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AVARAGE STATE'S DENSITY OF PSEUDOSPIN-ONE MATRIX EUGEN VALUES

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Abstract

Density of states of pseudospin-one disordered system has been considered. The random potential corresponds to the mosaic model when the system consists of domain with random values of energy inside each of two dimensional granule. The energy spectrum and states density relative to some realization of random potential were determined by Green's function method. Assuming normal distribution of random field the average density of states has been found Here was shown that the most sensitive to the disorder is the spectrum near the conic points.

Key words: pseudospon-one, density of states, Green function method, conic points.

Анотація

Досліджується густина станів невпорядкованої системи з псевдо спіном 1. Хаотичний потенціал відповідає мозаїчній моделі безпорядку, в якій система розглядається як сукупність двомірних областей, в кожній з яких енергія набуває хаотичних значень. Енергетичний спектр і густина станів, які відповідають деякій реалізації хаотичного потенціалу встановлені застосуванням техніки функцій Гріна. В припущенні про нормальний розподіл хаотичного поля знайдена усереднена густина станів. Показано, що найбільш чутливим до безпорядку є спектр в околі конічних точок

Ключові слова: псевдо спін 1, густина станів, метод функцій Гріна,конічні точки.

Pseudospin systems have become known beginning from early 2000-th when one layer Graphene was synthesized in its stable and reproducible form. Graphene energy spectrum in a tight-binding approximation consists [1] of two bands with linear dependence on wave vector as it is shown in the Fig.1
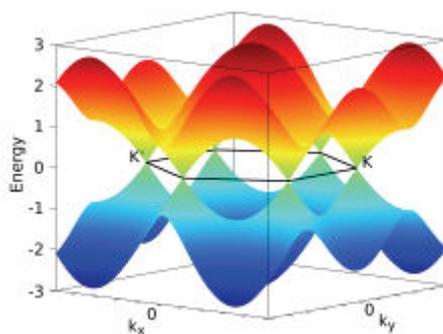
Fig.1 Graphene energy spectrum near the conic points[1]

These two-dimensional bands find out in touch in so-called conic points which represent Fermi-surface of system. Due to peculiarities of dispersion law the wave function shows the structure correspondent to spinor of the second order. That is why graphene and a number of similar materials with the Dirac-like Hamiltonian determined in terms of Pauli matrixes were named as Dirac systems with pseudospin 1/2.

Because of new physics manifested by Dirac systems searches of materials with pseudospin different of 1/2 have been undertaken. Concerning the natural material the Lieb lattice on the edge of some covalent organic frameworks[] can be considered as example of pseudospin 1. More promised candidates are linked with artificial compositions such as photonic Lieb crystal formed by periodic set of optical fibers or waveguides. Dispersion law manifests Diral-like behavior due to the some specific properties of metamaterials and features of system symmetry. Notable progress in technology based on scanning electron microscopy use allowed to elaborate two successful approaches in electronic Lieb lattices fabrication[3]. In each of them Cu crystal atomic surface is applied as the template. One case the Lieb lattice can be formed

with vacancies obtained by means of Cu(100) surface treatment by chlorine while in another case the Cu surface (111) has been covered with carbon monoxide molecular layers. Once more example[2] relates to the optical Lieb lattice that as a rule can be realized by arranging of ultra cold bosonic atoms in nodes of this lattice.

General peculiarity of the above enumerated examples refers to their energy spectrum structure. It consists of three bands. Two of them are similar to the graphene and other pseudospin 1/2 systems while the third is dispersless flat band. This energy scheme corresponds to the photonic crystal formed by periodical array of dielectric rods, borrowed from the reference[3], is shown on Fig.2. Because of specific values of geometrical parameters and permittivity of rods, threefold degeneracy take place in the Brilluine's zone centre. The degeneracy point manifest such the properties that dispersive bands have got a linear character in function of two dimensional wave vector.

Passing to the energy spectrum we have to begin from the matrix representation of wave equation determined as the following form:
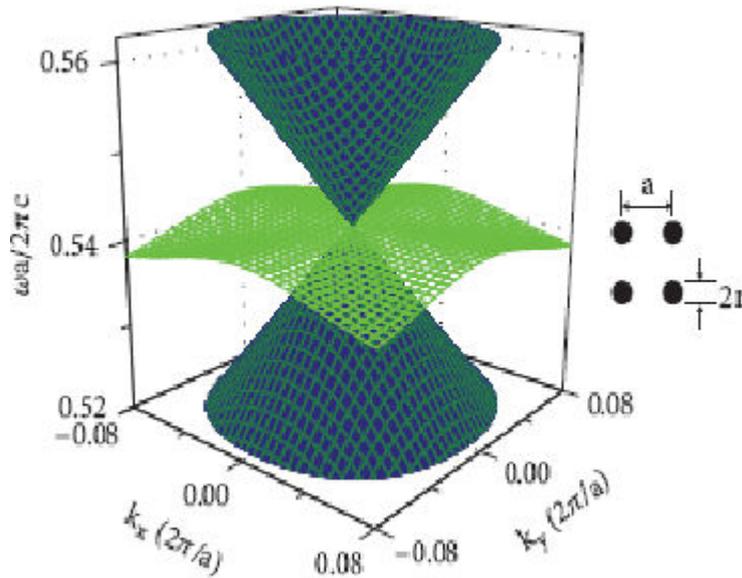
Fig.2. Typical band structure corresponds to the pseudospin-one system[3]

$$\hat{H} = v_F \hbar \begin{pmatrix} 0 & k_- + W^* & 0 \\ k_+ + W & 0 & k_- + W^* \\ 0 & k_+ + W & 0 \end{pmatrix} \quad (1)$$

where $k_\pm = k_x \pm k_y$. As for as the complex constant $W = U + iV$ it represents some random field. Some concrete value of this complex potential with the dimension of wave number determinates the energy band bottom position on energy scale.

We have intention to study one special model of pseudospin-one system that manifests some texture structure and consists of "clusters" in such a way that the complex potential has got different values within each of them. Because of technology reasons the potential W runs random realizations. If the number of particles in "cluster"of elementary cells in photonic crystal domain is about $N \square 1$ than the cluster" perimeter contains $\sqrt{N} \square N$ of structure unities. Hence, any correlation between "clusters" admits to be considered negligible small. Consequently, the matrixes introduced by equation (1) represent an ensemble of no correlated random objects. Than their eugenvalues depending on random variable W form the chaotic spectrum which can be sensible in relation to potential W distribution.

Therefore, assuming some distribution of W considered as given function of the complex variable $W = U + iV$ we will investigate the average density of states determined as eugenvalues of matrix (1). It is well-known that eugenvalues of the matrix (1) coincide with poles of correspondent resolvent Green function $G(\omega)$ introduced by expression

$$\hat{G}(\omega) = [\omega \hat{I} - \hat{H} / (v_F \hbar)]^{-1} \qquad (2)$$

where ω is eugenvalues normalized in unities of $v_F \hbar$ and $\hat{I}$ is unity matrix. Accordingly to the general theory[] the density of states for a certain realization of potential W results as a form

$$g(\omega) = \frac{1}{\pi} \operatorname{Im}[Tr\hat{G}(\omega - i\varepsilon)] \qquad (3)$$

In order to find the average density of states we have to integrate the formula (3) over the random potential distribution. Taking in account that .the trace of matrix is invariant in relation to representation we have recourses to representation based on matrix (1) eugenvector. They follow as solutions of the eugenvalues problem

$$\hat{H}_c \vec{b} = \lambda \vec{b} \qquad (4)$$

where $\hat{H}_c \equiv \hat{H} / (v_F \hbar)$ is rescaled Dirac-like Hamiltonian. Due to especial structure of this matrix one can directly find the solutions of equation (4). Some obvious and simple algebra allows to write the below given explicit results^

$$\varepsilon_1 = -\sqrt{2(k_+ + W)(k_- + W^*)} = -\sqrt{2\varepsilon_+ \varepsilon_-} = -\varepsilon \ ; \vec{b}_1^T = \frac{1}{\sqrt{2}}\left[-\varepsilon_-/\varepsilon \ ; \ 1 \ ; \ -\varepsilon_+/\varepsilon\right] \quad (5)$$

$$\varepsilon_2 = 0 \ ; \ \vec{b}_2^T = \frac{1}{\sqrt{2}}\left[1; \ 0 \ ; -\varepsilon_+/\varepsilon_-\right] \qquad (6)$$

$$\varepsilon_3 = \sqrt{2(k_+ + W)(k_- + W^*)} = \sqrt{2\varepsilon_+ \varepsilon_-} = \varepsilon \ ; \ \vec{b}_3^T = \frac{1}{\sqrt{2}}\left[\varepsilon_-/\varepsilon \ ; \ 1 \ ; \ \varepsilon_+/\varepsilon\right] \qquad (7)$$

We point out that usual treatment allows to prove that eugenvectors (5-7) represent the complete and hortonormal system of vectors. Then Green's matrix (2) assumes diagonal form if the eugenvectors set, given by expressions (5)-(7), is chosen as the basic frame. After procedure of projection has being carried out for the trace of Green function comes out the following result:

$$Tr\hat{G}(\omega - i\varepsilon) = \sum_{k_x, k_y}\left[\frac{1}{\omega - i\varepsilon} + \frac{1}{\omega - i\varepsilon + \omega_1(\vec{k})} + \frac{1}{\omega - i\varepsilon - \omega_3(\vec{k})}\right] \qquad (8)$$

By substitution (8) in (3) and passing from the sum to the integration over inverse space we can find:

$$g(\omega) = \frac{A}{(2\pi)^2}\left\{\Omega_0 \delta(\omega) + 2\int\left[\delta\left|\omega\right| - \sqrt{(k_x + U)^2 + (k_y + V)^2}\right]d\vec{k}\right\} \qquad (9)$$

where A − area of system and Ω_0 is the first Brillouin zone area.

In final, the random potential distribution depends on technology peculiarities which apriori are incognita. Therefore we accept the most simple version that the random potential is distributed accordingly to the normal distribution given as follows:

$$w(U,V) = \frac{1}{\pi \Delta^2} e^{-\frac{U^2 + V^2}{\Delta^2}} \qquad (10)$$

Putting attention the explicit form of distribution function(10) we are able to write the average state density:

$$\langle g(\omega)\rangle = \frac{A}{(2\pi)^2}\left\{\Omega_0\delta(\omega)+\right.$$

$$\left.+\frac{2}{\pi\Delta^2}\int e^{-\frac{U^2+V^2}{\Delta^2}}\left[\delta\left|\omega\right|-\sqrt{(k_x+U)^2+(k_y+V)^2}\right]d\vec{k}\,dU\,dV\right\} \qquad (11)$$

The integration conducts to the main result of this studies:

$$\langle g(\omega)\rangle = \frac{A}{(2\pi)^2}\left\{\Omega_0\delta(\omega)+\frac{8\pi}{\Delta^2}\left|\omega\right|e^{-\frac{\omega^2}{\Delta^2}}\int_0^{k_0}k\,e^{-\frac{k^2}{\Delta^2}}I_0\left(\frac{2k\omega}{\Delta^2}\right)dk\right\} \qquad (12)$$

where $I_0(x)$ − modified Bessel function and k_0 is the characteristic wave number that restricts the first Brillouin's zone linear size.

Unfortunately the last additive in formula (12) no admits its integration in concise expression. Then we consider two only limit cases. One of them corresponds to the high energy, that is $k_0\left|\omega\right|/\Delta^2 \square\ 1$ and can be determined by quite expected expression, namely:

$$\langle g(\omega)\rangle = \frac{A}{(2\pi)^2}\left\{\Omega_0\delta(\omega)+8\pi\left|\omega\right|\right\} \qquad (13)$$

This result is well clear due to the suggestion that highly energetic particle is insensitive in relation to the relief of random potential field. That is why the formula (13) reproduces the density of states related to the completely ordered system. The second case refers to the small energy, that is $k_0\left|\omega\right|/\Delta^2 \square\ 1$.

Assuming the approximation $I_0\left(\frac{2k\omega}{\Delta^2}\right)\approx 1$ we have found:

$$\langle g(\omega)\rangle = \frac{A}{(2\pi)^2}\left\{\Omega_0\delta(\omega)+4\pi\left|\omega\right|e^{-\frac{\omega^2}{\Delta^2}}\left(1-e^{-\frac{k_0^2}{\Delta^2}}\right)\right\} \qquad (14)$$

Comparing results expressed by equations (13) and (14) one can conclude that mosaic disorder considered here doesn't affect delta peak related to the flat band while the other two bands are more sensitive to the disorder especially in the vicinity of conic points where average states density is modified be exponential factor.

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