Knowledge Extraction in Fuzzy Relational Systems

Based on Genetic and Neural Approach

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Abstract In this paper, a problem of MIMO object identification expressed mathematically in terms of fuzzy relational equations is considered. We use the multivariable relational structure based on the modular fuzzy relational equations with the multilevel composition law. The identification problem consists of extraction of an unknown relational matrix and also of parameters of membership functions included in the fuzzy knowledge base, which can be translated as a set of fuzzy IF-THEN rules. In fuzzy relational calculus this type of the problem relates to inverse problem and requires resolution for the composite fuzzy relational equations. The search for solution amounts to solving an optimization problem using the hybrid genetic and neural approach. The genetic algorithm uses all the available experimental information for the optimization, i.e., operates off-line. The essence of the approach is in constructing and training a special neuro-fuzzy network, which allows on-line correction of the extracted relations if the new experimental data is obtained. The resulting solution is linguistically interpreted as a set of possible rules bases. The approach proposed is illustrated by the computer experiment and the example from medical diagnosis.

Keywords Knowledge Extraction, Fuzzy Relational Identification, Composite Fuzzy Relational Equations, Solving Modular Fuzzy Relational Equations, Hybrid Genetic And Neural Algorithm

1. Introduction

Fuzzy relational calculus [1, 2] provides a powerful theoretical background for knowledge extraction from data. Some fuzzy rule base is modelled by a fuzzy relational matrix, discovering the structure of the data set [3 – 5]. Fuzzy relational equations, which connect membership functions of input and output variables, are built on the basis of a fuzzy relational matrix and Zadeh's compositional rule of inference [6, 7]. The identification problem consists of extraction of an unknown relational matrix, which can be translated as a set of fuzzy IF-THEN rules. In fuzzy relational calculus this type of problem relates to inverse problem resolution for the composite fuzzy relational equations [2].

Inverse problem resolution is of interest to both simplified relational models and multivariable ones. Solvability and approximate solvability conditions of the simplified composite fuzzy relational equations are considered in [2, 8, 9]. The non-optimizing approach [10] is widely used for fuzzy relational identification. Such adaptive recursive techniques are of interest for the most of on-line applications [11 – 13]. Under general conditions, an optimization environment is the convenient tool for fuzzy relational identification [14]. An approach for identification of fuzzy relational models by fuzzy neural networks is proposed in [15 – 17].

In the case of multiple variable linguistic model, the inputs - outputs dependency is extended to the multidimensional fuzzy relational structure and requires modularization of the system of fuzzy relational equations [7, 18]. The insufficient use of the modular structures is stipulated through the lack of effective algorithms for solving fuzzy relational equations with multilevel composition law. When dealing with the multilevel structure, it is necessary to apply the optimization schemes rather than to make attempts to develop analytical solutions [18 – 21].

An approach for fuzzy relational identification expressed mathematically in terms of composite fuzzy relational equations is proposed in [22, 23]. We use two main types of relational structures built with the aid of simplified and modular multilevel fuzzy relational equations for MIMO object identification. In [22, 23], the genetic algorithm [24, 25] as a tool to solve the simplified composite fuzzy relational equations is adapted to identify the relational matrix of rules weights. In [26], we suggest some procedures of numerical solution of the modular multilevel relational equations using genetic algorithms. The procedures envisage

the optimal solution growing from a set of primary variants by extending the simplified solution set to the case of multidimensional fuzzy relational equations. In this paper, we use the genetic algorithm [26] as a tool to solve the modular composite fuzzy relational equations to identify the relational matrix of terms weights for the given inputs-outputs data set.

Following [22, 23], the algorithm for fuzzy relation matrix identification is accomplished in two stages. At the first stage, parameters of membership functions included in the fuzzy knowledge base and elements of the relational matrix are defined using the genetic algorithm. In this case, proximity of linguistic approximation results and experimental data is the criterion of extracted relations quality. At the second stage, the obtained null solution allows us to arrange the genetic search for the complete solution set for the relational matrix. After linguistic interpretation the resulting solution can be represented as a set of possible rules collections, discovering the structure of the given data.

The genetic algorithm [24 – 26] uses all the available experimental information for the optimization, i.e., operates off-line and becomes toilful and inefficient if the new experimental data is obtained, i.e., in the on-line mode. In this paper, the process of knowledge extraction is augmented by a hybrid genetic and neural approach to solving composite fuzzy relational equations [23, 27]. The essence of the approach is in constructing and training a special neuro-fuzzy network, which allows on-line correction of the extracted relations.

The approach proposed is illustrated by the computer experiment and the example of medical diagnosis.

2. «Multiple Inputs – Multiple Outputs» Object

Let us consider an object $\mathbf{Y} = f(\mathbf{X})$ with n inputs $\mathbf{X} = (x_1,...,x_n)$ and m outputs $\mathbf{Y} = (y_1,...,y_m)$, for which the following is known:

- intervals of inputs and outputs change $x_i \in [\underline{x}_i, \overline{x}_i]$, $i = \overline{1,n}$; $y_j \in [\underline{y}_j, \overline{y}_j]$, $j = \overline{1,m}$;

- classes of decisions e_{jp} for evaluation of output variable y_j, $j = \overline{1,m}$, formed by digitizing the range $[\underline{y}_j, \overline{y}_j]$

into q_j levels: $[\underline{y}_j, \overline{y}_j] = [\underbrace{\underline{y}_j, y_{j1}}_{e_{j1}})\cup...\cup[\underbrace{y_{jp-1}, y_{jp}}_{e_{jp}})\cup...\cup[\underbrace{y_{jq_j-1}, \overline{y}_j}_{e_{jq_j}}]$;

- training data in the form of L pairs of "inputs-outputs" experimental data $\left\langle \hat{\mathbf{X}}_s, \hat{\mathbf{Y}}_s \right\rangle$, $s = \overline{1,L}$, where

$\hat{\mathbf{X}}_s = (\hat{x}_1^s,...,\hat{x}_n^s)$ and $\hat{\mathbf{Y}}_s = (\hat{y}_1^s,...,\hat{y}_m^s)$ are the vectors of the values of the input and output variables in the experiment number s.

It is necessary to transfer the available training data into the following system of IF-THEN rule [7]

$$\text{Rule } k: \text{IF } x_1 = a_{1k} \text{ AND } ... \text{ } x_n = a_{nk} \text{ THEN } y_1 = b_{1k} \text{ AND } ... \text{ } y_m = b_{mk}, \text{ } k = \overline{1,N}, \qquad (1)$$

where a_{ik} (b_{jk}) is the fuzzy term describing a variable x_i (y_j) in rule k;

N is the number of rules.

3. Fuzzy Rules, Relations and Relational Equations

This fuzzy rule base is modelled by the fuzzy relational matrix presented in Table 1. Inputs-outputs interconnection is given by the system of SISO fuzzy relational matrices

$$\mathbf{R}_{ij} \subseteq c_{il} \times e_{jp} = [r_{il,jp}, \text{ } i = \overline{1,n}, \text{ } j = \overline{1,m}, \text{ } l = \overline{1,k_i}, \text{ } p = \overline{1,q_j}],$$

where c_{il} is the fuzzy term for a variable x_i evaluation, $i = \overline{1,n}$, $l = \overline{1,k_i}$;

$$r_{il,jp} \text{ is the relation } c_{il} \times e_{jp}, \text{ } j = \overline{1,m}, \text{ } p = \overline{1,q_j}.$$

We shall redenote the set of input and output terms-assessments in the following way $\{C_1,...,C_N\}$ =$\{c_{11},...,c_{1k_1},...,c_{n1},...,c_{nk_n}\}$, $\{E_1,...,E_M\}$ =$\{e_{11},...,e_{1q_1},...,e_{m1},...,e_{mq_m}\}$, where $N = k_1 + ... + k_n$, $M = q_1 + ... + q_m$. Thus, the collection of SISO fuzzy relational matrices is equivalent to the MIMO fuzzy relational

matrix $\mathbf{R} \subseteq C_I \times E_J = [r_{IJ}, \; I = \overline{1,N}, \; J = \overline{1,M}]$.

This relational matrix can be translated as a set of MISO rules

Rule k: IF $x_1 = a_{1k}$ (with weight $v_{1k,jp}$) … AND $x_n = a_{nk}$ (with weight $v_{nk,jp}$)

$$\text{THEN } y_j = e_{jp} \text{ (with weight } w_{k,jp}\text{), } j = \overline{1,m}, \quad p = \overline{1,q_j}, \; k = \overline{1,N}, \qquad (2)$$

where $v_{ik,jp}$ is the weight of the term a_{ik} in the rule k interpreted as a relation $a_{ik} \times e_{jp}$, $a_{ik} \in \{c_{i1},...,c_{ik_i}\}$;

$w_{k,jp}$ is the weight of the rule k determined as $w_{k,jp} = \min\limits_{i=1,n} v_{ik,jp}$.

Table 1.   Fuzzy knowledge base

IF inputs			THEN outputs					
			y_1			…	y_m	
			e_{11}	…	e_{1q_1}	…	e_{m1}	…	e_{mq_m}
			E_1			…		E_M
x_1	c_{11}	C_1	$\mathbf{R}_{11}$			…	$\mathbf{R}_{1m}$	
	…							
	c_{1k_1}							
…	…	…	…			…	…	
x_n	c_{n1}		$\mathbf{R}_{n1}$			…	$\mathbf{R}_{nm}$	
	…							
	c_{nk_n}	C_N						

Given the matrix $\mathbf{R}$, the inputs-outputs dependency can be described with the help of the extended compositional rule of inference [7]

$$(\mu^{A_1}(x_1),...,\mu^{A_n}(x_n)) * \begin{bmatrix} \mathbf{R}_{11} & ... & \mathbf{R}_{1m} \\ ... & ... & ... \\ \mathbf{R}_{n1} & ... & \mathbf{R}_{nm} \end{bmatrix} = (\mu^{B_1}(y_1),...,\mu^{B_m}(y_m)), \qquad (3)$$

where $(\mu^{A_1}(x_1),...,\mu^{A_n}(x_n)) = ((\mu^{c_{11}},...,\mu^{c_{1k_1}}),...,(\mu^{c_{n1}},...,\mu^{c_{nk_n}}))$ or $\mu^C = (\mu^{C_1},...,\mu^{C_N})$ is the vector of membership degrees of variables x_i to fuzzy terms c_{il}, $i = \overline{1,n}$, $l = \overline{1,k_i}$;

$(\mu^{B_1}(y_1),...,\mu^{B_m}(y_m)) = ((\mu^{e_{11}},...,\mu^{e_{1q_1}}),...,(\mu^{e_{m1}},...,\mu^{e_{mq_m}}))$ or $\mu^E = (\mu^{E_1},...,\mu^{E_M})$ is the vector of membership degrees of variables y_j to classes e_{jp}, $j = \overline{1,m}$, $p = \overline{1,q_j}$;

$*$ is the operation of $(\circ, \cap)$ [7].

The system of fuzzy relational equations is derived from relation (3):

$$\mu^{B_1} = \mu^{A_1} \circ \mathbf{R}_{11} \cap ... \cap \mu^{A_n} \circ \mathbf{R}_{n1}$$

$$\mu^{B_m} = \mu^{A_1} \circ \mathbf{R}_{1m} \cap ... \cap \mu^{A_n} \circ \mathbf{R}_{nm}. \qquad (4)$$

Since the operation $\circ$ is associated with *max-min* and the operation $\cap$ is replaced by *min* in fuzzy set theory [7], system (4) can be rearranged as:

$$\mu^{e_{jp}} = \min_{i=1,n} [\max_{l=1,k_i} (\min(\mu^{c_{il}}, r_{il,jp}))], \quad j = \overline{1,m}, \quad p = \overline{1,q_j}. \tag{5}$$

Here $\mu^{c_{il}}(x_i)$ is a membership function of a variable x_i to the fuzzy term c_{il}, $i = \overline{1,n}$, $l = \overline{1,k_i}$; $\mu^{e_{jp}}(y_j)$ is a membership function of a variable y_j to the class e_{jp}, $j = \overline{1,m}$, $p = \overline{1,q_j}$.

We use a bell-shaped membership function model of variable u to arbitrary term T in the form [23]:

$$\mu^T(u) = 1 / (1 + ((u - \beta)/\sigma)^2), \tag{6}$$

where β is a coordinate of function maximum, $\mu^T(\beta) = 1$; σ is a parameter of concentration-extension. The operation of defuzzification is defined as follows [23]:

$$y_j = \sum_{p=1}^{q_j} y_{jp} \cdot \mu^{e_{jp}}(y_j) \left/ \sum_{p=1}^{q_j} \mu^{e_{jp}}(y_j) \right. . \tag{7}$$

Correlations (5) - (7) define the generalized fuzzy model of the object as follows:

$$\mathbf{Y} = F_R(\mathbf{X}, \mathbf{R}, \mathbf{B}_C, \mathbf{\Omega}_C), \tag{8}$$

where $\mathbf{B}_C = (\beta^{C_1}, .., \beta^{C_N})$ and $\mathbf{\Omega}_C = (\sigma^{C_1}, ..., \sigma^{C_N})$ are the vectors of β - and σ - parameters for fuzzy terms $C_1, ..., C_N$ membership functions;

F_R is the operator of inputs-outputs connection, corresponding to formulae (5) – (7).

4. Optimization Problem for Fuzzy Relations Extraction

Let us impose limitations on the knowledge base (2) volume in the following form: $k_1 \le \overline{k_1}$, ..., $k_n \le \overline{k_n}$, where $\overline{k_i}$ is the maximum permissible total number of fuzzy terms describing a variable x_i, $i = \overline{1,n}$.

So as content and number of linguistic terms c_{il} ($i = \overline{1,n}$, $l = \overline{1,k_i}$) used in fuzzy knowledge base (2) are not known beforehand then we suggest to interpret them on the basis of membership functions (6) parameter values ($\beta^{c_{il}}, \sigma^{c_{il}}$). Therefore, knowledge base (1) synthesis is reduced to obtaining the matrix of knowledge base parameters (Table 1). This problem can be formulated as follows. It is necessary to find such a knowledge base parameters matrix, which satisfies the limitations imposed on knowledge base volume and provides the least distance between model and experimental outputs of the object:

$$\sum_{s=1}^{L} [F_R(\hat{\mathbf{X}}_s, \mathbf{R}, \mathbf{B}_C, \mathbf{\Omega}_C) - \hat{\mathbf{Y}}_s]^2 = \min_{\mathbf{R}, \mathbf{B}_C, \mathbf{\Omega}_C}. \tag{9}$$

5. Solving Composite Fuzzy Relational Equations

5.1. Optimization Problem

If $\mathbf{R}_0$ is a solution of the optimization problem (9), then $\mathbf{R}_0$ is the exact solution of the composite system of fuzzy relational equations

$$(\hat{\mathbf{\mu}}^{A_1}(\hat{x}_1^s), ..., \hat{\mathbf{\mu}}^{A_n}(\hat{x}_n^s)) * \begin{bmatrix} \mathbf{R}_{11} & ... & \mathbf{R}_{1m} \\ ... & ... & ... \\ \mathbf{R}_{n1} & ... & \mathbf{R}_{nm} \end{bmatrix} = (\hat{\mathbf{\mu}}^{B_1}(\hat{\mathbf{X}}_s), ..., \hat{\mathbf{\mu}}^{B_m}(\hat{\mathbf{X}}_s)), \tag{10}$$

where the experimental input and output matrices

$$\hat{\boldsymbol{\mu}}^{A_i} = \begin{bmatrix} \hat{\mu}^{c_{i1}}(\hat{x}_i^1) & ... & \hat{\mu}^{c_{ik_i}}(\hat{x}_i^1) \\ ... & ... & ... \\ \hat{\mu}^{c_{i1}}(\hat{x}_i^L) & ... & \hat{\mu}^{c_{ik_i}}(\hat{x}_i^L) \end{bmatrix}, \quad \hat{\boldsymbol{\mu}}^{B_j} = \begin{bmatrix} \hat{\mu}^{e_{j1}}(\hat{\mathbf{X}}_1) & ... & \hat{\mu}^{e_{jq_j}}(\hat{\mathbf{X}}_1) \\ ... & ... & ... \\ \hat{\mu}^{e_{j1}}(\hat{\mathbf{X}}_L) & ... & \hat{\mu}^{e_{jq_j}}(\hat{\mathbf{X}}_L) \end{bmatrix}$$

are obtained for the given training data.

The problem of solving fuzzy relational equations (10) is formulated as follows [24 – 26]. Fuzzy relation matrix $\mathbf{R} = [r_{IJ}]$, $I = \overline{1,N}$, $J = \overline{1,M}$, should be found which satisfies the constraints $r_{IJ} \in [0,1]$ and also provides the least distance between model and experimental outputs of the object; that is, the minimum value of the criterion (9).

In the general case, system (10) has a solution set $S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$, which is completely characterized by the set of upper solutions $\overline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \{\overline{\mathbf{R}}_k, \ k = \overline{1,T}\}$ and the set of lower solutions $\underline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \{\underline{\mathbf{R}}_l, \ l = \overline{1,H}\}$. This implies that the solution set $S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ contains the simplified subsets $D_k(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$, $k = \overline{1,T}$, each of which is determined by the unique greatest solution $\overline{\mathbf{R}}_k \in \overline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ and the set of lower solutions $\underline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \{\underline{\mathbf{R}}_l, \ l = \overline{1,H}\}$. The solution set $S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ is obtained by extending the simplified solution subsets $D_k(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ to the case of multidimensional composite fuzzy relational equations [26]:

$$S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B) = \bigcup_{\overline{\mathbf{R}}_k \in \overline{S}^*} \bigcup_{\underline{\mathbf{R}}_l \in \underline{S}^*} \left[\underline{\mathbf{R}}_l, \overline{\mathbf{R}}_k \right], \ l = \overline{1,H}, \ k = \overline{1,T}. \tag{11}$$

Here $\overline{\mathbf{R}}_k = [\overline{r}_{IJ}^k]$ and $\underline{\mathbf{R}}_l = [\underline{r}_{IJ}^l]$ are the matrices of the upper and lower bounds of the fuzzy relations r_{IJ}, where the union is taken over all $\overline{\mathbf{R}}_k \in \overline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ and $\underline{\mathbf{R}}_l \in \underline{S}^*(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$.

Following [24 – 26], formation of the intervals (11) is accomplished by way of solving a multiple optimization problem (9) and it begins with the search for its null solution $\mathbf{R}_0 = [r_{IJ}^0]$, where $r_{IJ}^0 \leq \overline{r}_{IJ}^k$, $I = \overline{1,N}$, $J = \overline{1,M}$. The lower bound $(\underline{r}_{IJ}^l)$ for $l = 1$ is found in the range $[0, r_{IJ}^0]$, and for $l > 1$ – in the range $[0, \min_{p=1,k} \overline{r}_{IJ}^p]$, where the minimal solutions $\underline{\mathbf{R}}_s$, $s < l$, are excluded from the search space. The upper bound $(\overline{r}_{IJ}^k)$ for $k = 1$ is found in the range $[r_{IJ}^0, 1]$, and for $k > 1$ – in the range $[\max_{s=1,l} \underline{r}_{IJ}^s, 1]$, where the maximal solutions $\overline{\mathbf{R}}_p$, $p < k$, are excluded from the search space.

Let $\mathbf{R}(t) = [r_{IJ}(t)]$ be some t-th solution of optimization problem (9), that is $F(\mathbf{R}(t)) = F(\mathbf{R}_0)$, since for all $\mathbf{R} \in S(\hat{\boldsymbol{\mu}}^A, \hat{\boldsymbol{\mu}}^B)$ we have the same value of criterion (9). While searching for upper bounds $(\overline{r}_{IJ}^k)$ it is suggested that $r_{IJ}(t) \geq r_{IJ}(t-1)$, and while searching for lower bounds $(\underline{r}_{IJ}^l)$ it is suggested that $r_{IJ}(t) \leq r_{IJ}(t-1)$. The definition of the upper (lower) bounds follows the rule: if $\mathbf{R}(t) \neq \mathbf{R}(t-1)$, then $\overline{r}_{IJ}^k (\underline{r}_{IJ}^l) = r_{IJ}(t)$, $l = \overline{1,H}$, $k = \overline{1,T}$. If $\mathbf{R}(t) = \mathbf{R}(t-1)$, then the search for the interval solution $[\underline{\mathbf{R}}_l, \overline{\mathbf{R}}_k]$ is stopped. Formation of intervals (11) will go on till the conditions $\overline{\mathbf{R}}_k \neq \overline{\mathbf{R}}_p$ and $\underline{\mathbf{R}}_l \neq \underline{\mathbf{R}}_s$, $p < k$, $s < l$, have been satisfied.

The hybrid genetic and neural approach is proposed for solving optimization problem (9).

5.2. Genetic Algorithm for Fuzzy Relations Extraction

The chromosome needed in the genetic algorithm for solving this optimization problem includes the real codes of parameters $\mathbf{R}$, $\mathbf{B}_C$, $\boldsymbol{\Omega}_C$. While searching for the complete solution set it is suggested that the chromosome includes only

the codes of parameters r_{IJ}. Parameters of membership functions are defined simultaneously with the null solution. The crossover operation is carried out by way of exchanging genes inside each variable r_{IJ}, β^{C_I}, σ^{C_I}, $I = \overline{1, N}$, $J = \overline{1, M}$. The multi-crossover operation provides a more accurate adjusting direction for evolving offsprings that allows to systematically reduce the size of the search region. The non-uniform mutation, the action of which depends on the age of the population, provides generation of the non-dominated solutions. We used the roulette wheel selection procedure giving priority to the best solutions. The greater the fitness function of some chromosome the greater is the probability for the given chromosome to yield offsprings. The fitness function is built on the basis of criterion (9). While performing the genetic algorithm the size of the population stays constant. That is why after crossover and mutation operations it is necessary to remove the chromosomes having the worst values of the fitness function from the obtained population [28, 29].

5.3. Neuro-Fuzzy Network for Fuzzy Relations Extraction

Limitations on the knowledge base (2) volume allow embedding system (10) into the special neuro-fuzzy network, which is able to extract knowledge (See Fig. 1)
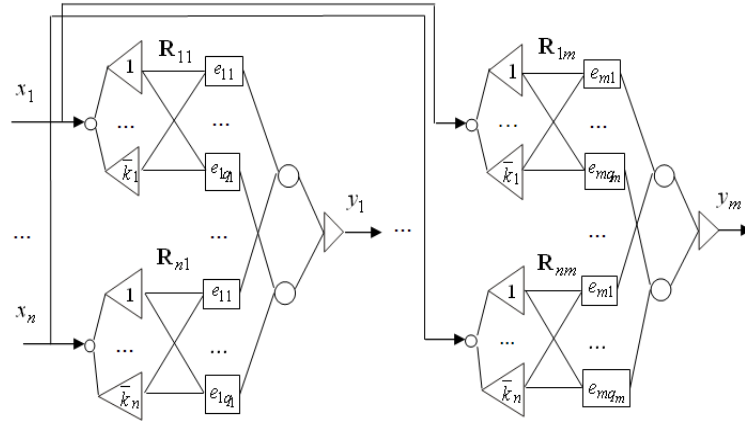
Figure 1. Neuro-fuzzy network for knowledge extraction

To train the parameters of the neuro-fuzzy network, the recurrent relations

$$r_{il,jp}(t+1) = r_{il,jp}(t) - \eta \frac{\partial \varepsilon_t}{\partial r_{il,jp}(t)};$$

$$\beta^{c_{il}}(t+1) = \beta^{c_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \beta^{c_{il}}(t)}; \quad \sigma^{c_{il}}(t+1) = \sigma^{c_{il}}(t) - \eta \frac{\partial \varepsilon_t}{\partial \sigma^{c_{il}}(t)}, \quad (12)$$

are used which minimize the criterion

$$\varepsilon_t = \frac{1}{2}(y_t - \hat{y}_t)^2,$$

where y_t and $\hat{y}_t$ are the model and the experimental outputs of the object at the t-th step of training; $r_{il,jp}(t)$ are fuzzy relations at the t-th step of training; $\beta^{c_{il}}(t)$, $\sigma^{c_{il}}(t)$ are parameters for the fuzzy terms membership functions at the t-th step of training; η is a parameter of training [30].

The partial derivatives appearing in recurrent relations (12) can be obtained according to the results from [27].

6. Computer Experiment

The aim of the experiment is to generate the system of IF-THEN rules for the target "two inputs (x_1, x_2) – two outputs (y_1, y_2)" model presented in Fig. 2:

$$y_1 = f_1(x_1, x_2) = \frac{1}{10}(2z - 0.9)\ (7z - 1)\ (17z - 19)\ (15z - 2),$$

$$y_2 = f_2(x_1, x_2) = -\frac{1}{2}y_1 + 1,$$

where $z = \dfrac{(x_1 - 3.0)^2 + (x_2 - 2.5)^2}{40}$.

The training data in the form of the interval values of input and output variables is presented in Table 2.

Table 2.   Training data ($\hat{\mathbf{X}}_s$, $\hat{\mathbf{Y}}_s$)

	Preliminary data			
	Inputs		Outputs	
s	x_1	x_2	y_1	y_2
1	[0, 2.0]	[0, 2.0]	[0.16, 1.85]	[0.07, 0.92]
2	[0, 2.0]	[2.0, 4.0]	[0.44, 1.85]	[0.07, 0.78]
3	[2.0, 4.0]	[0, 2.0]	[0.16, 1.85]	[0.07, 0.92]
4	[2.0, 4.0]	[2.0, 4.0]	[0.44, 3.40]	[-0.70, 0.78]
5	[4.0, 6.0]	[0, 2.0]	[0.16, 1.85]	[0.07, 0.92]
6	[4.0, 6.0]	[2.0, 4.0]	[0.44, 1.85]	[0.07, 0.78]
	Specified data			
	Inputs		Outputs	
s	x_1	x_2	y_1	y_2
1	[0.2, 1.2]	[0.3, 1.6]	[0, 1.0]	[0.5, 1.0]
2	[0.2, 1.2]	[1.3, 4.0]	[0, 0.8]	[0.6, 1.0]
3	[0.7, 3.0]	[0.3, 1.6]	[0, 2.3]	[-0.15, 1.0]
4	[0.7, 3.0]	[1.3, 4.0]	[0, 3.4]	[-0.7, 1.0]
5	[3.0, 5.3]	[0.3, 1.6]	[0, 2.3]	[-0.15, 1.0]
6	[3.0, 5.3]	[1.3, 4.0]	[0, 3.4]	[-0.7, 1.0]
7	[4.8, 5.8]	[0.3, 1.6]	[0, 1.0]	[0.5, 1.0]
8	[4.8, 5.8]	[1.3, 4.0]	[0, 0.8]	[0.6, 1.0]

The experiment methods consist of carrying out the following steps.

1^0. Given the training data, find the null matrix of fuzzy relations between the input and output fuzzy terms by solving the optimization problem (9).

2^0. For the composite system of fuzzy relational equations (10), generate the experimental input and output matrices $\hat{\boldsymbol{\mu}}^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}^B(\hat{\mathbf{X}}_s)$, $s = \overline{1, L}$, using the obtained parameters of the membership functions.

3^0. Given the null solution $\mathbf{R}_0$, find the complete solution set for the relational matrix of terms weights.

4^0. For the obtained relations between input and output fuzzy terns, generate the system of fuzzy rules.

5^0. Give some linguistic interpretation of the solution set for the relational matrix of rules weights.

We shall evaluate the quality of the model using the following root mean-squared errors RMSE_j:

$$\text{RMSE}_j = \sqrt{\frac{1}{L}\sum_{s=1}^{L}[y_j(\hat{\mathbf{X}}_s, \mathbf{R}_{ij}, \mathbf{B}_C, \mathbf{\Omega}_C) - \hat{y}_j^s]^2}, \;\; i = \overline{1, n}, \; j = \overline{1, m}.$$

The total number of fuzzy terms for input variables is limited to three.
The classes for output variables evaluation are formed as follows:

$$[\underline{y}_1, \overline{y}_1] = [\underbrace{0, 0.2}_{e_{11}}) \cup [\underbrace{0.2, 1.2}_{e_{12}})] \cup [\underbrace{1.2, 3.4}_{e_{13}}], \;\; [\underline{y}_2, \overline{y}_2] = [\underbrace{-0.7, 0}_{e_{21}}) \cup [\underbrace{0, 1.2}_{e_{22}}].$$
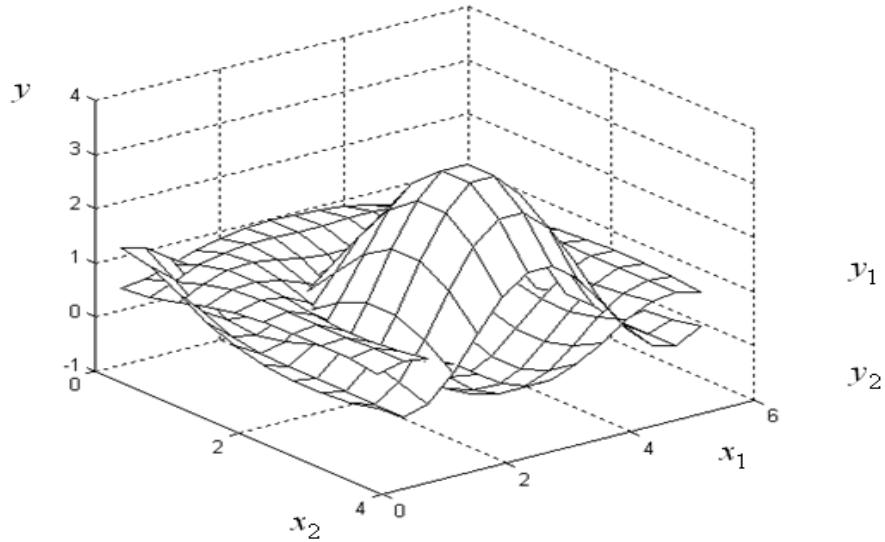
Figure 2. Inputs-outputs model-generator

The null solution $\mathbf{R}_0^g$ presented in Table 3 together with the parameters of the knowledge matrix is obtained using the genetic algorithm for the preliminary data.

Table 3. Fuzzy relational matrix (null solution) synthesized using the genetic algorithm

IF inputs				THEN outputs				
				y_1			y_2	
				e_{11}	e_{12}	e_{13}	e_{21}	e_{22}
x_1	c_{11}	(0.00, 0.70)		0.10	0.86	0.05	0.11	0.82
	c_{12}	(3.03, 0.90)		0.08	0.68	0.90	0.86	0.30
	c_{13}	(5.98, 0.72)		0.12	0.81	0.09	0.12	0.84
x_2	c_{21}	(0.01, 0.75)		0.12	0.74	0.10	0.14	0.76
	c_{22}	(3.00, 0.93)		0.12	0.39	0.92	0.88	0.48

The obtained null solution $\mathbf{R}_0^g$ allows us to arrange for the genetic search for the solution set of the system (10), where the matrices $\hat{\mathbf{\mu}}_g^A(\hat{\mathbf{X}}_s)$ and $\hat{\mathbf{\mu}}_g^B(\hat{\mathbf{X}}_s)$ for the preliminary data take the following form:

$$\hat{\mu}_g^A =$$

	$\hat{\mu}^{C_1}$	$\hat{\mu}^{C_2}$	$\hat{\mu}^{C_3}$	$\hat{\mu}^{C_4}$	$\hat{\mu}^{C_5}$
$\hat{X}_1$	[0.11, 1]	[0.08, 0.43]	0	[0.12, 1]	[0.09, 0.46]
$\hat{X}_2$	[0.11, 1]	[0.08, 0.43]	0	[0, 0.11]	[0.46, 1]
$\hat{X}_3$	[0, 0.11]	[0.43, 1]	[0, 0.12]	[0.12, 1]	[0.09, 0.46]
$\hat{X}_4$	[0, 0.11]	[0.43, 1]	[0, 0.12]	[0, 0.11]	[0.46, 1]
$\hat{X}_5$	0	[0.08, 0.46]	[0.12, 1.0]	[0.12, 1]	[0.09, 0.46]
$\hat{X}_6$	0	[0.08, 0.46]	[0.12, 1.0]	[0, 0.11]	[0.46, 1]

$$\hat{\mu}_g^B =$$

	$\hat{\mu}^{E_1}$	$\hat{\mu}^{E_2}$	$\hat{\mu}^{E_3}$	$\hat{\mu}^{E_4}$	$\hat{\mu}^{E_5}$
$\hat{X}_1$	0.10	[0.11, 0.74]	[0.08, 0.43]	[0.11, 0.43]	[0.11, 0.76]
$\hat{X}_2$	0.10	[0.11, 0.39]	[0.08, 0.43]	[0.11, 0.43]	[0.11, 0.48]
$\hat{X}_3$	[0, 0.12]	[0.12, 0.68]	[0.10, 0.46]	[0.12, 0.46]	[0.12, 0.48]
$\hat{X}_4$	[0, 0.12]	[0.11, 0.39]	[0.43, 0.90]	[0.43, 0.86]	[0.11, 0.48]
$\hat{X}_5$	0.12	[0.12, 0.74]	[0.09, 0.46]	[0.12, 0.46]	[0.12, 0.76]
$\hat{X}_6$	0.12	[0.12, 0.39]	[0.09, 0.46]	[0.12, 0.46]	[0.12, 0.48]

The complete solution set for the fuzzy relation matrix is presented in Table 4, where input x_1 is described by fuzzy terms *Low (L)*, *Average (A)*, *High (H)*; input x_2 is described by fuzzy terms *Low (L)*, *High (H)*; output y_1 is described by fuzzy terms *higher than Low (hL)*, *lower than Average (lA)*, *High (H)*; output y_2 is described by fuzzy terms *Low (L)*, *lower than Average (lA)*.

Table 4. Fuzzy relational matrix (complete solution set) synthesized using the genetic algorithm

IF inputs		THEN outputs					
		y_1			y_2		
		hL	lA	H	L	lA	
x_1	L	0.10 ∪ [0, 0.10]	0.74 ∪ [0.74, 1]	0.74 ∪ [0.74, 1]	[0, 0.43]	[0.11, 0.43]	0.76 ∪ [0.76, 1]	0.76 ∪ [0.76, 1]
	A	[0, 0.10] ∪ 0.10	0.68	[0.12, 0.68]	0.90 ∪ [0.90, 1]	0.86 ∪ [0.86, 1]	[0.12, 0.48]	0.48
	H	0.12	0.74 ∪ [0.74, 1]	0.74 ∪ [0.74, 1]	[0, 0.46]	[0.12, 0.46]	0.76 ∪ [0.76, 1]	0.76 ∪ [0.76, 1]
x_2	L	0.12	[0.74, 1] ∪ 0.74	[0.74, 1] ∪ 0.74	[0, 0.46]	[0.12, 0.46]	[0.76, 1] ∪ 0.76	[0.76, 1] ∪ 0.76
	H	0.12	[0.12, 0.39]	0.39	[0.90, 1] ∪ 0.90	[0.86, 1] ∪ 0.86	0.48	[0.12, 0.48]

The obtained solution provides the approximation of the object shown in Fig. 3.
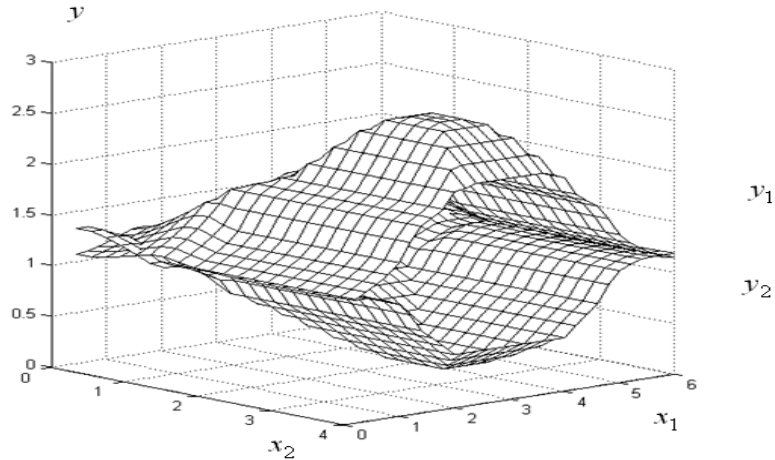
Figure 3. Inputs-outputs model synthesized using the genetic algorithm

The matrix of rules (terms combinations) weights, which corresponds to the preliminary solution, is presented in Table 5. The preliminary matrix of rules weights can be linguistically interpreted as (See Table 9):

● the set of the four possible rules bases for output y_1, which differ in the fuzzy terms describing output y_1 in rules 4 and 6 with overlapping weights;
● the set of the eight possible rules bases for output y_2, which differ in the fuzzy terms describing output y_2 in rules 2, 4, 6 with overlapping weights.

Table 5. Matrix of rules weights for the preliminary solution

IF inputs		THEN outputs				
		y_1			y_2	
x_1	x_2	hL	lA	H	L	lA
L	L	0.10 ∪ [0, 0.10]	0.74 ∪ [0.74, 1]	[0, 0.43]	[0.11, 0.43]	0.76 ∪ [0.76, 1]
A	L	0.10 ∪ [0, 0.10]	0.68 ∪ [0.12, 0.68]	[0, 0.46]	[0.12, 0.46]	[0.12, 0.48]
H	L	0.12	0.74 ∪ [0.74, 1]	[0, 0.46]	[0.12, 0.46]	0.76 ∪ [0.76, 1]
L	H	0.10 ∪ [0, 0.10]	0.39 ∪ [0.12, 0.39]	[0, 0.43]	[0.11, 0.43]	0.48 ∪ [0.12, 0.48]
A	H	0.10 ∪ [0, 0.10]	0.39 ∪ [0.12, 0.39]	0.90 ∪ [0.90, 1]	0.86 ∪ [0.86, 1]	0.48 ∪ [0.12, 0.48]
H	H	0.12	0.39 ∪ [0.12, 0.39]	[0, 0.46]	[0.12, 0.46]	0.48 ∪ [0.12, 0.48]

For the specified data, a neural adjustment of the null solution (Table 3) has yielded the new solution $\mathbf{R}_0^n$ presented in Table 6 together with the parameters of the knowledge matrix.

Table 6. Fuzzy relational matrix (null solution) specified using neural adjustment

IF inputs			THEN outputs				
			y_1			y_2	
			e_{11}	e_{12}	e_{13}	e_{21}	e_{22}
x_1	c_{11}	(0.00, 0.73)	0.86	0.83	0.10	0.08	0.84
	c_{12}	(3.00, 0.91)	0.14	0.20	0.93	0.89	0.25
	c_{13}	(6.00, 0.76)	0.85	0.88	0.12	0.10	0.80
x_2	c_{21}	(0.02, 0.75)	0.12	0.77	0.11	0.10	0.72
	c_{22}	(3.00, 0.90)	0.82	0.09	0.92	0.90	0.63

The obtained null solution $\mathbf{R}_0^n$ has allowed adjustment of the bounds in the interval solution (Table 4) and generation of the complete solution set of the system (10), where the matrices $\hat{\boldsymbol{\mu}}_n^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}_n^B(\hat{\mathbf{X}}_s)$ for the specified data take the following form:

$\hat{\boldsymbol{\mu}}^A =$

	$\hat{\mu}^{C_1}$	$\hat{\mu}^{C_2}$	$\hat{\mu}^{C_3}$	$\hat{\mu}^{C_4}$	$\hat{\mu}^{C_5}$
$\hat{\mathbf{X}}_1$	[0.27, 0.93]	[0.10, 0.20]	0	[0.18, 0.88]	[0.10, 0.29]
$\hat{\mathbf{X}}_2$	[0.27, 0.93]	[0.10, 0.20]	0	[0, 0.26]	[0.22, 1]
$\hat{\mathbf{X}}_3$	[0.06, 0.52]	[0.14, 1]	[0, 0.06]	[0.18, 0.88]	[0.10, 0.29]
$\hat{\mathbf{X}}_4$	[0.06, 0.52]	[0.14, 1]	[0, 0.06]	[0, 0.26]	[0.22, 1]
$\hat{\mathbf{X}}_5$	[0, 0.06]	[0.14, 1]	[0.06, 0.54]	[0.18, 0.88]	[0.10, 0.29]
$\hat{\mathbf{X}}_6$	[0, 0.06]	[0.14, 1]	[0.06, 0.54]	[0, 0.26]	[0.22, 1]
$\hat{\mathbf{X}}_7$	0	[0.10, 0.20]	[0.29, 0.94]	[0.18, 0.88]	[0.10, 0.29]
$\hat{\mathbf{X}}_8$	0	[0.10, 0.20]	[0.29, 0.94]	[0, 0.26]	[0.22, 1]

$\hat{\boldsymbol{\mu}}^B =$

	$\hat{\mu}^{E_1}$	$\hat{\mu}^{E_2}$	$\hat{\mu}^{E_3}$	$\hat{\mu}^{E_4}$	$\hat{\mu}^{E_5}$
$\hat{\mathbf{X}}_1$	[0.12, 0.29]	[0.18, 0.77]	[0.10, 0.20]	[0.10, 0.20]	[0.18, 0.72]
$\hat{\mathbf{X}}_2$	[0.22, 0.82]	[0.09, 0.26]	[0.10, 0.20]	[0.10, 0.20]	[0.22, 0.63]
$\hat{\mathbf{X}}_3$	[0.12, 0.29]	[0.14, 0.52]	[0.11, 0.29]	[0.10, 0.29]	[0.14, 0.52]
$\hat{\mathbf{X}}_4$	[0.14, 0.52]	[0.09, 0.26]	[0.14, 0.92]	[0.14, 0.89]	[0.14, 0.52]
$\hat{\mathbf{X}}_5$	[0.12, 0.29]	[0.14, 0.54]	[0.11, 0.29]	[0.10, 0.29]	[0.14, 0.54]
$\hat{\mathbf{X}}_6$	[0.14, 0.54]	[0.09, 0.26]	[0.14, 0.92]	[0.14, 0.89]	[0.14, 0.54]
$\hat{\mathbf{X}}_7$	[0.12, 0.29]	[0.18, 0.77]	[0.11, 0.20]	[0.10, 0.20]	[0.18, 0.72]
$\hat{\mathbf{X}}_8$	[0.22, 0.82]	[0.09, 0.26]	[0.12, 0.20]	[0.10, 0.20]	[0.22, 0.63]

The complete solution set for the fuzzy relation matrix is presented in Table 7.

Table 7.   Fuzzy relational matrix (complete solution set) specified using neural adjustment

IF inputs		THEN outputs				
		y_1			y_2	
		hL	lA	H	L	lA
x_1	L	[0.82, 1] $\cup$ 0.82	[0.77, 1] $\cup$ 0.77	[0, 0.20]	[0, 0.20]	[0.72, 1] $\cup$ 0.72
	A	[0.12, 0.52]	[0.14, 0.52]	[0.92, 1] $\cup$ 0.92	0.89 $\cup$ [0.89, 1]	[0.14, 0.52]
	H	[0.82, 1] $\cup$ 0.82	[0.77, 1] $\cup$ 0.77	[0, 0.20]	[0, 0.20]	[0.72, 1] $\cup$ 0.72
x_2	L	[0.12, 0.29]	0.77 $\cup$ [0.77, 1]	[0, 0.29]	[0, 0.29]	0.72 $\cup$ [0.72, 1]
	H	0.82 $\cup$ [0.82, 1]	[0.09, 0.26]	0.92 $\cup$ [0.92, 1]	[0.89, 1] $\cup$ 0.89	0.63

The obtained solution provides the approximation of the object shown in Fig. 4.

For output variables y_1 and y_2, the root mean-squared errors take the values of: RMSE_1=0.82 and RMSE_2=0.47

after 10000 iterations of the genetic algorithm; $RMSE_1$ =0.60 and $RMSE_2$ =0.35 after 3000 iterations of the neuro-fuzzy network, respectively (100 min on Intel Core 2 Duo P7350 2.0 GHz).

The matrix of rules (terms combinations) weights, which corresponds to the resulting solution, is presented in Table 8. The resulting matrix of rules weights can be linguistically interpreted as the unique set of IF-THEN rules (See Table 9).

Table 8. Matrix of rules weights for the specified solution

IF inputs		THEN outputs				
		y_1			y_2	
x_1	x_2	hL	lA	H	L	lA
L	L	[0.12, 0.29]	[0.77, 1] $\bigcup$ 0.77	[0, 0.20]	[0, 0.20]	[0.72, 1] $\bigcup$ 0.72
A	L	[0.12, 0.29]	[0.14, 0.52]	[0, 0.29]	[0, 0.29]	[0.14, 0.52]
H	L	[0.12, 0.29]	0.77 $\bigcup$ [0.77, 1]	[0, 0.20]	[0, 0.20]	[0.72, 1] $\bigcup$ 0.72
L	H	[0.82, 1] $\bigcup$ 0.82	[0.09, 0.26]	[0, 0.20]	[0, 0.20]	0.63
A	H	[0.12, 0.52]	[0.09, 0.26]	[0.92, 1] $\bigcup$ 0.92	0.89 $\bigcup$ [0.89, 1]	[0.14, 0.52]
H	H	0.82 $\bigcup$ [0.82, 1]	[0.09, 0.26]	[0, 0.20]	[0, 0.20]	0.63
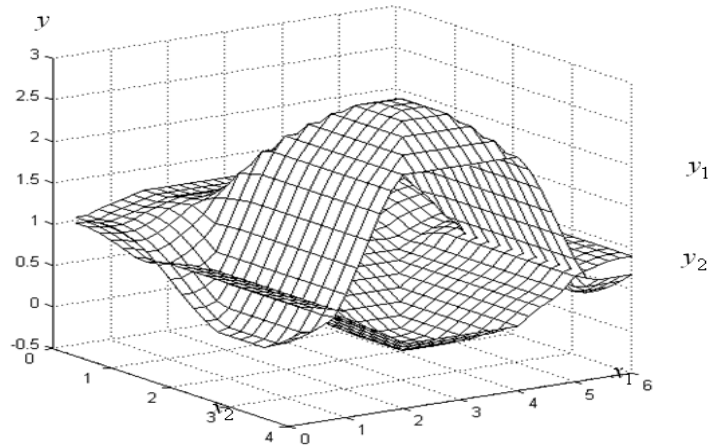
Figure 4. Inputs-outputs model specified using neural adjustment

Table 9. System of IF-THEN rules synthesized using the genetic and neural algorithm

Rule	IF inputs		THEN outputs			
	x_1	x_2	y_1		y_2	
			Genetic	Neural	Genetic	Neural
1	L	L	lA		lA	
2	A	L	lA		L or lA	lA
3	H	L	lA		lA	
4	L	H	lA or H	hL	L or lA	lA
5	A	H	H		L	
6	H	H	lA or H	hL	L or lA	lA

7. Fuzzy Relations Extraction for Heart Diseases Diagnosis

The aim is to generate the system of IF-THEN rules for diagnosis of heart diseases.

Input parameters are (variation ranges are indicated in parentheses): x_1 – aortic valve size (0.75 – 2.5 cm^2); x_2 – mitral valve size (1 – 2 cm^2); x_3 – tricuspid valve size (0.5 – 2.7 cm^2); x_4 – lung artery pressure (65 – 100 mm Hg).

Output parameters are: y_1 – left ventricle size (11–14 mm); y_2 – left auricle size (40–70 mm); y_3 – right ventricle size (36–41 mm); y_4 – right auricle size (38–45 mm).

The training data obtained in the Vinnitsa Clinic of Cardiology is represented in Table 10.

In current clinical practice, the valve sizes $x_1 \div x_3$ are described by fuzzy terms *stenosis* (*S*) and *insufficiency* (*I*); pressure x_4 is described by fuzzy terms *normal* (*N*) and *lung hypertension* (*H*).

The classes for output variables y_j evaluation are formed as follows:

$$[\underline{y}_1, \overline{y}_1] = [\underbrace{11,12}_{e_{11}}) \cup [\underbrace{13,14}_{e_{12}}], \quad [\underline{y}_2, \overline{y}_2] = [\underbrace{41,50}_{e_{21}}) \cup [\underbrace{50,70}_{e_{22}}],$$

$$[\underline{y}_3, \overline{y}_3] = [\underbrace{36,38}_{e_{31}}) \cup [\underbrace{38,41}_{e_{32}}], \quad [\underline{y}_4, \overline{y}_4] = [\underbrace{38,40}_{e_{41}}) \cup [\underbrace{40,45}_{e_{42}}].$$

In clinical practice these classes correspond to the types of diagnoses e_{j1} *low inflation* and e_{j2} *dilation* (*hypertrophy*) of heart sections $y_1 \div y_4$. The aim of the diagnosis is to translate a set of specific parameters $x_1 \div x_4$ into decision e_{jp} for each output $y_1 \div y_4$.

Table 10.   Training data

s	Input parameters				Output parameters			
	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
Preliminary data								
1	0.75-2.5	2	2	65-69	12-13	40-44	36	38
2	2.0-2.5	1-2	2	71-80	11	40-60	37-39	39-44
3	2.0-2.5	1-2	0.5-2.7	72-90	11-12	50-70	40-41	40-43
4	2.0-2.5	1-2	2-2.7	80-100	11	45-55	38-41	38-40
Specified data								
1	0.75-2.0	2	2	65-69	12-14	41-44	36	38
2	2.0-2.5	2	2	65-69	11-13	40-41	36	38
3	2.0-2.5	1-2	2	71-80	11	40	38-40	40-45
4	2.0-2.5	2	2	71-80	11	50-70	37-38	38-40
5	2.0-2.5	2	0.5-2	72-90	11-12	60-70	40-41	40-45
6	2.0-2.5	1-2	2-2.7	80-90	11-12	40	40-41	38
7	2.0-2.5	2	2	80-100	11	50-60	36	38
8	2.0-2.5	1-2	2-2.7	80-100	11	40	40-41	38-40

The null solution $\mathbf{R}_0^g$ presented in Table 11 together with the parameters of the knowledge matrix is obtained using the genetic algorithm for the preliminary data. The obtained null solution $\mathbf{R}_0^g$ allows us to arrange for the genetic search for the solution set of the system (10), where the matrices $\hat{\boldsymbol{\mu}}_g^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}_g^B(\hat{\mathbf{X}}_s)$ for the preliminary data take the following form:

$$\hat{\mu}_g^A =$$

	$\hat{\mu}^{c_{11}}$	$\hat{\mu}^{c_{12}}$	$\hat{\mu}^{c_{21}}$	$\hat{\mu}^{c_{22}}$	$\hat{\mu}^{c_{31}}$	$\hat{\mu}^{c_{32}}$	$\hat{\mu}^{c_{41}}$	$\hat{\mu}^{c_{42}}$
$\hat{X}_1$	[0.29, 1]	[0.24, 0.99]	0.47	1	0.29	0.79	[0.97, 1]	[0.28, 0.33]
$\hat{X}_2$	[0.29, .45]	[0.83, 0.99]	[0.47, 0.99]	[0.38, 1]	0.29	0.79	[0.58, 0.91]	[0.36, 0.56]
$\hat{X}_3$	[0.29, 0.45]	[0.83, 0.99]	[0.47, 0.99]	[0.38, 1]	[0.16, 0.99]	[0.22, 0.99]	[0.32, 0.88]	[0.38, 0.87]
$\hat{X}_4$	[0.29, 0.45]	[0.83, 0.99]	[0.47, 0.99]	[0.38, 1]	[0.16, 0.29]	[0.79, 0.99]	[0.19, 0.58]	[0.56, 0.99]

$$\hat{\mu}_g^B =$$

	$\hat{\mu}^{e_{11}}$	$\hat{\mu}^{e_{12}}$	$\hat{\mu}^{e_{21}}$	$\hat{\mu}^{e_{22}}$	$\hat{\mu}^{e_{31}}$	$\hat{\mu}^{e_{32}}$	$\hat{\mu}^{e_{41}}$	$\hat{\mu}^{e_{42}}$
$\hat{X}_1$	[0.24, 0.56]	[0.24, 0.52]	[0.24, 0.72]	0.29	[0.24, 0.75]	[0.24, 0.52]	[0.24, 0.79]	[0.24, 0.33]
$\hat{X}_2$	[0.36, 0.56]	[0.30, 0.52]	[0.38, 0.72]	0.29	[0.32, 0.75]	[0.38, 0.56]	[0.38, 0.79]	[0.36, 0.56]
$\hat{X}_3$	[0.22, 0.87]	[0.22, 0.52]	[0.22, 0.72]	[0.16, 0.80]	[0.22, 0.75]	[0.22, 0.87]	[0.22, 0.88]	[0.22, 0.75]
$\hat{X}_4$	[0.47, 0.92]	[0.19, 0.52]	[0.19, 0.58]	[0.16, 0.29]	[0.19, 0.58]	[0.47, 0.90]	[0.38, 0.88]	[0.47, 0.75]

Table 11. Fuzzy relational matrix (null solution) synthesized using the genetic algorithm

	IF inputs	THEN outputs							
		y_1		y_2		y_3		y_4	
		e_{11}	e_{12}	e_{21}	e_{22}	e_{31}	e_{32}	e_{41}	e_{42}
x_1	(0.76, 1.12)	0.15	0.14	0.21	0.35	0.10	0.24	0.08	0.17
	(2.43, 0.94)	0.96	0.36	0.39	0.87	0.76	0.91	0.95	0.79
x_2	(1.06, 0.88)	0.93	0.58	0.84	0.90	0.22	0.98	0.17	0.82
	(1.97, 0.76)	0.48	0.19	0.32	0.26	0.89	0.30	0.93	0.12
x_3	(0.53, 0.95)	0.12	0.07	0.16	0.98	0.37	0.22	0.24	0.05
	(2.58, 1.12)	0.98	0.76	0.91	0.15	0.88	0.64	0.96	0.77
x_4	(65.90, 16.54)	0.34	0.58	0.83	0.72	0.39	0.48	0.61	0.24
	(97.85, 20.27)	0.67	0.19	0.40	0.29	0.56	0.92	0.73	0.69

The complete solution set for the fuzzy relation matrix is presented in Table 12. The obtained solution provides the preliminary results of diagnosis presented in Table 16 for 57 patients.

The matrix of rules (terms combinations) weights, which corresponds to the preliminary solution, is presented in Table 13. The preliminary matrix of rules weights can be linguistically interpreted as (See Table 18):

- the set of the sixteen possible rules bases for output y_1, which differ in the fuzzy terms describing output y_1 in rules 13 – 16 with overlapping weights;
- the set of the two possible rules bases for output y_2, which differ in the fuzzy terms describing output y_2 in rule 9 with overlapping weights;
- the set of the four possible rules bases for output y_3, which differ in the fuzzy terms describing output y_3 in rules 14 and 16 with overlapping weights;
- the set of the sixteen possible rules bases for output y_4, which differ in the fuzzy terms describing output y_4 in rules 10, 12, 14, 16 with overlapping weights.

While interpreting the preliminary results in Table 13, the outputs $y_1 \div y_4$ are described by the additional fuzzy term *Normal* (N) in the rules 1 – 8 with low weights for both types of diagnoses e_{j1} and e_{j2}.

Table 12. Fuzzy relational matrix (complete solution set) synthesized using the genetic algorithm

IF inputs		THEN outputs							
		y_1		y_2		y_3		y_4	
		L	D	L	D	L	D	L	D
x_1	S	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0.29, 1]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
	I	[0.56, 0.92] $\cup$ [0.92, 1]	[0.52, 1] $\cup$ [0.30, 0.52]	[0.38, 0.72] $\cup$ [0.72, 1]	[0.80, 1] $\cup$ [0.29, 0.80]	[0.75, 1] $\cup$ [0.32, 0.75]	[0.90, 1] $\cup$ [0.56, 0.90]	[0.88, 1] $\cup$ [0.30, 0.88]	[0.75, 1] $\cup$ [0.56, 0.75]
x_2	S	[0.92, 1] $\cup$ [0.56, 0.92]	{[0, 0.52] $\cup$ [0.30, 0.52]} $\cup$ [0.52, 1]	{[0, 0.72] $\cup$ [0.38, 0.72]} $\cup$ [0.72, 1]	{[0.29, 0.80] $\cup$ [0, 0.80]} $\cup$ [0.80, 1]	{[0.32, 0.75] $\cup$ [0, 0.75]} $\cup$ [0, 1]	[0.56, 0.90] $\cup$ [0.90, 1]	{[0, 0.88] $\cup$ [0.30, 0.88]} $\cup$ [0, 1]	[0.56, 0.75] $\cup$ [0.75, 1]
	I	[0, 0.56]	{[0.30, 0.52] $\cup$ [0, 0.52]} $\cup$ [0, 0.52]	{[0.38, 0.72] $\cup$ [0, 0.72]} $\cup$ [0, 0.72]	{[0, 0.80] $\cup$ [0.29, 0.80]} $\cup$ [0, 1]	[0.75, 1] $\cup$ {[0, 0.75] $\cup$ [0.32, 0.75]}	[0, 0.52]	[0.88, 1] $\cup$ {[0.30, 0.88] $\cup$ [0, 0.88]}	[0, 1] $\cup$ [0, 0.75]
x_3	S	[0, 1] $\cup$ [0, 0.92]	[0, 1] $\cup$ [0, 0.52]	[0, 1] $\cup$ [0, 0.72]	[0.29, 0.80] $\cup$ [0.80, 1]	[0, 1] $\cup$ [0, 0.75]	[0, 1] $\cup$ [0, 0.90]	[0, 1] $\cup$ [0, 0.88]	[0, 1] $\cup$ [0, 0.75]
	I	[0.92, 1] $\cup$ [0.56, 0.92]	[0.52, 1] $\cup$ [0.30, 0.52]	[0.72, 1] $\cup$ [0.38, 0.72]	[0, 0.29]	[0.75, 1] $\cup$ [0.32, 0.75]	[0.90, 1] $\cup$ [0.56, 0.90]	[0.88, 1] $\cup$ [0.30, 0.88]	[0.75, 1] $\cup$ [0.56, 0.75]
x_4	N	[0, 0.56]	{[0, 0.52] $\cup$ [0.30, 0.52]} $\cup$ [0.52, 1]	[0.72, 1] $\cup$ [0.38, 0.72]	[0.29, 1] $\cup$ [0.29, 0.80]	[0.32, 0.75] $\cup$ [0.75, 1]	[0.38, 0.52]	[0.30, 0.88] $\cup$ [0.88, 1]	[0, 0.33]
	H	[0.92, 1] $\cup$ [0.56, 0.92]	{[0.30, 0.52] $\cup$ [0, 0.52]} $\cup$ [0, 0.52]	[0, 0.58]	[0.80, 1] $\cup$ [0.29, 0.80]	[0, 0.58]	[0.56, 0.90] $\cup$ [0.90, 1]	[0.56, 0.88]	[0.56, 0.75] $\cup$ [0.75, 1]

Table 13. Matrix of rules weights for the preliminary solution

Rule	IF inputs				THEN outputs							
	x_1	x_2	x_3	x_4	y_1		y_2		y_3		y_4	
					L	D	L	D	L	D	L	D
1	S	S	S	N	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0.29, 1]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
2	S	S	S	H	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0.29, 1]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
3	S	S	I	N	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.29]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
4	S	S	I	H	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.29]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
5	S	I	S	N	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0.29, 1]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
6	S	I	S	H	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0.29, 1]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
7	S	I	I	N	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.29]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
8	S	I	I	H	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.29]	[0, 0.24]	[0, 0.24]	[0, 0.24]	[0, 0.24]
9	I	S	S	N	[0, 0.56]	[0, 1]	[0, 1]	[0.29, 1]	[0, 1]	[0, 0.52]	[0, 1]	[0, 0.33]
10	I	S	S	H	[0, 1]	[0, 0.52]	[0, 0.58]	[0.80, 1]	[0, 0.58]	[0, 1]	[0, 0.88]	[0, 1]
11	I	S	I	N	[0, 0.56]	[0.52, 1]	[0.72, 1]	[0, 0.29]	[0.32, 1]	[0.38, 0.52]	[0.30, 1]	[0, 0.33]
12	I	S	I	H	[0.92, 1]	[0.30, 0.52]	[0, 0.58]	[0, 0.29]	[0, 0.58]	[0.90, 1]	[0.30, 0.88]	[0.75, 1]
13	I	I	S	N	[0, 0.56]	[0, 0.52]	[0, 0.72]	[0.29, 1]	[0, 1]	[0, 0.52]	[0, 1]	[0, 0.33]
14	I	I	S	H	[0, 0.56]	[0, 0.52]	[0, 0.58]	[0.29, 1]	[0, 0.58]	[0, 0.52]	[0, 0.88]	[0, 1]
15	I	I	I	N	[0, 0.56]	[0.30, 0.52]	[0.38, 0.72]	[0, 0.29]	[0.32, 1]	[0, 0.52]	[0.88, 1]	[0, 0.33]
16	I	I	I	H	[0, 0.56]	[0.30, 0.52]	[0, 0.58]	[0, 0.29]	[0, 0.58]	[0, 0.52]	[0.56, 0.88]	[0, 1]

  For the specified data, a neural adjustment of the null solution (Table 11) has yielded the new solution $\mathbf{R}_0^n$ presented in Table 14 together with the parameters of the knowledge matrix.

Table 14. Fuzzy relational matrix (null solution) specified using neural adjustment

IF inputs		THEN outputs							
		y_1		y_2		y_3		y_4	
		e_{11}	e_{12}	e_{21}	e_{22}	e_{31}	e_{32}	e_{41}	e_{42}
x_1	(0.75, 1.08)	0.23	0.95	0.74	0.35	0.68	0.29	0.43	0.32
	(2.46, 0.96)	0.98	0.61	0.25	0.79	0.17	0.86	0.59	0.80
x_2	(1.04, 0.83)	0.59	0.48	0.11	0.28	0.39	0.18	0.32	0.41
	(1.95, 0.78)	0.36	0.89	0.82	0.64	0.90	0.47	0.96	0.85
x_3	(0.51, 0.97)	0.17	0.32	0.44	0.80	0.26	0.12	0.61	0.58
	(2.58, 1.16)	0.89	0.94	0.63	0.47	0.85	0.91	0.87	0.14
x_4	(65.73, 19.22)	0.05	0.75	0.96	0.14	0.92	0.17	0.91	0.43
	(98.14, 20.68)	0.92	0.29	0.37	0.79	0.53	0.98	0.32	0.96

The obtained null solution $\mathbf{R}_0^n$ has allowed adjustment of the bounds in the interval solution (Table 12) and generation of the complete solution set of the system (10), where the matrices $\hat{\boldsymbol{\mu}}_n^A(\hat{\mathbf{X}}_s)$ and $\hat{\boldsymbol{\mu}}_n^B(\hat{\mathbf{X}}_s)$ for the specified data take the following form:

$\hat{\boldsymbol{\mu}}_n^A =$

	$\hat{\mu}^{c_{11}}$	$\hat{\mu}^{c_{12}}$	$\hat{\mu}^{c_{21}}$	$\hat{\mu}^{c_{22}}$	$\hat{\mu}^{c_{31}}$	$\hat{\mu}^{c_{32}}$	$\hat{\mu}^{c_{41}}$	$\hat{\mu}^{c_{42}}$
$\hat{\mathbf{X}}_1$	[0.43,1]	[0.24,0.81]	0.43	1	0.30	0.80	[0.97, 1]	[0.28, 0.33]
$\hat{\mathbf{X}}_2$	[0.28, 0.43]	[0.81,1]	0.43	1	0.30	0.80	[0.97, 1]	[0.28, 0.33]
$\hat{\mathbf{X}}_3$	[0.28, 0.43]	[0.81,1]	[0.43, 1]	[0.40, 1]	0.30	0.80	[0.64, 0.93]	[0.37, 0.56]
$\hat{\mathbf{X}}_4$	[0.28, 0.43]	[0.81,1]	0.43	1	0.30	0.80	[0.64, 0.93]	[0.37, 0.56]
$\hat{\mathbf{X}}_5$	[0.28, 0.43]	[0.81, 1]	0.43	1	[0.30, 1]	[0.24, 0.80]	[0.38, 0.90]	[0.38, 0.87]
$\hat{\mathbf{X}}_6$	[0.28, 0.43]	[0.81, 1]	[0.43, 1]	[0.40, 1]	[0.16, 0.30]	[0.80, 0.99]	[0.38, 0.64]	[0.56, 0.87]
$\hat{\mathbf{X}}_7$	[0.28, 0.43]	[0.81, 1]	0.43	1	0.30	0.80	[0.24, 0.64]	[0.56, 0.99]
$\hat{\mathbf{X}}_8$	[0.28, 0.43]	[0.81, 1]	[0.43, 1]	[0.40, 1]	[0.16, 0.30]	[0.80, 0.99]	[0.24, 0.64]	[0.56, 0.99]

$\hat{\boldsymbol{\mu}}_n^B =$

	$\hat{\mu}^{e_{11}}$	$\hat{\mu}^{e_{12}}$	$\hat{\mu}^{e_{21}}$	$\hat{\mu}^{e_{22}}$	$\hat{\mu}^{e_{31}}$	$\hat{\mu}^{e_{32}}$	$\hat{\mu}^{e_{41}}$	$\hat{\mu}^{e_{42}}$
$\hat{\mathbf{X}}_1$	[0.28, 0.33]	[0.43, 0.80]	[0.24, 0.78]	[0.24, 0.33]	[0.24, 0.80]	[0.28, 0.33]	[0.43, 0.80]	0.30
$\hat{\mathbf{X}}_2$	[0.28, 0.33]	0.61	[0.24, 0.78]	[0.28, 0.33]	[0.24, 0.80]	[0.28, 0.33]	[0.43, 0.80]	0.30
$\hat{\mathbf{X}}_3$	[0.37, 0.56]	[0.40, 0.61]	[0.24, 0.78]	[0.37, 0.56]	[0.24, 0.80]	[0.37, 0.56]	[0.40, 0.80]	0.30
$\hat{\mathbf{X}}_4$	[0.37, 0.56]	0.61	[0.24, 0.78]	[0.37, 0.56]	[0.24, 0.80]	[0.37, 0.56]	[0.43, 0.80]	0.30
$\hat{\mathbf{X}}_5$	[0.38, 0.87]	[0.30, 0.61]	[0.24, 0.78]	[0.30, 0.65]	[0.24, 0.80]	[0.24, 0.80]	[0.24, 0.80]	[0.30, 0.80]
$\hat{\mathbf{X}}_6$	[0.43, 0.87]	[0.38, 0.61]	[0.24, 0.64]	[0.30, 0.56]	[0.24, 0.64]	[0.38, 0.80]	[0.38, 0.64]	[0.16, 0.30]
$\hat{\mathbf{X}}_7$	[0.43, 0.80]	[0.24, 0.61]	[0.24, 0.64]	[0.30, 0.56]	[0.24, 0.64]	[0.38, 0.80]	[0.24, 0.64]	0.30
$\hat{\mathbf{X}}_8$	[0.43, 0.87]	[0.24, 0.61]	[0.24, 0.64]	[0.30, 0.56]	[0.24, 0.64]	[0.38, 0.80]	[0.24, 0.64]	[0.16, 0.30]

The complete solution set for the fuzzy relation matrix is presented in Table 15. The resulting solution provides the final

results of diagnosis presented in Table 16 for 57 patients.

Table 15. Fuzzy relational matrix (complete solution set) specified using neural adjustment

IF inputs		y_1 L	y_1 D	y_2 L	y_2 D	y_3 L	y_3 D	y_4 L	y_4 D
x_1	S	[0, 1]	[0.43, 0.80] ∪ [0.80, 1]	{[0.24, 0.78] ∪ [0, 0.78]} ∪ [0, 1]	[0, 0.65]	{[0.24,0.80] ∪ [0, 0.80]} ∪ [0, 0.80]	[0.28, 1]	[0.43,0.80] ∪ [0.80, 1]	[0.30, 1]
	I	[0.43,0.87] ∪ [0.87, 1]	0.61	{[0, 0.78] ∪ [0.24,0.78]} ∪ [0.78, 1]	[0.65, 1] ∪ [0.30,0.65]	{[0, 0.80] ∪ [0.24,0.80]} ∪ [0.80, 1]	[0.80, 1] ∪ [0.38,0.80]	[0.43,0.80]	[0.80, 1] ∪ [0.30,0.80]
x_2	S	[0.87, 1] ∪ [0.43,0.87]	[0.40,0.61]	[0, 0.78]	[0, 0.65]	[0, 0.64]	[0, 0.80]	[0, 0.64]	[0.30, 1]
	I	[0, 0.87]	[0.80, 1] ∪ [0.43,0.80]	[0.78, 1] ∪ [0.24, 0.78]	[0.65, 1] ∪ [0.30,0.65]	[0.80, 1] ∪ [0.24, 0.80]	[0.80, 1] ∪ [0.38,0.80]	[0.80, 1] ∪ [0.43,0.80]	[0.80, 1] ∪ [0.30,0.80]
x_3	S	[0, 1]	[0.30,0.61]	[0, 0.78]	[0.30, 0.65] ∪ [0.30, 1]	[0, 0.80]	{[0, 0.80] ∪ [0.24,0.80]} ∪ [0, 1]	[0, 0.80]	[0.30,0.80] ∪ [0.80, 1]
	I	[0.87, 1] ∪ [0.43,0.87]	[0.80, 1] ∪ [0.43,0.80]	[0.78, 1] ∪ [0.24,0.78]	[0.37, 0.56]	[0.80, 1] ∪ [0.24, 0.80]	{[0.38,0.80] ∪ [0, 0.80]} ∪ [0.80, 1]	[0.80, 1] ∪ [0.43,0.80]	[0, 0.30]
x_4	N	[0, 0.33]	[0.80, 1] ∪ [0.43,0.80]	[0.78, 1] ∪ [0.24,0.78]	[0, 0.33]	[0.80, 1] ∪ [0.24, 0.80]	[0, 0.33]	[0.80, 1] ∪ [0.43,0.80]	[0.30,0.80]
	H	[0.87, 1] ∪ [0.43,0.87]	[0, 0.64]	[0, 0.64]	[0.65, 1] ∪ [0.30,0.65]	[0, 0.64]	[0.38, 0.80] ∪ [0.80, 1]	[0, 0.64]	[0.80, 1] ∪ [0.30,0.80]

Table 16. Genetic and neural algorithm efficiency characteristics

Output parameter	Type of diagnosis	Number of cases	Probability of the correct diagnosis Genetic algorithm	Probability of the correct diagnosis Neural network
y_1	e_{11}	20	18/20=0.90	19/20=0.95
	e_{12}	37	34/37=0.92	36/37=0.97
y_2	e_{21}	26	22/26=0.84	24/26=0.92
	e_{22}	31	27/31=0.87	29/31=0.93
y_3	e_{31}	28	25/28=0.89	27/28=0.96
	e_{32}	29	26/29=0.90	28/29=0.96
y_4	e_{41}	40	34/40=0.85	38/40=0.95
	e_{42}	17	14/17=0.82	16/17=0.94

Table 17. Matrix of rules weights for the specified solution

Rule	IF inputs				THEN outputs							
	x_1	x_2	x_3	x_4	y_1		y_2		y_3		y_4	
					L	D	L	D	L	D	L	D
1	S	S	S	N	[0, 0.33]	[0.30,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.64]	[0, 0.33]	[0, 0.64]	[0.30,0.80]
2	S	S	S	H	[0, 1]	[0, 0.61]	[0, 0.64]	[0, 0.65]	[0, 0.64]	[0, 0.80]	[0, 0.64]	[0.30, 1]
3	S	S	I	N	[0, 0.33]	[0.40,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.64]	[0, 0.33]	[0, 0.64]	[0, 0.30]
4	S	S	I	H	[0, 1]	[0, 0.61]	[0, 0.64]	[0, 0.56]	[0, 0.64]	[0, 0.80]	[0, 0.64]	[0, 0.30]
5	S	I	S	N	[0, 0.33]	[0.30,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.80]	[0, 0.33]	[0, 0.80]	[0.30,0.80]
6	S	I	S	H	[0, 0.87]	[0, 0.61]	[0, 0.64]	[0, 0.65]	[0, 0.64]	[0.24, 1]	[0, 0.64]	[0.30, 1]
7	S	I	I	N	[0, 0.33]	[0.80, 1]	[0.24, 1]	[0, 0.33]	[0.24,0.80]	[0, 0.33]	[0.80, 1]	[0, 0.30]
8	S	I	I	H	[0, 0.87]	[0, 0.64]	[0, 0.64]	[0, 0.56]	[0, 0.64]	[0.28, 1]	[0, 0.64]	[0, 0.30]
9	I	S	S	N	[0, 0.33]	[0.30,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.64]	[0, 0.33]	[0, 0.64]	[0.30,0.80]
10	I	S	S	H	[0, 1]	[0, 0.61]	[0, 0.64]	[0, 0.65]	[0, 0.64]	[0, 0.80]	[0, 0.64]	[0.30, 1]
11	I	S	I	N	[0, 0.33]	[0.40,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.64]	[0, 0.33]	[0, 0.64]	[0, 0.30]
12	I	S	I	H	[0.87, 1]	[0, 0.61]	[0, 0.64]	[0, 0.56]	[0, 0.64]	[0, 0.80]	[0, 0.64]	[0, 0.30]
13	I	I	S	N	[0, 0.33]	[0.30,0.61]	[0, 0.78]	[0, 0.33]	[0, 0.80]	[0, 0.33]	[0, 0.80]	[0.30,0.80]
14	I	I	S	H	[0, 0.87]	[0, 0.61]	[0, 0.64]	[0.30, 1]	[0, 0.64]	[0.24, 1]	[0, 0.64]	[0.80, 1]
15	I	I	I	N	[0, 0.33]	[0.43,0.61]	[0.78, 1]	[0, 0.33]	[0.80, 1]	[0, 0.33]	[0.43,0.80]	[0, 0.30]
16	I	I	I	H	[0, 0.87]	[0, 0.61]	[0, 0.64]	[0.37, 0.56]	[0, 0.64]	[0.80, 1]	[0, 0.64]	[0, 0.30]

Table 18. System of IF-THEN rules synthesized using the genetic and neural algorithm

Rule	IF inputs				THEN outputs							
	x_1	x_2	x_3	x_4	y_1		y_2		y_3		y_4	
					Genetic	Neural	Genetic	Neural	Genetic	Neural	Genetic	Neural
1	S	S	S	N	N	D	D	L	N	L	N	D
2	S	S	S	H	N	L	D	L or D	N	D	N	D
3	S	S	I	N	N	D	N	L	N	L	N	L
4	S	S	I	H	N	L	N	L	N	D	N	L
5	S	I	S	N	N	D	D	L	N	L	N	L or D
6	S	I	S	H	N	L	D	L or D	N	D	N	D
7	S	I	I	N	N	D	N	L	N	L	N	L
8	S	I	I	H	N	L	N	L	N	D	N	L
9	I	S	S	N	D		L or D	L	L		L	D
10	I	S	S	H	L		D	L or D	D		L or D	D
11	I	S	I	N	D		L		L		L	
12	I	S	I	H	L		L		D		L or D	L
13	I	I	S	N	L or D	D	D	L	L		L	L or D
14	I	I	S	H	L or D	L	D		L or D	D	L or D	D
15	I	I	I	N	L or D	D	L		L		L	
16	I	I	I	H	L or D	L	L		L or D	D	L or D	L

The matrix of rules (terms combinations) weights, which corresponds to the resulting solution, is presented in Table 17. The resulting matrix of rules weights can be linguistically interpreted as (See Table 18):

- the unique set of IF-THEN rules for outputs y_1 and y_3;
- the set of the eight possible rules bases for output y_2, which differ in the fuzzy terms describing output y_2 in rules 2, 6, 10 with overlapping weights;
- the set of the four possible rules bases for output y_4, which differ in the fuzzy terms describing output y_4 in rules 5 and 13 with overlapping weights.

For output variables $y_1 \div y_4$, the root mean-squared errors take the values of: RMSE_1=0.37, RMSE_2=4.82, RMSE_3=0.59, RMSE_4=1.28 after 50000 iterations of the genetic algorithm; RMSE_1=0.21, RMSE_2=3.17, RMSE_3=0.25, RMSE_4=0.43 after 10000 iterations of the neuro-fuzzy network (250 min on Intel Core 2 Duo P7350 2.0 GHz).

For heart diseases diagnosis, we obtain the average accuracy rates of 87% and 95% after genetic and neural training, respectively.

8. Conclusion

This paper proposes a method based on fuzzy relational equations to identify MIMO systems. In experimental data analysis rules generation combined with solving fuzzy relational equations is a promising technique to restore and identify the relational matrix together with some rules base explanation. The method proposed is focused on generating accurate and interpretable fuzzy rule-based systems. The obtained results depend on the randomness of the training data initialization, e.g., on the generation of the training intervals during the execution. In some cases the model may have the highest rule performance only with the special test and training data partition, which is used to build and test the model.

For the practical applications the initial and specified training intervals are derived directly from the problem. In this paper, the issue of adaptation of the resulting solution, while the samples of experimental data (training intervals) are changing, is considered. The genetically guided global optimization is augmented by more refined gradient-based adaptation mechanisms to provide the invariability of the generated fuzzy rule-based systems. Such an adaptive approach envisages the development of a hybrid genetic and neural algorithm for solving composite fuzzy relational equations. Using our new hybrid approach it is possible to avoid random effects caused by different partitions of training and test data by detecting a representative set of rules bases.

REFERENCES

[1] Di Nola A, Sessa S, Pedrycz W, Sancez E. Fuzzy relational equations and their applications to knowledge engineering. Dordrecht: Kluwer; 1989. 430 p.

[2] Peeva K, Kyosev Y. Fuzzy relational calculus. Theory, applications and software. New York: World Scientific; 2004. 291 p.

[3] Higashi M, Klir GJ. Identification of fuzzy relation systems. IEEE Trans Syst Man Cybern 1984;14:349-355.

[4] Pedrycz W. An identification algorithm in fuzzy relational systems. Fuzzy Sets Syst 1984;13:153-167.

[5] Pedrycz W. Fuzzy models and relational equations. Math Model 1987;9(6):427-434.

[6] Zadeh L. The concept of linguistic variable and its application to approximate decision making. Moscow: Mir; 1976. 165 p. (in Russian)

[7] Yager RR, Filev DP. Essentials of fuzzy modeling and control. New York: John Wiley & Sons; 1994. 388 p.

[8] Pedrycz W, Gomide F. An introduction to fuzzy sets: analysis and design. Cambridge: The MIT Press; 1998. 465 p.

[9] Bourke MM, Fisher DG. Calculation and application of a minimum fuzzy relational matrix. Fuzzy Sets Syst 1995;74(2):225-236.

[10] Bourke MM, Fisher DG. Identification algorithms for fuzzy relational matrices. Part 1: Non-optimizing algorithms. Fuzzy Sets Syst 2000;109(3):305-320.

[11] Yi SY, Chung MJ. Identification of fuzzy relational model and its application to control. Fuzzy Sets Syst 1993;59(1):25-33.

[12] Branco PJ, Dente JA. A fuzzy relational identification algorithm and its application to predict the behaviour of a motor drive system. Fuzzy Sets Syst 2000;109(3):343–354.

[13] Wong CH, Shah SL, Fisher DG. Fuzzy relational predictive identification. Fuzzy Sets Syst 2000;113(3):417-426.

[14] Bourke MM, Fisher DG. Identification algorithms for fuzzy relational matrices. Part 2: Optimizing algorithms. Fuzzy Sets Syst 2000;109(3):321-341.

[15] Blanco A, Delgado M, Requena I. Identification of fuzzy relational equations by fuzzy neural networks. Fuzzy Sets Syst 1995;71(2):215-226.

[16] Chakraborty D, Pal NR. Integrated feature analysis and fuzzy rule-based system identification in a neuro-fuzzy paradigm. IEEE Trans Syst Man Cybern. Part B: Cybern 2001;31(3):391-400.

[17] Ciaramella A, Tagliaferri R, Pedrycz W, Di Nola A. Fuzzy relational neural network. Int J Approx Reason 2006;41(2):146-163.

[18] Pedrycz W, Vasilakos A. Modularization of fuzzy relational equations. Soft Comput 2002;6:33-37.

[19] Pedrycz W. Computational intelligence: An introduction. Boca Raton: CRC Press; 1998. 284 p.

[20] Pedrycz W. Fuzzy relational calculus. In: Ruspini E, Bonnissone P, Pedrycz W, editors. Handbook of fuzzy computation. Bristol: IOP Publishing; 1998. pp 1-14.

[21] Pedrycz W. Fuzzy relational equations: bridging theory, methodology and practice. Int J Gen Syst 2000;29:529-554.

[22] Rotshtein A, Rakytyanska H. Fuzzy genetic object identification: multiple-inputs multiple-outputs case. In: Hippe ZS, Kulikowski JL, Mroczek T, editors. Human-computer systems interaction. Heidelberg: Springer; 2012. pp 375-394.

[23] Rotshtein A, Rakytyanska H. Fuzzy evidence in identification, forecasting and diagnosis. Heidelberg: Springer; 2012. 330 p.

[24] Rotshtein A, Rakytyanska H. Fuzzy relation-based diagnosis. Autom Remote Control 2007;68(12):2198-2213.

[25] Rotshtein A, Rakytyanska H. Diagnosis problem solving using fuzzy relations. IEEE Trans Fuzzy Syst 2008;16(3):664-675.

[26] Rotshtein A, Rakytyanska H. Fuzzy logic and the least squares method in diagnosis problem solving. In: Sarma RD, editor. Genetic diagnoses. New York: Nova Science Publishers; 2011. pp 53-97.

[27] Rotshtein A, Rakytyanska H. Adaptive diagnostic system based on fuzzy relations. Cybern Syst Anal 2009;45(4):623-637.

[28] Gen M, Cheng R. Genetic algorithms and engineering design. New York: John Wiley & Sons; 1997. 352 p.

[29] Cordon O, Herrera F, Hoffmann F, Magdalena L. Genetic fuzzy systems. Evolutionary tuning and learning of fuzzy knowledge bases. New York: World Scientific; 2001. 462 p.

[30] Tsypkin YZ. Information theory of identification. Moscow: Nauka; 1984. 320 p. (in Russian).