Proceedings of the

XIII INTERNATIONAL CONFERENCE
“ELECTRONICS AND APPLIED PHYSICS”

October 24-27, 2017, Kyiv, Ukraine

Taras Shevchenko National University of Kyiv
Faculty of Radio Physics, Electronics and Computer Systems

ONE EXAMPLE OF EXACTLY SOLVABLE QUANTUM MECHANICS PROBLEM OF POSITION DEPENDENT MASS

V.M.Burdeynyy
Vinnytsia National Technical University, Vinnytsya, Hmelnytsky street,95,e-mail:brdnvldmr@ukr.net

This communication has been dedicated to quantum dynamics of particle whose mass depends on coordinate. We considered one dimensional model which admits to obtain the exact solution of wave equation. The position dependent mass was represented as the periodic function. Inside of period the mass varies accordingly to the inverse proportionality. We have found wave functions in their explicit form as well as energy eiguen values. It has been shown that the energy spectrum manifests properties typical for periodic nanostructures.

Here we consider the one-dimensional motion of a particle with mass, which in general depends on the coordinate x according to the formula:

$$m(x) = m_0 f(x) \tag{1}$$

where $f(x)$ is a function whose choice will be specified in the below given text. As far as of liner momentum operator and a mass ordering we use the suggestion of the review[], that is:

$$-\frac{\hbar^2}{2m_0}\partial_x \frac{1}{f(x)}\partial_x \Psi = E\Psi \tag{2}$$

Since we try to find an explicit and exact solution of equation (2) further consideration demands to specify the function $f(x)$ which describes coordinate dependence of particle's mass. There are a lot references [1,2] where some general forms of functions which provide the exact solutions of wave equation (2) have been established. However in the extensive bibliography we have found a few examples where these exact solutions would have been written in their explicit forms and corresponding energy eigenvalues would have been determined. That is why in this paper we try at least slightly to expand the list of exact solutions of position dependent mass problem considering the model which in our opinion could be relevant to nanostructures and other applications. Thus we treat the one dimensional quantum dynamics of the particle with position dependent effective mass represented by the periodical function given as follows:

$$f(x) = \left[\frac{\kappa(x-nL)}{L}+1\right]^{-1} \;;\; x\in[nL,(n+1)L] \;;\; n\in Z \tag{3}$$

Due to the coefficients of Schrödinger equation (2) are periodical functions with the period L its solutions have to satisfy to certain boundary conditions. Having recourse to Fluquet theorem [3] we can write the first boundary condition as follows:

$$\Psi(x)\big|_{x=0} = e^{iqL}\,\Psi(x)\big|_{x=L} \tag{4}$$

Concerning the second one it comes up of probability's flux density continuity, that is:

$$\left.\frac{1}{m_0 f(x)}\partial_x\Psi(x)\right|_{x=0} = e^{iqL}\left.\frac{1}{m_0 f(x)}\partial_x\Psi(x)\right|_{x=L} \tag{5}$$

Here q is the wave vector which runs quase continuous values into the first Brilluiene zone.

Now we introduce a dimensionless variable y determining it by the expression

$$y = \frac{\kappa}{L}x + 1 \tag{6}$$

By substituting the formula (6) into the equation (2) we rewrite it as follows:

$$y\partial_y^2\Psi + \partial_y\Psi + Q^2\Psi = 0 \tag{7}$$

where we assume the notice $Q^2 = \frac{2m_0E}{\hbar^2}\left(\frac{L}{\kappa}\right)^2$.

The system of fundamental solutions of problem (7) can be expressed in terms of the Bessel $J_\nu(\xi)$ and Neumann $N_\nu(\xi)$ functions. According to the reference book [3] we have such general solution:

$$\Psi(y) = AJ_0\left(2Qy^{1/2}\right) + BN_0\left(2Qy^{1/2}\right) \tag{8}$$

Substitution (8) into the boundary conditions (4) and (5) leads to the linear system of equations for coefficients A , B and eigenvalues equation in the form

$$\begin{aligned}\frac{2}{\pi}CosqL = Q\left[J_1(2Q)N_0\left(2Qy_L^{1/2}\right) - J_0\left(2Qy_L^{1/2}\right)N_1(2Q)\right] + \\ + Q\sqrt{y_L}\left[J_1\left(2Qy_L^{1/2}\right)N_0(2Q) - J_0(2Q)N_1\left(2Qy_L^{1/2}\right)\right]\end{aligned} \tag{9}$$

Approximate solutions of equation (9) can be obtained for two asymptotic cases, namely, $Q \gg 1$ and $Qy_L^{1/2} \gg 1$. In the first case, taking into account the asymptotic expansions of the Bessel and Neumann functions[4], we come to the equation:

$$\frac{1}{2}(y_L^{1/4} + y_L^{-1/4})Cos\left[2Q(y_L^{1/2} - 1)\right] = CosqL \tag{10}$$

where the terms of order Q^{-1} and superior ones have been neglected. Let us point out that multiplicative factor in equation (14) satisfies to the condition $(y_L^{1/4} + y_L^{-1/4})/2 \geq 1$ for all the actual values of parameter κ. Hence the equation (10) always has solutions. By resolving the equation (10), we have obtained:

$$E_n(q) = \frac{\hbar^2}{8m_0}\left(\frac{\kappa}{L}\right)^2 \frac{1}{(y_L^{1/2} - 1)^2}\left(\pi n + \arccos\frac{2CosqL}{y_L^{1/4} + y_L^{-1/4}}\right)^2 \tag{11}$$

It is interesting to note that neglecting by the dispersion one can obtain the results identical to ones corresponding to the well known problem of the particle in the potential rectangular walls[5]

In the second case, that is, if $Qy_L^{1/2} \ll 1$, after applying the expansion of cylindrical functions in series and restricting us by the main terms only, we have

$$E(q) = \frac{\hbar^2}{2m_0}\left(\frac{\kappa}{L}\right)^2 \frac{1 + \sqrt{1+k} - 2\sqrt{1+k}CosqL}{(\sqrt{1+k} - 1)\left[(1+k)^2 - 1 + 2\sqrt{1+k}(2 + k + \sqrt{1+k})\ln(1+k)\right]} \tag{12}$$

Than lowly lied branches of energy spectrum is analogous to the energy bands peculiar to the nearest neighbors' approximation.

References

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