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Influence of imperfections of polarization elements on measurement errors in three probing polarizations method

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ABSTRACT

In this paper an influence of imperfections of polarizing elements imaging Muller-polarimeter on accuracy of measurement is investigated. The operating of polarimeter is based on three probing polarization method (). The optimal scheme of polarimeter was chosen and recommendations on a selection of its parameters were produced.

1. INTRODUCTION

Polarimetric methods of investigation of properties and structures of different objects now is extensively develops. It is caused by their high sensitivity and possibility to study object without its destruction. Recently because of fast growth of computer facilities it turns to measure polarizing characteristics of objects in image mode. In this case if it means not a simple getting image of object in polarized light but analysis of each "active" pixel of image is necessary (for example, during Muller matrix measurements) then number of necessary calculations increases considerably. As a result a lot of time is required and impossible to make a repeatedly averaging.

In the given thesis we present the simple polarimeter which allows making real-time measurements of Stokes vector of polarized light and Muller matrices of objects with a comprehensible error. Also the analysis of influence of possible imperfections of working elements of polarimeter on accuracy of Mueller matrix measurement of objects is carried out.

So in scope of elastic interaction it is possible to note:

$$\mathbf{S}^{out} = \mathbf{M} \cdot \mathbf{S}^{in}$$

where $\mathbf{S}^{in}$, $\mathbf{S}^{out}$ – denote 4x1 Stokes vectors of incident and output polarized light beams correspondingly; $\mathbf{M}$ - 4x4 Mueller matrix of investigated object.

It is important to note that all elements of $\mathbf{M}$ can be independent in general. However more often it has independent elements that number is less then eleven [1,2]. The last can be used to simplify a measurement setup scheme or/also to simplification of measurement algorithm and increase of accuracy of measurements [8]. In [3,4] by us a three probing polarization method (TPPM) was developed. The method has three different realizations which allow to measure incomplete structure of Mueller matrices of object. In [5] it is shown, the elements of these structures are enough for the complete description of the wide class of objects which are widely widespread in the nature.

TPPM consists that investigated object consistently probe by light with three different polarizations the certain kind. Analyzing the changes of polarizations of light after its interaction with object, one of three possible parts of a matrix of Muller of object [4] has been derived. This method is interesting from that point of view that allows to reduce time and to raise accuracy of measurements of elements of a Muller matrix, in comparison with a similar of four probing polarizations method [6] where all elements of Muller matrix is measured. Increase of accuracy thus is reached only due to optimization of methodology of measurement and calculations.

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Moreover TPPM allows using in the polarimeter's probing channel a linear polarizer to generate different probing polarizations. It is important to note that polarizer can be made high quality with enough big apertures. Last fact is very important in image polarimetry where it is necessary to form wide bunches of light homogeneous on polarization. By development of the scheme polarimeter we take into account its universality. Meaning that with its help we shall have an opportunity to define both Stokes parameters of polarized light and elements of Muller matrix of investigated object.

The receiving channel of polarimeter is realized under the scheme (Fig.1) which is in details described and optimized in [7]. It consists of a photodetector 1, the analyzer 2 and a wave plate 3 which can rotate.

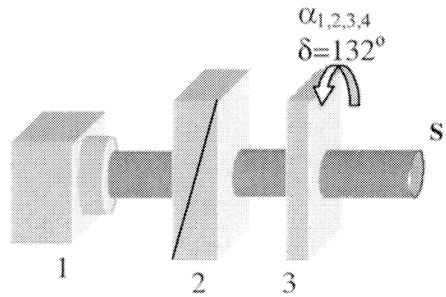

Fig.1. Receiving channel – Stoks-polarimeter.
1 - detector (CCD camera); 2 - analyzer;3 - rotating wave plate.

According to [7], the given scheme of a reception part of polarimeter allows to measure all four elements Stokes vector with the minimum error at use of a wave plate 3 with birefringence $\delta = 132^{\circ}$. Thus it should turn on the certain angular positions: $\alpha_1 = 15^{\circ}$, $\alpha_2 = -15^{\circ}$, $\alpha_3 = 51^{\circ}$, $\alpha_4 = -51^{\circ}$. When four intensities of light $\mathbf{I} = [I_1 \quad I_2 \quad I_3 \quad I_4]^T$ at the set positions of a wave plate 3 are measured by the detector 1 then elements of Stokes vector can be calculated by the expression (1):

$$\mathbf{S} = \mathbf{A}^{-1} \cdot \mathbf{I}, \tag{1}$$

where **A** – a characteristic matrix of the receiving channel:

$$\mathbf{A} = \begin{bmatrix} 1 & \cos^2(2\alpha_1) + \cos(\delta)\sin^2(2\alpha_1) & \sin^2(\delta/2)\sin(4\alpha_1) & -\sin(\delta)\sin(2\alpha_1) \\ 1 & \cos^2(2\alpha_2) + \cos(\delta)\sin^2(2\alpha_2) & \sin^2(\delta/2)\sin(4\alpha_2) & -\sin(\delta)\sin(2\alpha_2) \\ 1 & \cos^2(2\alpha_3) + \cos(\delta)\sin^2(2\alpha_3) & \sin^2(\delta/2)\sin(4\alpha_3) & -\sin(\delta)\sin(2\alpha_3) \\ 1 & \cos^2(2\alpha_4) + \cos(\delta)\sin^2(2\alpha_4) & \sin^2(\delta/2)\sin(4\alpha_4) & -\sin(\delta)\sin(2\alpha_4) \end{bmatrix}; \tag{2}$$

δ, α - birefringence and an azimuth of orientation of a wave plate 3 accordingly.

The full scheme of polarimeter offered for imaging Mueller matrix measurements is presented on Fig.2:

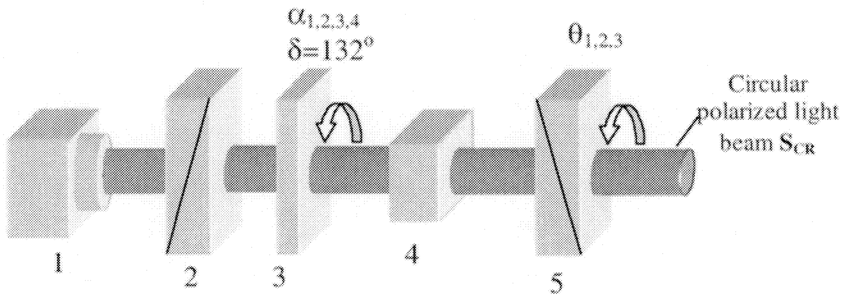

Fig.2. Image Mueller-polarimeter.
1 - detector (CCD camera); 2 - analyzer;
3 - rotating wave plate; 4 - sample; 5 - forming polarizer.

By using of scheme Fig.2 it is possible to measure a part of a matrix $\mathbf{M}^{4\times3}$ without the fourth column [4]. Process of measurement of an incomplete matrix $\mathbf{M}^{4\times3}$ consists of following steps: firstly, without studied object 4 three Stokes vectors of probing light formed in series by a polarizer 5 from incident circular polarization S_{CR} are measured. These vectors are accepted as probing $\mathbf{S}_{\mathbf{in}}^{(\mathbf{i})}$. In general $\mathbf{S}_{\mathbf{in}}^{(\mathbf{i})}$ looks like $\left[S_1^{(i)} \quad S_2^{(i)} \quad S_3^{(i)} \quad 0\right]^T$ (i =1÷3 – number of probing polarization). Zero value of the fourth element of Stokes vectors $\mathbf{S}_{\mathbf{in}}^{(\mathbf{i})}$ causes that only the part of a Muller matrix without the fourth column can be measured. The next step is the measurement the output Stokes vectors $\mathbf{S}_{\mathbf{out}}^{(\mathbf{i})}$ after interaction with studied object. Then, at third step, the system of the equations for calculation of required elements of a Mueller is obtained:

$$\begin{aligned}
S_{1,out}^{i} &= M_{11}S_{1,in}^{i} + M_{12}S_{2,in}^{i} + M_{13}S_{3,in}^{i};\\
S_{2,out}^{i} &= M_{21}S_{1,in}^{i} + M_{22}S_{2,in}^{i} + M_{23}S_{3,in}^{i};\\
S_{3,out}^{i} &= M_{31}S_{1,in}^{i} + M_{32}S_{2,in}^{i} + M_{33}S_{3,in}^{i};\\
S_{4,out}^{i} &= M_{41}S_{1,in}^{i} + M_{42}S_{2,in}^{i} + M_{43}S_{3,in}^{i};
\end{aligned}
\quad i = 1 \div 3. \tag{3}$$

In [3,4] it has been shown, that for the maximal accuracy of measurement of elements of a Muller matrix in the given scheme (Fig.2), azimuths probing light polarizations should differ on a 60°. It is clear, that imperfection of different elements of the polarimeter Fig.2 will be influence on accuracy of measurement of matrices of objects. Level of this influence, most likely, will be also different. As it was mentioned marked above, making measurements in image mode, we shall not have a possibility to do repeated averaging. Therefore, for reduction of the measurement error we should concern with the maximal attention to selection of elements of polarimeter.

2. SIMULATION

Following simulation experiment has allowed us to find out, which of elements of the given scheme polarimeter are governing for maintenance of comprehensible accuracy of measurement of elements of a Muller matrix. The essence of experiment consists in that we, in accordance to the scheme Fig.2, had been created the mathematical model of polarimeter which emulated process of measurement of a Mueller matrix of model objects. The expression for getting of all necessary parameters for simulation measurement of elements of Muller matrices is :

$$I^{\det} = \left[\mathbf{M}(\mathrm{P2},0)\mathbf{M}(\delta \pm \Delta\delta_{mxn}, \alpha \pm \Delta\alpha)\mathbf{M}^{samlpe}\mathbf{M}(\mathrm{P1}, \theta 1 \pm \Delta\theta)\mathbf{S}(I \pm \Delta I, \varepsilon, 0)\right]_1 \tag{4}$$

where $\mathbf{S}(I,\varepsilon,\varphi)$ – Stokes vector which corresponds to the polarized light with intensity I, ellipticity ε and an azimuth of orientation of an ellipse of polarization φ; $\mathbf{M}(P,\theta)$ – Muller matrix of a linear polarizer with suppression P ($0 \le P \le 1$, $P = 0$ – corresponds to an ideal linear polarizer) and an azimuth of orientation of an axis maximal transmittance θ; $\mathbf{M}(\delta,\alpha)$ – Muller matrix of a linear wave plate with birefringence δ and orientation of a fast axis α; $\mathbf{M}^{sample}$ – Muller matrix of the investigated sample; $I^{\det}$ – intensity that registered by the photodetector 1 (Fig.2); $[\ldots]_1$ – denotes the fact that the first element (intensity) of resumed Stokes vector which corresponds to polarization of incident light on detector 1 of polarimeter Fig.2.

As it can be seen from Eq.(3) the model of polarimeter considered allows to simulate the next types of imperfections:

- instability of intensity of a light source (laser) ΔI;
- imperfect circular polarization $\mathbf{S}_{CR}$ impinging onto polarizer 5 (i.e. $\varepsilon \neq 45^0$). It results in that the polarization $\mathbf{S}_{\mathbf{in}}^{(\mathbf{i})}$ will have different intensities;
- imperfection of polarizer 5;
- imperfection of wave plate 3 in the receiving channel of polarimeter;
- imperfection of analyzer 2 in the receiving channel of polarimeter.

Each of the mentioned imperfection could be considered both separately and in combinations. With use of the expression Eq.(4) we simulated the measurements of a Muller matrices of a test objects (as an example we used the object with circular phase anisotropy). The test matrix of the given type of anisotropy is the following:

$$\mathbf{M}^{sample} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\varphi) & \sin(2\varphi) & 0 \\ 0 & -\sin(2\varphi) & \cos(2\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (5)$$

where φ - value of circular phase anisotropy.

In our case, distribution of value of anisotropy was set in such a way that corresponding elements of an incomplete Muller matrix $\mathbf{M}^{4\times3}$ formed images presented at Fig.3. At Fig.3 shade of grey color of each point corresponds to the certain value of an element of a Mueller matrix which describes anisotropic properties of object in the given point of cross-section. The cross-section dimension is $m \times n = 100 \times 100$ pixels. In simulation experiment each measurement is a result of 100 averaging under given set of polarization elements imperfections of polarimeter Fig.2.

The results of simulation experiments at different combinations of imperfections of polarizing elements are shown in Figures $(4) \div (13)$. In particular, it is presented the differential images which obtained as a comparison of initial ideal images of cross-section (Fig.3) and images received in simulation measurements in each considered case. For example, Fig.4 corresponds to the case when Muller matrix is measured under following conditions: alterations of intensity of a light is $\Delta I = 2\%$; imperfections of polarizer 5 are approximately $P1 = 0.001$; orientation of the polarizer 5 and wave plate 3 are $\Delta\theta = 0^0$ and $\Delta\delta = 0^0$; the wave plate 3 and the analyzer 2 are ideal.

In the bottom of each image Fig.$(4) \div (13)$ it is presented the following information: a scale of gradation; the maximal value deviations of the measured elements of matrix $\mathbf{M}_{ij}$ from initial values $\mathbf{M}_{ij}^0$; averaged value of a deviation in all cross-sections $\overline{\Delta M_{ij}}$ which is calculated by Eq.(6); the averaged estimation of deviation of the measured Muller matrix from ideal matrix, $\overline{\delta \mathbf{M}}$, which is defined according to Eq.(7).

$$\overline{\Delta M_{ij}} = \frac{1}{m \times n} \sum_{m,n} \left(\left(M_{ij} \right)_{m,n} - \left(M_{ij}^{0} \right)_{m,n} \right); \tag{6}$$

$$\overline{\delta \mathbf{M}} = \frac{1}{m \times n} \sum_{m,n} \left(\frac{\| \mathbf{M} - \mathbf{M}^{0} \|}{\| \mathbf{M}^{0} \|} \right)_{m,n} \qquad \| \mathbf{M} \| = \sqrt{\sum_{i=1;\, j=1}^{4;3} \left(M_{ij} \right)^{2}} \tag{7}$$

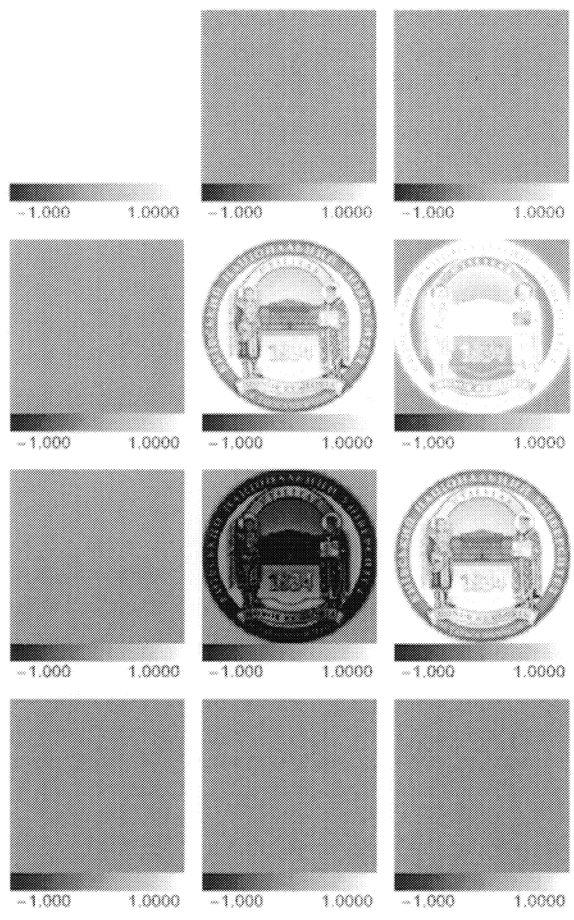

Fig.3. Initial distribution of the values of the incomplete Mueller matrix $\mathbf{M}_{4x3}$ elements.

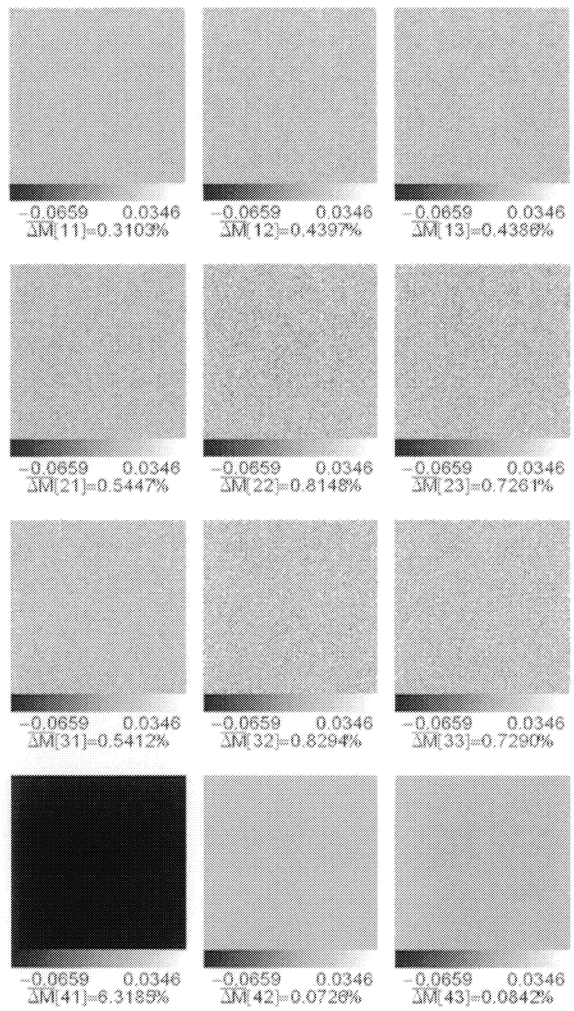

Fig.4. ΔI =2 %; P1 =0.001; P2 =0; $\Delta\theta$ =0°; ε =45°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°; $\overline{\delta \mathbf{M}} = 3.36\%$.

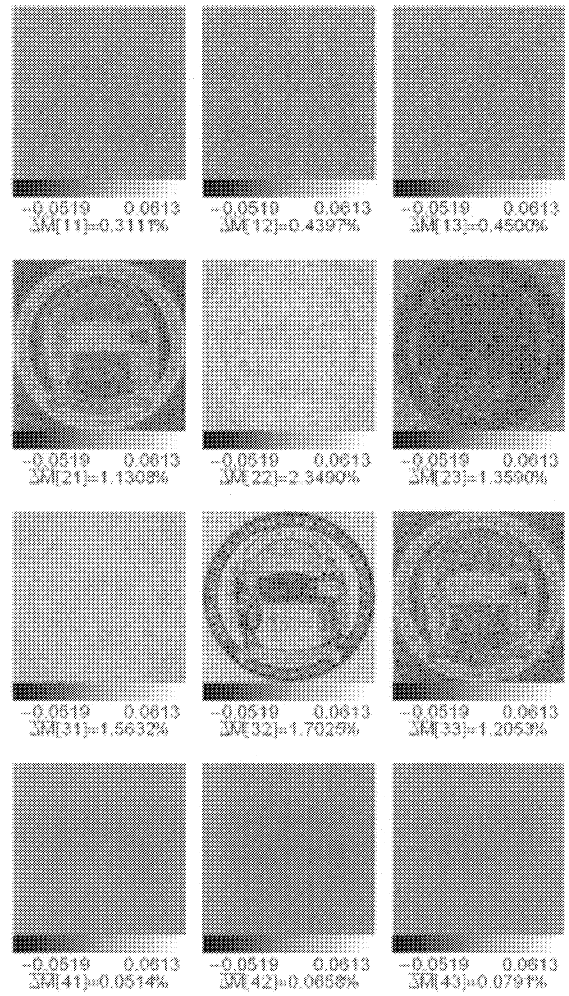

Fig.5. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =1°; ε =45°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°; $\overline{\delta \mathbf{M}} = 2.25\%$.

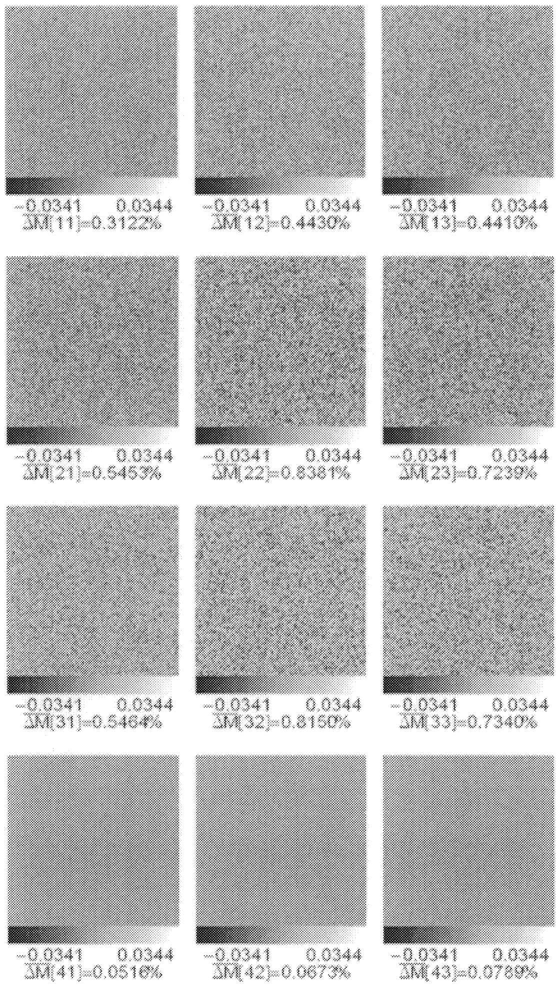

Fig.6. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =0°; ε =35°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°; $\overline{\delta \mathbf{M}} = 1.13\%$.

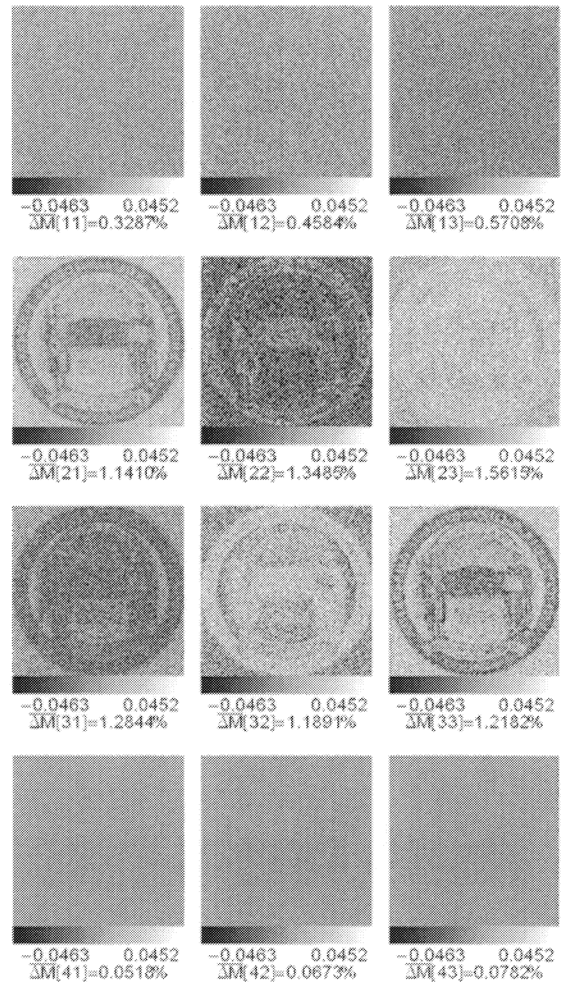

Fig.7. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =1°; ε =35°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°; $\overline{\delta \mathbf{M}} = 1.90\%$.

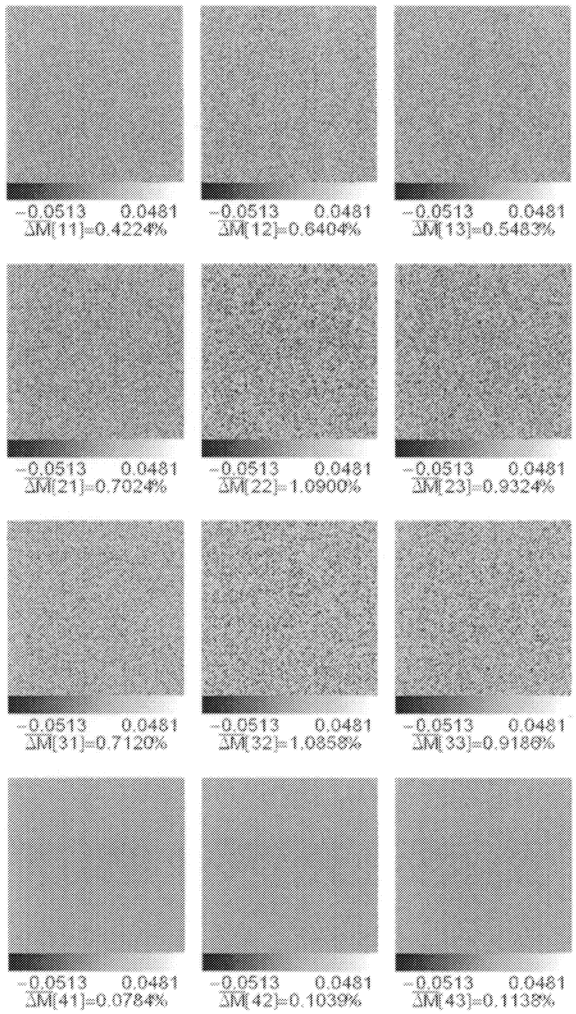

Fig.8. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =0°; ε =45°; $\Delta\delta$ =3°; $\Delta\alpha$ =0°;

$$\overline{\delta \mathbf{M}} = 1.48\%$$

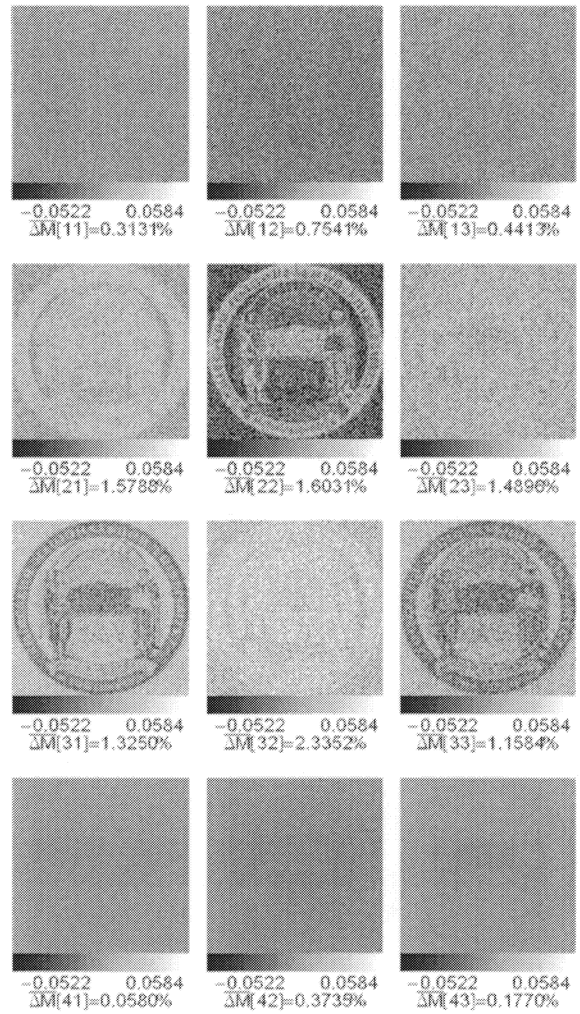

Fig.9. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =0°; ε =45°; $\Delta\delta$ =0°; $\Delta\alpha$ =1°;

$$\overline{\delta \mathbf{M}} = 2.32\%$$

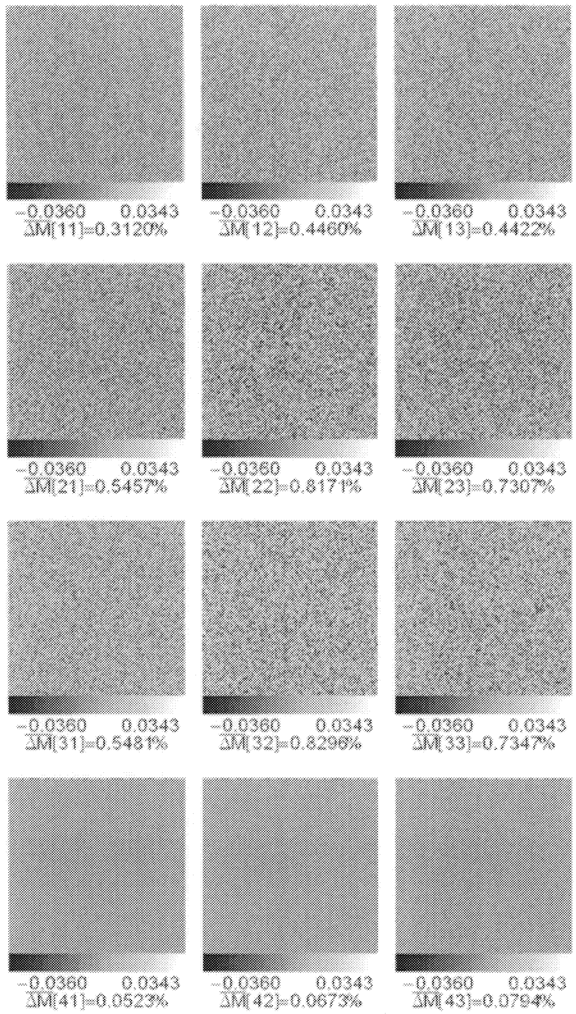

Fig.10. ΔI =2 %; P1 =0; P2 =0.001; $\Delta\theta$ =0°; ε =45°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°. $\overline{\delta\mathbf{M}} = 1.13\%$

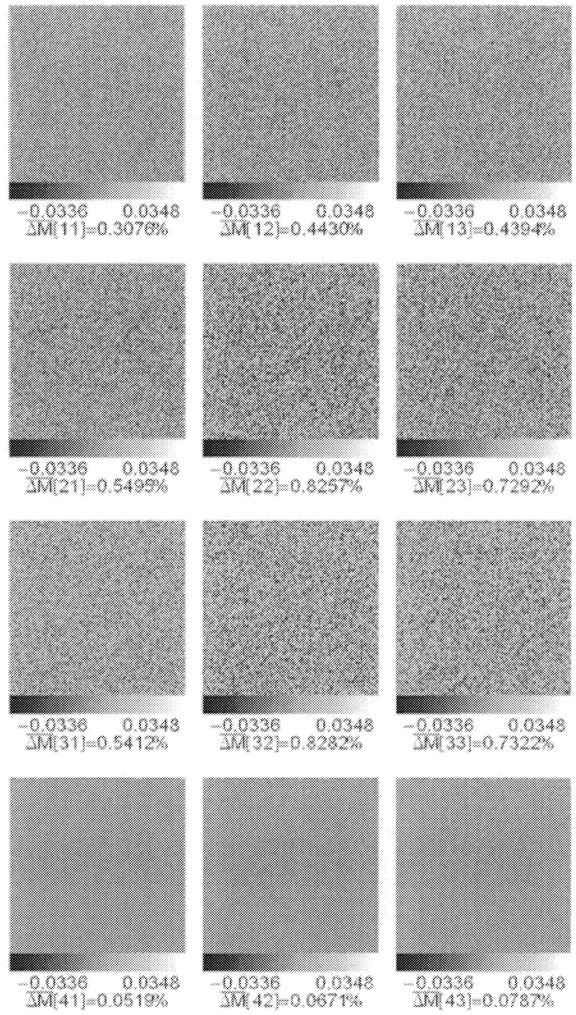

Fig.11. ΔI =2 %; P1 =0; P2 =0; $\Delta\theta$ =0°; ε =0°; $\Delta\delta$ =0°; $\Delta\alpha$ =0°; $\overline{\delta\mathbf{M}} = 1.13\%$

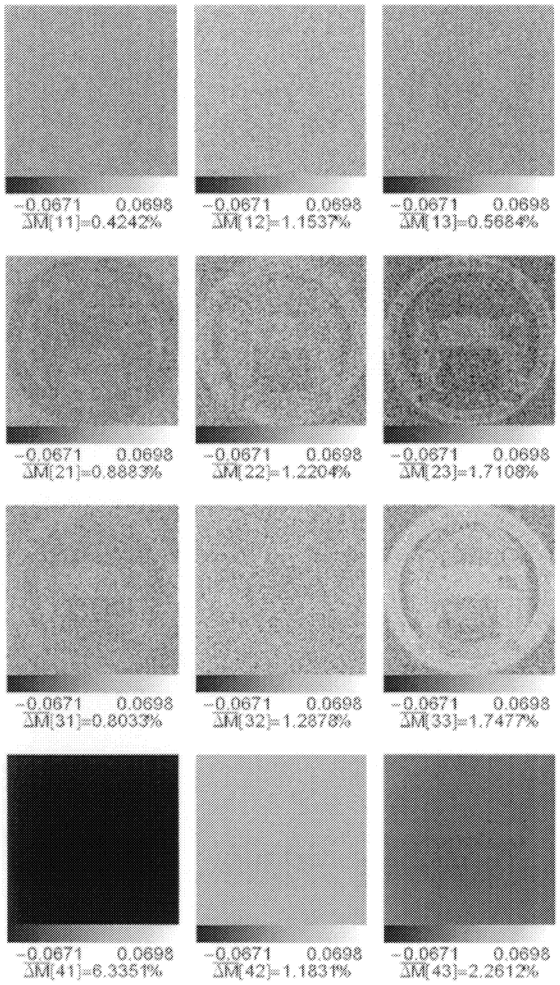

Fig.12. ΔI =2 %; P1 =0.001; P2 =0.001; $\Delta\theta$ =1°; ε =35°; $\Delta\delta$ =3°; $\Delta\alpha$ =1°; $\overline{\delta \mathbf{M}} = 4.03\%$

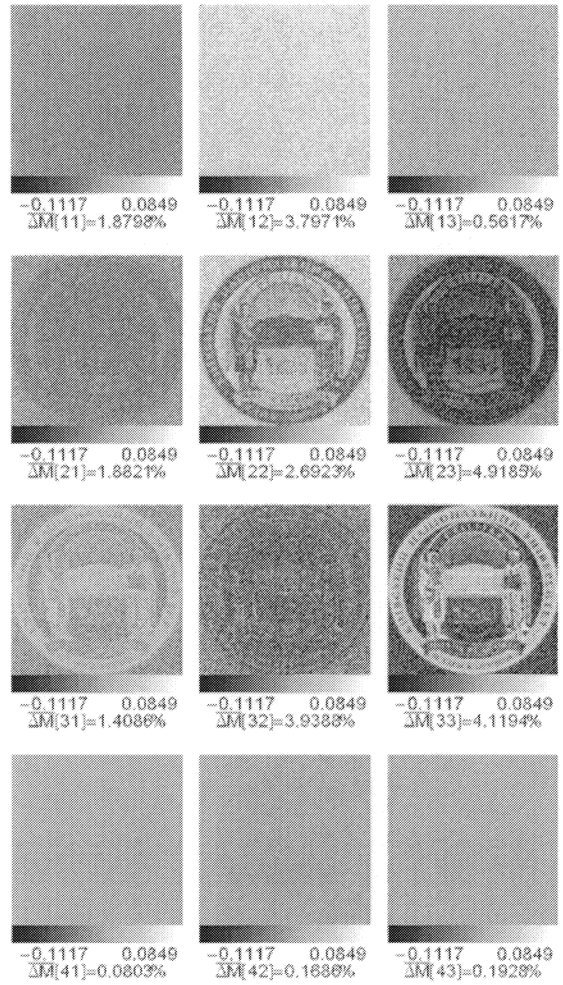

Fig.13. ΔI =2 %; P1 =0.001; P2 =0.001; $\Delta\theta$ =1°; ε =0°; $\Delta\delta$ =3°; $\Delta\alpha$ =1°; $\overline{\delta \mathbf{M}} = 5.13\%$

3. CONCLUSIONS AND DISCUSSIONS

From Fig.(4) ÷ (13) it is possible to stress that from all kinds of imperfections the alterations in orientations of polarizer 5 and of wave plate 3 in the receiving channel (Fig.3) have most significantly affected the results of measurements. The imperfections of polarizer 5 also lead to appreciable increase the value of measurement errors for determination of the matrix element M_{41}. The last leads by itself to increasing of the averaged error of Mueller matrix measurements more than six times. The matter is that in this case the input polarization differs from linear (the fourth element of Stokes vector not equal to zero). This disturbs the equalities in system Eq.(3).

Other kinds of imperfections (imperfections of analyzer, non circular polarization of light impinging onto polarizer 5, inhomogeneity of a wave plate 3 in the receiving channel etc.) lead to the values of measurement error, which are comparable with value of laser intensity alterations.

Thus, from preceding analysis we can make the following conclusion: for polarimeter built by scheme Fig.3 the most attention should be paid to the quality of polarizer 5 and to the accuracy of alignment of this polarizer and of the wave plate 3 in the receiving channel of polarimeter.

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