ANIMATION OF THREE-DIMENSIONAL OBJECTS USING ITERATIVE METHODS
АНІМАЦІЯ ТРИВИМІРНИХ ОБ'ЄКТІВ З ВИКОРИСТАННЯМ ІНТЕРАТИВНИХ МЕТОДІВ

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Annotation: *The article deals with iterative methods for animation of three-dimensional objects. The features of the methods are determined. The obtained results can be used in the formation of dynamic images.*
Keywords: *animation, inverse kinematics, iterative methods*

Introduction

One of the tasks of computer graphics is the animation of three-dimensional scene objects, in particular, the problem of inverse kinematics.

Inverse kinematics is a widely used method of model animation. It is used to create motion in both simple and complex hierarchical models. When using inverse kinematics, it is not necessary to animate each individual node of a hierarchically connected chain to obtain its motion as a whole. To do this, you can set the necessary parameters, and the calculation of the chain motion taking into account the connectivity will be performed automatically on each frame [1].

The inverse kinematics chain is a hierarchy where the interaction between objects is carried out "from the bottom up", from the child object to the parent object. For example, take the classic model of a man - a bipod. If you move the body (parent object) in space, the arms, legs, and head (child objects) will move with it as if they were rigidly fixed. This is a chain of direct kinematics, where the impact on the parent object affects its child objects. If the reverse kinematics chain is implemented in this bipod, the movement in the space of a child object, for example, a hand, will lead to the movement of the parent objects: forearm, shoulder, trunk [2].

For the algebraic solution of the inverse kinematics problem, it is required to solve the equation for 2N independent variables [3]. Since the dimension of the matrices is an element, it is possible to obtain four linearly independent equations, which makes it possible to find four variables. In fact, I would like to have a solution for an arbitrary number of variables, because the greater the number of degrees of freedom involved, the more objects can be in the chain, the more universal the manipulator [4]. The algebraic method gives solutions for manipulators with no more than six degrees of freedom. The ability to find six variables at four linearly independent equations appears because the local matrices of objects in the chain as a

whole are strongly sparse. This allows you to get a small number of solutions, and then choose from them using a certain criterion the most acceptable and reasonable. In General, six degrees of freedom allow you to create a full-fledged three-wheeled (manipulator of three objects in the chain) manipulator that meets most of the tasks of robotics, where manipulators are usually used. The disadvantages are a small number of degrees of freedom (no more than six) and difficulties with the control of restrictions on the degree of freedom.

Iterative methods

The main thing for inverse kinematics is the speed and accuracy of the solution. The most significant and intractable problem is the problem of achieving a given accuracy [5].

The theoretical foundations of the methods do not guarantee global convergence, since iterative methods refer to methods for finding a local extremum rather than a global one. The condition of global convergence is the monotonicity of the function taken over the entire domain of definition. In the case of inverse kinematics, the form of the function is not obvious, especially with a large number of variables. This is the main theoretical problem of such methods. The second problem is accuracy. Unlike the real world, the virtual world has finite accuracy, which inevitably leads to errors in the calculation of both the values of functions and their derivative [6].

Theoretical issues of this kind are difficult to solve and are among the fundamental problems. Regarding the type of inverse kinematics function (IK function), we can say the following. From the geometry of the chain, it can be seen that the global minimum corresponds to a set of vectors from the domain of definition, and there is a problem of monotony of the function. The paper [7] shows an extended Rosenbrock function, an extended generalized Powell function, a spiral chute type function, and a wood function. In addition, if the method successfully copes with the minimization of functions of the wood, there is still no guarantee that on the other test functions are the solution. In this regard, the only way to guarantee the correct operation of the method for an arbitrary function is to test it on the largest possible set of nonlinear functions with a global minimum [8]. Only after testing the method on all the functions proposed in [9], it is possible to set a sufficient degree of confidence in the correct operation of the method for an arbitrary function.

In addition to the successful operation of the method, it is required to qualitatively implement the calculation of the function and its derivatives. Unlike test functions that have analytical form, IK function [10] does not have this type, which significantly reduces the accuracy of the calculation of the derivative by finite differences. Thus, no less significant problem than the implementation of the method, there is a problem of accurate calculation of derivatives, in terms of significant loss of accuracy in the calculation of the function. Hamming method can be used for these purposes [11]. A striking example of how important it is to be able to correctly calculate the derivative of finite differences is the same wood function. IK-function also requires selection of optimal parameters for the algorithm.

There are several methods to improve the efficiency of iterative approaches.

This is a transition from the BFGS algorithm to its modification L-BFGS [12], which can significantly improve the speed of the algorithm for a large number of variables [10], [13]. Transition of the usual linear search algorithm to a linear search algorithm with a condition on the derivative [1]. The transition from conventional algorithm selection step in terms of arbitrary functions of private, dynamic step. Transition from a single-pass algorithm to a multi-pass one, with dynamic analysis of solution development and with restrictions on the total number of iterations

It should also be noted that a linear search with a derivative condition generally improves the convergence of the algorithm on test functions, but does not affect the convergence of the IK function. Most of the time is spent in the neighborhood where the current value of the objective function is close to the desired value, but has not yet reached the desired accuracy. Theoretically, the BFGS algorithm has two main terminal codes (completion code):

* the relative gradient of the function is less than the allowed value:

$$\text{relgrad} = \max_{1 \leq i \leq n} \left| \frac{\nabla f(x)_i \max\{|(x_+)_i|, typ\, x_i\}}{\max\{|f(x_+)|, typf\}} \right| \leq gradtol$$

* the relative change in the successive values of the variable vector is less than the allowed value:

$$\text{rel}_{x_i} = \frac{|((x_+)_i - (x_c)_i)|}{\max\{|(x_+)_i|, typ\, x_i\}} \leq steptol$$

$typf$, $typ\, x_i$ are the characteristic function values *and* x_i .

Conclusion

The main difficulty, as mentioned above, is that it is impossible to fully test the cases of the algorithm until the end. In particular, when minimizing the objective function in the IK problem, a certain initial vector is set on each frame, the objective function is calculated on it and then it is minimized, thereby driving the chain of IK - objects in motion. It is clear that the display of the IK chain object is moving erratically, often not reaching the goal. This means that on some frames the algorithm does not fit into the specified accuracy, completing the work on the values of the objective function outside the permissible accuracy of the neighborhood.

The cyclic coordinate descent method is faster, which is important for real-time, as well as easier to implement. Iterative numerical methods have their advantages. This is the universality of numerical methods, both in terms of the number of degrees of freedom, and the possibility of implementing the entire range of parameters required to configure the chain. The increase in the capacity of personal computers at this stage allows us to calculate the inverse kinematics in real time using numerical methods, which is confirmed by some data from the modern literature on this topic. Analysis [1, 10, 11, 13, 14] of the current state of Affairs in the field of minimization of nonlinear functions showed that there are several competing algorithms that solve such problems. However, in recent publications on this topic, preference is given to the algorithm BFGS (Broyden-Fletcher-Goldfarb-Shanno).

References

1. Kwan W. Chin "Closed-form and generalized Inverse Kinematics solutions for animating the human articulated structure". 1996. http://www.citeulike.org/user/hurricane/article/1370370

2. Paolo Baerlocher, "Inverse Kinematics Techniques for the interactive posture control of articulated figures". Dissertation N 2383, Swiss Federal Institute of Technology, Lausanne, EPFL, Switzerland 2001.

3. Lee, G.C.S. (1982). Robot arm kinematics, dynamics and control. *Computer*, 15(12), 62-79.

4. Anon (1992).*SEA Mathematical Formulae and Statistical Tables Book.* Secondary Education Authority.

5. Craig, J.J. (1989). Introduction to Robotics: *Mechanics and Control.* Addison-Wesley.

6. Sasaki, S. (1995). Feasibility studies of kinematics problems of general serial-link robot manipulators. *Robotica*, 12, 309-322.

7. Richard H. Byrd, P. Lu, J.Nocedal and C. Zhu. "A limited memory algorithm for bound constrained optimization", SIAM Journal on Scientific Programming Computing, 16, 5 pp.1190-1208.1994.

8. Richard H. Byrd, J.Nocedal and R.B.Schnabel, "Representation of quasi-Newton matrices and their use in limited memory methods", Mathematical Programming 63, 4, 1994, pp.129-156.

9. D. C. Liu and J.Nocedal, "On the limited memory BFGS method for large scale optimization methods", Mathematical Programming 45 (1989), pp. 503-528.

10. J.Nocedal, "Updating quasi-Newton matrices with limited storage", Mathematics of Computation 35 (1980), pp. 773-782.

11. Richard H. Byrd, Jorge Nocedal and Ciyou Zhu "Towards a Discrete Newton Method with Memory for Large Scale Optimization"/Di Pillo, Gianni (ed.) et al., Nonlinear optimization and applications. Proceedings of the 21st workshop, Erice, Italy, June 13-21, 1995. New York, NY: Plenum Press. 1-12 (1996).

12. Ciyou Zhu, Richard H. Byrd, Peihuang Lu and Jorge Nocedal. L-BFGS-B: Fortran Subroutines for large-scale bound constrained optimization. SIAM Journal on Physical Review B, 53(4):1740-1743, 1996.

13. D. C. Liu and J.Nocedal, "On the limited memory BFGS method for large scale optimization methods", Mathematical Programming 45 (1989), pp. 503-528.

14. O. N. Romaniuk, A.V. Chorniy High-performance methods and means of coloring three-dimensional graphic objects. Monograph. - Vinnytsya: UNIVESUM-Vinnitsa, 2006.