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THE ALGORITHM FOR SIMULATION OF A NONLINEAR DYNAMIC LORENCE SYSTEM

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Abstract

An algorithm is proposed for numerically solving the Lorentz mathematical model in the form of a nonlinear autonomous system of ordinary third-order differential equations. This algorithm made it possible to describe chaotic processes in the behavior of nonlinear dynamical systems based on a compact subset of the phase space of a dynamical system in the form of an attractor.

Анотація

Запропоновано алгоритм чисельного розв'язку математичної моделі Лоренца, у вигляді нелінійної автономної системи звичайних диференціальних рівнянь третього порядку. Даний алгоритм дозволив описати хаотичні процеси в поведінці нелінійних динамічних системах на основі компактної підмножини фазового простору динамічної системи у вигляді атрактора.

Introduction

A number of such dynamic systems as oscillations of a dissipative harmonic oscillator with inertial nonlinearity [1], convective processes in a toroidal chamber [2], rotation of a water wheel [3], etc., are quite accurately described by the well-known Lorenz mathematical model of [4, 5] a third-order autonomous system of ordinary differential equations (ODE). This nonlinear mathematical model is unstable (sensitive) to initial conditions and parameters, which leads to the emergence of deterministic chaos [6], which complicates the forecast of future states of a dynamic system. Due to the lack of accurate methods of solving nonlinear general-purpose differential equations, the development of algorithms for their numerical solution becomes relevant. It will allow us to analyze the behavior of dynamic systems in the form of identification of compact subsets of phase space (attractors) [5, 6].

Research results

Dynamic systems that exhibit chaotic dynamics [6, 7], whose trajectories cannot be described in terms of analytic functions, are described by the Lorentz mathematical model in the form of a third-order ODE system. [8] for $\bar{x} = \left(x_1, x_2, x_3\right)^T \in R^3$:

$$\begin{cases} \dot{x}_1 = \sigma\left(x_2 - x_1\right); \\ \dot{x}_2 = Rx_1 - x_2 - x_1x_3; \\ \dot{x}_3 = x_1x_2 - bx_3, \end{cases} \qquad (1)$$

where, for example, the model of convective processes in the toroidal chamber, the parameters: x_1 – is an analog of the velocity component of the fluid flow; x_2, x_3 – distribution of liquid temperature horizontally and vertically; R – the Rayleigh normalization number [9]; σ – the Prandtl number [10]; b – geometric parameters of the convective computation cell [11].

The solution of the ODE system (1) cannot be found analytically, but there is a mathematical proof of its sole existence by imposing some conditions on the right-hand side, such as the Picard-Lindelöff theorem, which requires that the function be Lipshitz-continuous [7]. Under this condition and the existence of a single solution - as is the case with all practical problems - it can be applied to find a numerical approximation of this solution using the following algorithm implemented in the C ++ programming language [11].

For this purpose, the type of system state is determined, the right part of the Lorentz ODE system (1) is implemented and the Runge-Kut modular algorithm is used 4th order [8]:

```
typedef std::vector<double> state_type;
typedef runge_kutta4<state_type> rk4_type;

struct lorenz {
    const double sigma, R, b;
    lorenz (const double sigma, const double R, const double b)
        : sigma (sigma) , R(R), b(b) {}

    void operator () (const state_type& x, state_type& dxdt, double t)
    {
        dxdt[0] = sigma *(x[l] -x[0]);
        dxdt[l] = R*x[0] - x[l] - x[0] *x[2];
        dxdt[2] = -b*x[2] + x[0] *x[l];
     }
}.
```

Implementation of the ODE system solution (1):

```
int main () {
    const int steps = 5000;
    const double dt = 0.01;

    rk4_type stepper;
    lorenz system (10.0, 28.0, 8.0/3.0);
    state_type x (3, 1.0);
    x[0] =10.0;     // Початкова умова
    for (size_t n =0; n < steps; ++n) {
        stepper.do_step(system, x, n*dt, dt) ;
        std: :cout << n*dt << ' ';
        std::cout << x[0] << ' ' << x[l] << ' '
                  <<  x[2]  << std: :endl;
    }
}.
```

The figure shows a diagram of changing the parameters x_1, x_2, x_3 for typical parameter values σ=10, R=28 i b=10/3. For these values, the system exhibits a chaotic attractor.

According to Birkhoff's theorem [8], the Lorenz attractor contains recurrent trajectories, and each recurrent motion is Poisson-stable [8]. This means that there will be any large values of moments of time that the point of the system trajectory is found in any vicinity of its initial position. Such a recurrent motion can be a cycle, but we cannot make a conclusion based on the found trajectory return to some vicinity of the initial conditions.

As the calculations showed (see Fig. 1), by the Lorenz system, the dynamics of the solutions' behavior at the attractor are quite complex - the recurrent trajectories contained in it can, for example, be described by almost periodic solutions or have a more complex structure.

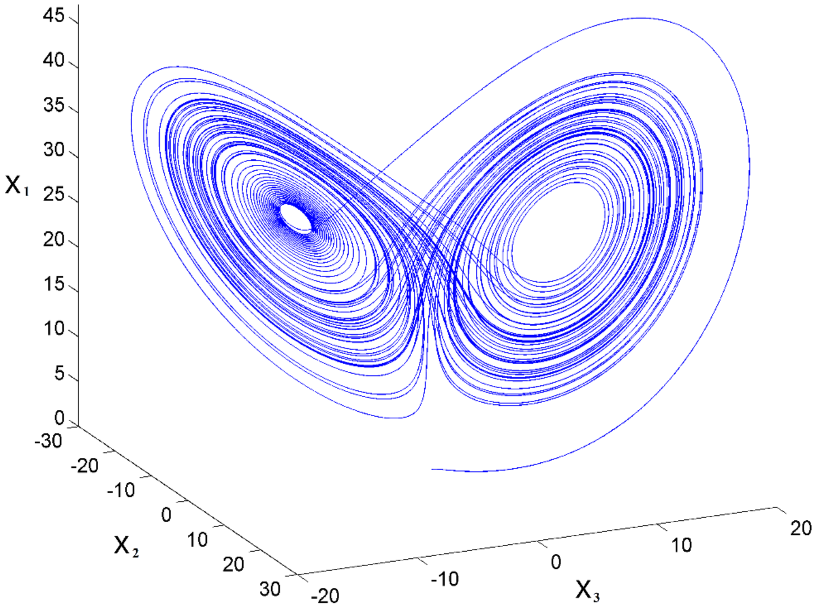

Figure 1 – Diagram of changes in the parameters of a nonlinear dynamic system

Conclusions

For any solution of the Lorentz system, there is a point in time when the corresponding phase trajectory is permanently immersed in the sphere of a fixed radius. Therefore, there is a boundary set - the Lorentz attractor, which attract all the trajectories of the dynamical system when $t \to \infty$ Thus, the attractor determines the behavior of the solutions of system (1) over large intervals of time.

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