$	…
Cauchy sequence: $\varepsilon > 0)(\exists N \epsilon \mathbb{N}): (\forall m, n > N)(\lvert a_n - a_m \rvert <$	…

Task 2. *Read some equations and write them in formulae* (Van, Phuong 2003, p. 162)

a. F equals one over two pi times the square roof of LC. . . . . .
b. E equals sigma T to the power of four . . . . . .
c. Gamma equals W subscript oh over four pi R all times F. . . . . .
d. C equals L over R squared plus omega squared L squared . . . . . .
e. V subscript two equals the squared root of open brackets, two *e* over *m* times capital V subscript two, close brackets . . . . . .
f. Gamma equals four Q over three pi R squared times, open brackets, R squared minus gamma squared, close brackets . . . . . .

It should be taken into account that mathematical symbols can have several variants of their reading, some of them being proper academic terms while others are rather colloquial. Students should understand that they can read such characters either one way or another, the choice depends on the sphere of usage. Table 1 shows some cases of different variants of the reading for the most common mathematical symbols:

Table 1. Some cases of different variants of the reading for the most common mathematical symbols

Character	How to Read in English
× or ✕	Is multiplied by / multiplies / times
=	equals / equal to / is / eq
≠	does not equal / not equal to / not eq
(	open parenthesis / left parenthesis / initial bracket / left bracket
)	closed parenthesis / right parenthesis / closing bracket / right bracket
√	the square root of x / square root x / root x
a^n	a superscript n / a to the power n / a to the n / a to the nth
a_n	a subscript n / a sub n / a n
d/dx	the derivative with respect to **x** / d over d **x** / d by d x / d to dx
∂/∂x	the partial derivative with respect to x / partial over partial **x**
…	dot, dot, dot / ellipsis / etc.
:	is to / ratio
â	a hat or a circumflex

Mathematics has a significant position among other sciences due to its strictly axiomatic structure, deductive reasoning and very precise terminology. Each term in mathematics has an accurate definition so that no ambiguities are acceptable. This can be explained by the fact that mathematics anywhere in the world describes and interprets the same abstract reality, regardless of external (socially determined) influences (Mikuláš, 2011, p. 834–848).

It can be a real challenge for educators to teach future mathematicians to read complex expressions in the foreign language. To achieve good results, language teachers can practice doing different exercises aimed at the formation of stable and deep connections between English terms and those in the mother tongue. Table 2 shows the most common mathematical expressions and the ways of their reading in English.

Table 2. The most common mathematical expressions and the ways of their reading in English

Character	How to Read in English
$\sum_{i=1}^{\infty} x$	summation from i equals one to infinity of x / the sum from i equals one to infinity x
$\int_a^b f(x)dx$	integral from a to b of f of $x\ dx$
$\prod_{i=1}^{\infty} y_i$	product from i equals one to inifinity of y sub i
$\lim_{n-\infty} t_n$	limit inferior as n approaches infinity of t sub n
$\left(\frac{f(x)}{g(x)}\right)'$	the quantity f of x over g of x, that quantity prime

The foreign language teacher should teach the students to pronounce such expressions. To reach the level of automatization, a set of specially selected exercises should be done on a regular basis during the lesson.

Task 3. *How are these values pronounced?*

a) x^3; b) x^{-n}; c) $\sqrt[n]{x-a}$; d) $\sqrt[3]{x^5}$

Task 4. Are the statements True (T) or False (F)? Correct the false sentences (Van, Phuong 2003, p. 162).

a. A point is an idea about any dot on a surface.
b. A point does not have exact dimension and location.
c. We can easily measure the length, the thickness and the width of a line.
d. A line is limited by two endpoints.
e. A line segment is also a subset of a line.
f. Although a ray has an endpoint, we cannot define its length.

If teachers want to build strong bonds between mathematical notions and language skills, they must teach students to analyze academic language in terms of its functions and forms, and reflect it in their planning process.

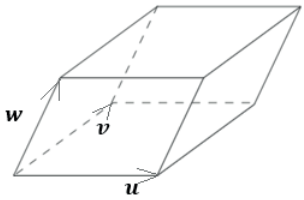
An effective approach to the English language instruction begins with the analysis of the linguistic demands of instruction and the choice of possible assignments. This means instructional planning should extend beyond analysis of curricular content and skills to include a careful analysis of the function language plays in that lesson. It is important to have a well-developed strategy for introducing and reinforcing the specific language forms, structures and vocabulary needed for each task. To develop fluency the teacher must consider how to provide opportunities for practicing the newly acquired language forms.

For instance, it would be very useful for the first year Math students to practice doing translations of the mathematical problems, facts, theorems from Ukrainian into English during their English languages lessons. The language teacher can ask them to explain the geometrical application of the triple scalar product of three vectors $\boldsymbol{u, v, w}$ in English. Here is the example of such translations task:

Task 5. Translate into Ukrainian

Given three vectors $\boldsymbol{u, v, w}$ in R^3, the parallelepiped determined by the three vectors is the "squished" box whose edges are parallel to $\boldsymbol{u, v}$, and $\boldsymbol{w}$ as depicted in Figure 1.

Figure 1. A parallelepiped



From calculus, we know that the volume of this object is $|\boldsymbol{u}(\boldsymbol{v} \times \boldsymbol{w})|$. This is the same as expansion by minors of the matrix whose columns are $\boldsymbol{u, v, w}$. Then: Volume $=|\boldsymbol{det\ u\ v\ w}|$ [8].

Moreover, it may be suggested to prove the linear convergence of three vectors, using the definition. The translation tasks can be performed either orally or in a written from, they can be accompanied with simultaneous topical discussions or disputes. On the whole, they are a good way for developing the communicative language competences. Besides, they help improve the knowledge of Math notions.

Task 6. Translate into Ukrainian

Definition. We say that the vectors $v_1, v_2, \dots v_n$, are linearly dependent if there exist constants $c_1, c_2, \dots c_n$ not all zero such that $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$. Otherwise, the vectors $v_1, v_2, \dots v_n$ are linearly independent (usually our vector spaces are defined over R, but in general we can have vector spaces defined over different base fields such as C or Z_2. The coefficients c_i should come from whatever our base field is usually R).

Task 7. Translate into Ukrainian

Example. Consider the following vectors in R^3:

$$v_1 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} -3 \\ 7 \\ 4 \end{pmatrix}, v_3 = \begin{pmatrix} 5 \\ 12 \\ 17 \end{pmatrix}, v_4 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

Are these vectors linearly independent?

Solution. No, since $3v_1 + 2v_2 - v_3 + v_4 = 0$, the vectors are linearly dependent (Cherney et al., 2013).

The appropriately selected teaching methods can have good results with students of mathematical specialties. It may seem that little resemblance can be found between linguistics and mathematics. However, a number of similarities exist between these two subjects. The building of difficult syntactical structures demands the developed logic. The study of grammar needs abstract thinking. The ability to take decisions quickly and to defend your point of view are vital for communication processes especially when leading disputes and discussions. People with significant logical/mathematical intelligence are often good at logical reasoning and scientific investigation. They may ask many questions as they organize and categorize new information about language. While working with mathematicians, English language teachers can use the inductive approach, where learners find rules themselves from examples of the language.

Asking questions is a good way to practice spoken language. The what, when, where and which questions encourage lower order thinking skills, while the why and how questions require the use of more complex language, thus developing higher order thinking skills.

Task 8. Answer the questions (Van, Phuong 2003, p. 162).

a. What is your favorite field in modern Math?
b. Why should everyone study Math?
c. What do you know about the famous Fermat's theorem?
d. How many axioms did the Italian mathematician Peano give?
e. How useful is Freund's system of 12 postulates?
f. What is an inequality in Math?
g. What does the following mean: a > b?
h. Which symbol do we use to signify an expression "is either greater than or equal to"?
i. How can you pronounce the symbol "∞"? What does it mean?

The organization of the process of multilateral communication using such methods as discussions and disputes is perfect for developing creative abilities of the students.

Mathematics discussion topics:

a. How many geometries are discovered?
b. What is the geometry of our planet (Losyeva 2006)?
c. Interesting curves in our life (Losyeva, Gubar 2011, p. 98–105).

They stimulate action, make students work towards the achievement of certain goals. Methods of case study and problem solving can also be very beneficial as they lead to the formation of important professional qualities and abilities needed for future specialists in Mathematics such as:

1. develop the ability to think logically;
2. facilitate interpersonal communication;
3. develop imagination and creativity;
4. develop reflection and introspection;
5. form communicability and ability to work in a team;
6. teach to solve problems quickly;
7. teach to apply the received knowledge in practice;
8. develop the ability to set professional and personal goals.

A target group of students that had classes in which the abovementioned tasks were applied showed much better results in general foreign language competences. And the level of understanding of the specialized mathematical notions and texts presented in English has increased more than 38% in comparison with the students which were trained in an ordinary manner.

It should be added that educators must take into account various factors related to the peculiarities of the language competence development. It is very important to create a supportive learning environment so that to engage the mathematics students into the learning process. To achieve better results, English language teachers should:

1. build background knowledge by connecting what students already know about a topic to new skills and concepts;
2. present new concepts in context and use visual aids to help students understand instruction;
3. give students opportunities to participate in lessons actively;
4. teach students specific skills they will need to know to complete learning activities and assignments, specifically those which they can apply to learning new math concepts;
5. use a variety of tools, such as manipulative objects, real-life math connections, specific terminology to establish firm connections between native language notions and the foreign language ones;
6. establish cultural connections.

Conclusion

On the basis of the personal experience of teaching English to future mathematicians and according to the theoretical studies of educators, mathematicians, linguists, and psychologists, we can come to the conclusion that to achieve good results in teaching English to the given category of students, language teachers must be ready to take into account a number of factors and implement a set of methods for language teaching that have shown good practical results. The peculiar learning style and comprehension mechanisms of students studying Math must be regarded and language learning tasks and methods must be chosen to suit the abstract and rational type of thinking predominant to the given category. It is highly recommended to present and explain concepts in the subject area clearly and accurately. Categorization, algoritmization and application of schemes and formulae while explaining new material will help to give new information logically, schematically and algorithmically and will lead to the enhancement of understanding among students. Language teachers have to know how to explain mathematical ideas and procedures in ways that will deepen learners' understanding; they should build definite parallels between English Math terms and Ukrainian ones; they should be prepared to answer learners' questions about subject notions that may be unfamiliar to them;

and they should be ready to widen their own knowledge of mathematical academic vocabulary. In addition, it should be born in mind that the language under study can be acquired in a natural way through meaningful interactions, so the role of communicative exercises, disputes and discussions on topics related to mathematics cannot be neglected.

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