Reconstruction of Acoustic Surfaces from Incomplete Data as an Identification Problem Based on Fuzzy Relations

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Abstract

The model of the acoustic surface in the form of the system of fuzzy relational equations (SFRE) is proposed. The relationship matrix connects fuzzy locations of sources or their groups and sound energy levels. The problem of acoustic surface reconstruction from incomplete data is reduced to the problem of identifying the matrix of fuzzy relations by solving the composite SFRE. Properties of the solution set allow avoiding the generation and selection of the source distribution parameters. The method for reconstructing acoustic surfaces by solving the composite SFRE is proposed. To reconstruct the set of solutions in the form of fuzzy if-then rules, the genetic-gradient algorithm is used. The reconstruction process is simplified due to ability to parallelize the process of numerical solution of the composite SFRE, that allows to increase the frequency of reconstruction when processing acoustic data streams. To minimize processing time, the number of microphones is limited, provided that the risk of incorrect reconstruction remains acceptable. For the testing set of acoustic images, the risk of incorrect reconstruction is evaluated by the comparison of the extracted rules and the rules which describe the real acoustic surface. The risk of incorrect reconstruction of the acoustic level is defined as the ratio of the number of rules from the contiguous and remote power classes to the total number of rules in the actual power class. The risk of incorrect reconstruction of the acoustic surface is defined as the average risk of incorrect reconstruction over all sound energy levels.

Keywords

Inverse problems in acoustics, risk of incorrect reconstruction of the acoustic surface, identification based on fuzzy relations, solving fuzzy relational equations

1. Introduction

Microphone arrays are the standard technology for acoustic field visualization in terrain monitoring systems [1]. Physical principles of the construction of arrays with a limited number of microphones cause the problem of sparse data through the irregular distribution of focal points at the intersection of rays. Reconstruction of the acoustic field from incomplete data is based on the retrospective propagation of sound pressure by solving the inverse problem [2]. The problem

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of increasing the acoustic image resolution consists in finding the coordinates and powers of sound sources, provided that the number of sources and their configuration are unknown. The model of acoustic field is built on the basis of the fundamental laws of sound energy propagation [2]. The use of classical regularization methods for solving the inverse problem of sound field reconstruction is limited to cases of a known number of sparse sources [3, 4]. Under data uncertainty, the source parameters are estimated using statistical methods [5, 6]. Such methods require significant computational costs for conducting a series of expensive experiments in order to specify the position of sources and their powers.

To minimize processing time, the number of microphones is limited, provided that the risk of incorrect reconstruction remains acceptable. The problem of acoustic field reconstruction from incomplete data can be considered as an identification problem based on fuzzy relations [7, 8]. The acoustic surface is described by the fuzzy rule base, which is modeled by the fuzzy relationship matrix. In the theory of fuzzy relational equations, the problem of extraction of the relationship matrix from experimental data belongs to the class of inverse problems [9]. The set of solutions to the inverse problem corresponds to the set of variants of the acoustic field reconstruction in the form of the set of if-then rules [8]. The risk of incorrect reconstruction of the acoustic surface is defined as the ratio of the number of incorrect rules to the total number of rules. Therefore, it is advisable to use intelligent identification technologies that will provide the permissible risk of incorrect reconstruction of the acoustic field without resorting to a series of costly experiments [7, 8].

2. Literature Review

The problem of incomplete data is solved by increasing the number of measurements, where estimates of source parameters are determined by beamforming methods [10]. In contrast to the classical beamforming, a series of experiments are performed from different positions of the array with a variable distribution of sensors (microphones). In [10], the sequence of array positions around the sources is chosen randomly or experiments are performed with deterministic patterns of measurement positions. The concept of forming the average beam, which consists in choosing a stationary field based on the results from different measurement positions, is proposed in [11]. In [12], the method of beamforming based on the most frequently repeated observations is derived. In [12, 13], the optimal array position and beamforming mode are determined on the basis of the Monte Carlo method.

Further processing of the experimental results is aimed to estimate the position of the sources and their contribution to the overall distribution of sound energy of the field by statistical inference methods [14]. The inference method [14] is based on maximizing the likelihood function, which can be interpreted as the measure of agreement between the statistical model and uncertain measurement data. To estimate the parameters of sound sources, an algorithm of expectation maximization is used, which iteratively maximizes the criterion by sequentially estimating the contribution of each source to the overall distribution of the sound field energy [14]. The reliability of the source parameters estimation is determined by the degree of data sparseness [15]. In general, the contribution of each source is estimated by simulation modeling [16]. The candidate source field is removed from the field generated by the cumulative effect of the sources until the noise level is less than the threshold value [14, 16, 17].

Refinement of the acoustic image by the method of Bayesian inference is called Bayesian focusing [10, 18, 19]. Resolution of the reconstruction based on the Bayesian model is increased due to a priori information on the sources distribution. This makes it possible to consistently estimate the position of sound sources by selecting a model of sound energy distribution according to the measured data. As a result of solving the inverse problem, the scan parameters are automatically estimated together with the source distribution parameters. In this case, the current estimates of the source parameters are used to update the tuning functions at the next iteration.

Statistically optimized field reconstruction requires significant costs. The cost of experiments rises with increasing requirements for equipment [10, 13, 20], which in contrast to empirical tuning provides optimal beamforming parameters in accordance with source distribution parameters. However, processing of acoustic stream data eliminates the possibility of conducting a series of experiments for multiple initialization of iterative search for sound sources parameters. In this case, derivation of a three-dimensional model of the acoustic field is carried out using alternative technologies. Separation of closely located sources on the terrain is not advisable in terms of computational costs, as excessive detail does not improve the general idea of real events in the acoustic field. Therefore, 3D visualization of complex acoustic scenes uses the method of an equivalent source, which reproduces the sound pressure similar to that measured in some virtual acoustic volume using the method of least squares [15, 16, 21–23]. In this case, spatial reconstruction is simplified and adapted to real measurement scenarios without the need to reconstruct the parameters of all sources in the acoustic field [10].

In conditions of incomplete data, the equivalent source method leads to the development of the sound field models with interval or fuzzy parameters [24, 25]. The location of the sources is estimated by fuzzy clustering methods based on the analysis of the density of the peak values in the sound field [26–28]. The contribution of an individual source or a group of sources to the total sound energy level is modeled by relationship matrices. Finally, for the found number of sources or their clusters, the field parameters are restored by regularization methods [3, 4]. Reconstruction of real acoustic scenes on the terrain is carried out by generating possible scenarios based on cluster analysis, followed by genetic selection of sources that provide a minimum distance between the model and observed sound pressure levels [28]. As a result, reconstruction of the field generated by multiple sources requires significant computational costs, as it is reduced to the generation and selection of relationship matrices that determine the distribution of the sources in the sound field.

3. Problem Statement

This paper proposes a model of the acoustic surface based on the system of fuzzy relational equations (SFRE). The coordinates of a sound source or a group of sources are described by fuzzy terms, and the matrix of fuzzy relations connects the locations of the sources and the sound energy levels of the field. The SFRE connects membership functions of the source locations and the sound energy levels using the compositional rule of inference [29]. Then the problem of acoustic surface reconstruction is reduced to solving the composite SFRE obtained for the available measurement results "coordinates - field strength" [8, 9]. The method for extracting fuzzy relations from experimental data by solving the composite SFRE was proposed in [8, 30, 31]. The method [8, 30, 31] is based on the numerical resolution of the SFRE using genetic and neural technology aimed at adapting the solution as new experimental data becomes available [32, 33]. Following [8, 30, 31], solving the composite SFRE is accomplished in two stages. At the first stage, the null solution for the relationship matrix and the membership functions parameters is determined. The null solution ensures the minimum difference between the results of linguistic approximation and experimental data. At the second stage, the null solution allows to organize the parallel search for the solution set in the form of the lower and upper bounds of fuzzy relations. Linguistic interpretation involves the transition from a set of solutions for the matrix of fuzzy relations to a set of fuzzy IF-THEN rules [34].

Following [8, 30, 31], at the first stage of reconstruction, the null distribution of sound energy levels in the form of the relational matrix is obtained. The parameters of membership functions of the fuzzy terms, which describe the coordinates of sound sources, are determined simultaneously with the null distribution of the field energy. At the second stage, the set of solutions in the form of boundary sound energy levels of the field for the matrix of fuzzy relations is determined. In this case, the results of fuzzy logic inference are the lower and upper acoustic surfaces. The number of variants for sound field reconstruction is defined by transforming the relationship matrix to a set of IF-THEN rules describing the acoustic surface.

Risk of incorrect reconstruction is evaluated by the comparison of the extracted rules and the rules which describe the real acoustic surface. The total number of rules is distributed according to the sound energy levels. Due to incomplete data, the interval rule is considered correct if the actual acoustic level is embedded within the lower and upper acoustic surfaces. The probability of correct reconstruction of the acoustic level is defined as the ratio of the number of correct rules to the total number of rules in the power class. The rule is considered incorrect if a different acoustic level is reconstructed instead of the actual acoustic level. Incorrect rules are divided into rules from the contiguous and remote power classes. The risk of incorrect reconstruction of the acoustic level is defined as the ratio of the number of incorrect rules to the total number of rules in the certain power class. The probability of correct reconstruction of the acoustic surface is defined as the average probability of correct reconstruction over all sound energy levels, and the risk of incorrect reconstruction – as the average risk of reconstruction of the contiguous and remote acoustic levels.

Properties of the solution set of the composite SFRE allow avoiding the generation and selection of the source distribution parameters based on relationship matrices "location - sound energy level" [26–28]. The process of recovering the field generated by multiple sources is simplified due to ability to parallelize the process of numerical resolution of the composite SFRE, that allows to increase the frequency of reconstruction when processing acoustic data streams. The genetic-neural algorithm of solving the SFRE for the problems of acoustic field reconstruction was proposed in [35], where the field model was built on the basis of the fundamental laws of the sound theory [1, 2], and the source parameters were determined by crisp values. Unlike [35], in this work the number of sound sources is not limited. An acoustic surface in the form of the fuzzy knowledge base is subject to reconstruction, where the number of input terms is limited by the size of the controlled area. The proposed approach does not require a series of cost experiments and allows to restrict experimental conditions with equipment that implements classical beamforming methods for processing acoustic data streams under incomplete measurement results.

The aim of the work is to develop the method based on solving the composite SFRE for reconstructing acoustic surfaces from incomplete data. The method should provide the minimum processing time while preserving the permissible risk of incorrect reconstruction of the acoustic field.

4. Model and Method for Sound Surfaces Reconstruction

4.1. Problem of knowledge extraction for recovering acoustic surfaces

In order to ensure the safety of mass events which involve the participation of people and equipment, the open area with coordinates $x_1 = x_2 \in [0, 250]$ m is controlled by acoustic vision. It is assumed that there is no effect of repeated reflection of acoustic signals. Following the fundamental laws of the sound theory [1, 2], the levels of sound field energy $y(x_1, x_2)$ are determined as follows:

$$y(x_1, x_2) = 10\, lg\left(\frac{1}{I_0} \cdot \sum_{p=1}^{n} \frac{w_p}{4\pi\ [(x_1-u_p)^2+(x_2-v_p)^2]}\right), \quad (1)$$

where $I_0 = 10^{-12}$ Wt/m^2 is the intensity of the audibility threshold; n is the number of sources; u_p, v_p and w_p are the coordinates and power of the p-th sound source.

The real acoustic image $y(x_1, x_2)$ was generated by $n = 300$ sources with a sound power range of $w_p \in [10^{-8},\ 10^{-1}]$ Wt, $p = \overline{1, n}$, which corresponds to acoustic levels of $y \in$ [40, 110] dB [1, 2]. The real acoustic image (1) at the input of the microphone array is shown in Figure 1.

The acoustic image (1) at the output of the microphone array is shown in Figure 2. The microphone array is formed by the matrix of 32*32 microphones with the distance of 25 m and the scanning step of 1^0 [35]. Image resolution which is Q =3615 points decreases with the growth of the distance from the center of the array.

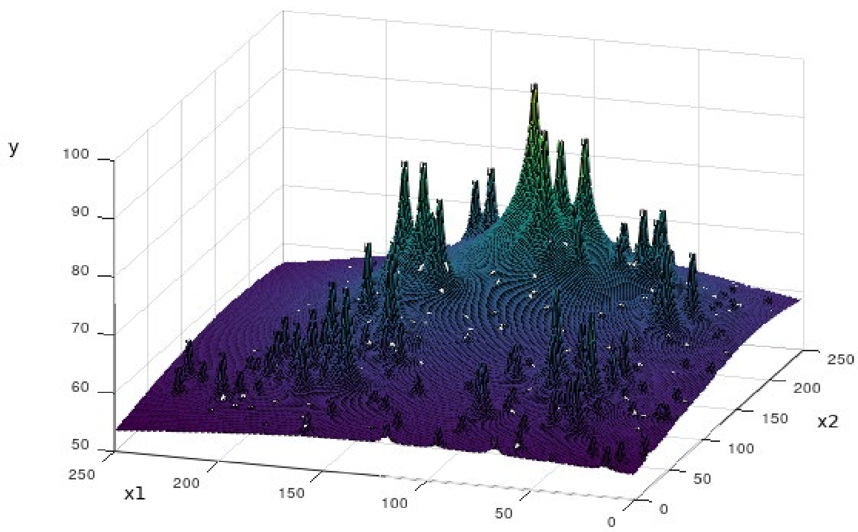

Figure 1: The real acoustic image for $n = 300$

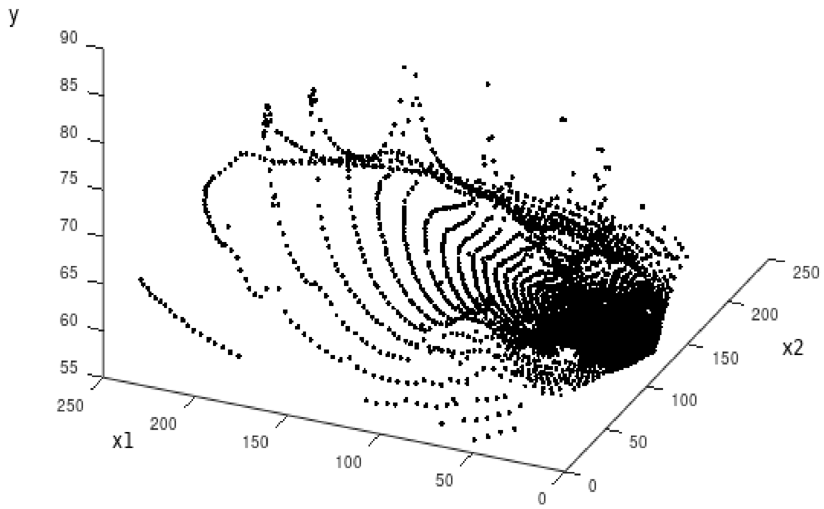

Figure 2: The observed acoustic image at the output of the microphone array

The problem of sound field reconstruction consists in the following. For the observed image in the form of the Q =3615 measurement results "coordinates $\hat{x}_1^s, \hat{x}_2^s$ - level of acoustic energy $\hat{y}_s$", $s = \overline{1, Q}$, it is necessary to restore the real image $y(x_1, x_2)$ at the input of the microphone array in order to increase the resolution to $250\times250 = 62500$ points. For this purpose, using the available measurement results, it is necessary to extract knowledge about the acoustic surface in the form of IF-THEN rules [7, 8]:

Rule K: IF $x_1 = a_{1K}$ AND $x_2 = a_{2K}$ THEN $y = d_K$, $K = \overline{1, Z}$, (2)

where $a_{1K} \in \{c_{11}, \ldots, c_{1k_1}\}$ and $a_{2K} \in \{c_{21}, \ldots, c_{2k_2}\}$ are the fuzzy terms for estimating the variables x_1 and x_2 in the rule K; $d_K \in \{E_1, \ldots, E_M\}$ is the decision class for estimating the variable y in the rule K; Z is the number of rules.

4.2. Assessing the risk of incorrect reconstruction of the acoustic surface

To evaluate the risk of incorrect reconstruction of the acoustic surface, it is necessary to make a distribution of Z rules according to the acoustic levels $\{E_1, \ldots, E_M\}$ [7].

We shall denote:

z_J is the number of rules demanding the acoustic level E_J, that is $Z = z_1 + \ldots + z_M$;

z_{JL} is the number of rules reconstructed by fuzzy inference for the acoustic level E_L instead of the acoustic level E_J, that is $z_J = z_{J1} + \ldots + z_{JM}$.

For the power classes of sound sources, quality of reconstruction is evaluated as follows:

$$P_J^1 = \frac{z_{JJ}}{z_J},\ P_J^0 = 1 - P_J^1 = \frac{1}{z_J} \sum_{\substack{L=1 \\ L\neq J}}^{M} z_{JL},$$

where P_J^1 is the probability of correct reconstruction of the acoustic level E_J; P_J^0 is the risk of incorrect reconstruction of the acoustic level E_J.

For the acoustic surface, quality of reconstruction is evaluated as follows:

$$P^1 = \frac{1}{Z} \sum_{J=1}^{M} z_{JJ}\ ,\ P^0 = 1 - P^1 = \frac{1}{Z} \sum_{J=1}^{M} \sum_{\substack{L=1 \\ L\neq J}}^{M} z_{JL},$$

where P^1 is the probability of correct reconstruction of the acoustic surface; P^0 is the risk of incorrect reconstruction of the acoustic surface.

4.3. Fuzzy relational model of the acoustic surface

We shall redenote:

$\{c_{11},\ldots,c_{1k_1},c_{21},\ldots,c_{2k_2}\}$=$\{C_1,\ldots,C_N\}$, $N=k_1+k_2$, is the set of fuzzy terms for estimating the coordinates of the sound field x_1 and x_2.

The fuzzy knowledge base (2) is modeled using the system of one-dimensional relation matrices «location c_{il} – sound energy level E_J»

$$\boldsymbol{R}_i \subseteq c_{il}\times E_J = [r_{il,J},\, i=\overline{1,2},\, l=\overline{1,k_i},\, J=\overline{1,M}].$$

The equivalent relation matrix

$$\boldsymbol{R}\subseteq C_I\times E_J = [r_{IJ},\, I=\overline{1,N},\, J=\overline{1,M}]$$

defines the fuzzy distribution of the sound energy levels in the field. The element $r_{IJ}\in[0,\ 1]$ of the matrix $\boldsymbol{R}$ is interpreted as the measure of manifestation of the sound energy level E_J at the location C_I.

Given matrices $\boldsymbol{R}_i$, the acoustic surface can be described with the help of the SFRE [29]:

$$\boldsymbol{\mu}^E=\boldsymbol{\mu}^{A_1}\circ\boldsymbol{R}_1\cap\boldsymbol{\mu}^{A_2}\circ\boldsymbol{R}_2, \quad (3)$$

where $\boldsymbol{\mu}^{A_i}(x_i)=(\mu^{c_{i1}},\ldots,\mu^{c_{ik_i}})$ is the vector of membership degrees of the variable x_i to the fuzzy locations c_{il}, $i=\overline{1,2}$, $l=\overline{1,k_i}$; $\boldsymbol{\mu}^E=(\mu^{E_1},\ldots,\mu^{E_M})$ is the vector of membership degrees of the variable y to the sound energy levels E_J, $J=\overline{1,M}$; $\circ$ is the operation of *max-min* composition [29].

Following [8, 30, 31], the matrices of membership degrees

$$\hat{\boldsymbol{\mu}}^{A_i}(\hat{x}_i^s)=\begin{bmatrix}\hat{\mu}^{c_{i1}}(\hat{x}_i^1) & \ldots & \hat{\mu}^{c_{ik_i}}(\hat{x}_i^1)\\ \ldots & \ldots & \ldots\\ \hat{\mu}^{c_{i1}}(\hat{x}_i^Q) & \ldots & \hat{\mu}^{c_{ik_i}}(\hat{x}_i^Q)\end{bmatrix},\ \hat{\boldsymbol{\mu}}^E(\hat{y}_s)=\begin{bmatrix}\hat{\mu}^{E_1}(\hat{y}_1) & \ldots & \hat{\mu}^{E_M}(\hat{y}_1)\\ \ldots & \ldots & \ldots\\ \hat{\mu}^{E_1}(\hat{y}_Q) & \ldots & \hat{\mu}^{E_M}(\hat{y}_Q)\end{bmatrix}$$

can be obtained according to the microphone array measurement results "field coordinates $\hat{x}_1^s,\hat{x}_2^s$ – acoustic energy level $\hat{y}_s$", $s=\overline{1,Q}$.

Given matrices $\hat{\boldsymbol{\mu}}^{A_i}$, $\hat{\boldsymbol{\mu}}^E$, the acoustic surface can be described with the help of the composite SFRE [8, 30, 31]:

$$\hat{\boldsymbol{\mu}}^E(\hat{y}_s)=\hat{\boldsymbol{\mu}}^{A_1}(\hat{x}_1^s)\circ\boldsymbol{R}_1\cap\hat{\boldsymbol{\mu}}^{A_2}(\hat{x}_2^s)\circ\boldsymbol{R}_2. \quad (4)$$

For each sound energy level $E_J, J=\overline{1,M}$, the SFRE (4) can be rewritten in the form [9]:

$$\hat{\boldsymbol{\mu}}^{E_J}(\hat{y}_s)=\hat{\boldsymbol{\mu}}^{A_1}(\hat{x}_1^s)\circ\boldsymbol{r}_1^J\cap\hat{\boldsymbol{\mu}}^{A_2}(\hat{x}_2^s)\circ\boldsymbol{r}_2^J,\ s=\overline{1,Q}, \quad (5)$$

where $\hat{\boldsymbol{\mu}}^{E_J}=(\hat{\mu}^{E_J}(\hat{y}_1),\ldots,\hat{\mu}^{E_J}(\hat{y}_Q))^T$ and $\boldsymbol{r}_i^J$=$(r_{i1,J},\ldots,r_{ik_i,J})^T$ are the vector-columns of the matrix of observed values $\hat{\boldsymbol{\mu}}^E$ and the fuzzy relation matrix $\boldsymbol{R}_i$ for the sound energy level E_J.

To obtain the degree of membership of the coordinate x to the fuzzy location c, we will use the membership function of the form [7]:

$$\mu^c(x)=\frac{1}{1+\left(\frac{x-\beta}{\sigma}\right)^2}, \quad (6)$$

where β is the coordinate of the function maximum; σ is the concentration parameter.

To obtain the crisp values of acoustic energy, the defuzzification operation is performed according to the centroid method [29].

Correlations (3)–(6) define the fuzzy model of the acoustic surface as follows:

$$y = f_R(x_1, x_2, \boldsymbol{B}_C, \boldsymbol{\Omega}_C, \boldsymbol{R}_1, \boldsymbol{R}_2), \quad (7)$$

where for each sound energy level E_J, $J = \overline{1,M}$, fuzzy relations are restored by solving the composite SFRE

$$\widehat{\boldsymbol{\mu}}^{E_J}(\hat{y}_s) = \hat{f}_R^J(\hat{x}_1^s, \hat{x}_2^s, \boldsymbol{B}_C, \boldsymbol{\Omega}_C, \boldsymbol{r}_1^J, \boldsymbol{r}_2^J),\ s = \overline{1,Q}, \quad (8)$$

obtained according to the microphone array measurement results $(\hat{x}_1^s, \hat{x}_2^s, \hat{y}_s)$, $s = \overline{1,Q}$.
Here $\boldsymbol{B}_C = (\beta^{C_1}, \ldots, \beta^{C_N})$ and $\boldsymbol{\Omega}_C = (\sigma^{C_1}, \ldots, \sigma^{C_N})$ are the vectors of β- and σ- parameters for the fuzzy locations $C_1, \ldots, C_N$ membership functions; f_R and $\hat{f}_R^J$ are the operators of inputs-output connection, corresponding to formulas (3), (6) and (5), (6), respectively.

4.4. Method of acoustic surfaces reconstruction based on solving composite SFRE

Following [8, 30, 31], the problem of acoustic surface reconstruction is reduced to finding the null solution and the solution set for the fuzzy matrix of energy distribution **R**.

When searching for the null distribution, the problem of tuning the fuzzy model (7) is formulated as follows. It is necessary to find the vectors of fuzzy locations parameters $\boldsymbol{B}_C$, $\boldsymbol{\Omega}_C$, and the fuzzy relation matrix **R**, which provide the least distance between the model and the observed acoustic images:

$$F = \sum_{s=1}^{Q} [f_R(\hat{x}_1^s, \hat{x}_2^s, \boldsymbol{B}_C, \boldsymbol{\Omega}_C, \boldsymbol{R}_1, \boldsymbol{R}_2) - \hat{y}_s]^2 = \min_{\boldsymbol{B}_C, \boldsymbol{\Omega}_C, \boldsymbol{R}_1, \boldsymbol{R}_2}. \quad (9)$$

When searching for the reconstruction set, the problem of solving the composite SFRE (8) is formulated as follows [8, 30, 31]. Given fuzzy locations parameters $\boldsymbol{B}_C$, $\boldsymbol{\Omega}_C$, the fuzzy relation matrix $\boldsymbol{R} = [r_{IJ}]$, $I = \overline{1,N}$, $J = \overline{1,M}$, should be found which satisfies the constraints $r_{IJ} \in [0,\ 1]$ and provides the least distance between the model and the observed vectors of membership degrees to the sound energy levels E_J; that is, the minimum value of the criterion (9):

$$F = \sum_{J=1}^{M} \left[\hat{f}_R^J(\hat{x}_1^s, \hat{x}_2^s, \boldsymbol{B}_C, \boldsymbol{\Omega}_C, \boldsymbol{r}_1^J, \boldsymbol{r}_2^J) - \widehat{\boldsymbol{\mu}}^{E_J}(\hat{y}_s)\right]^2 = \min_{\boldsymbol{r}_1^J, \boldsymbol{r}_2^J},\ s = \overline{1,Q}. \quad (10)$$

Following [8, 30, 31], the composite SFRE (8) has the solution set, that defines the set of variants for the sound field reconstruction in the form of the lower and upper acoustic surfaces. The solution to the SFRE (8) can be represented in the form of intervals [32, 33]:

$$r_{IJ} = [\underline{r}_{IJ}, \overline{r}_{IJ}] \subset [0,1],\ I = \overline{1,N},\ J = \overline{1,M}, \quad (11)$$

which correspond to the set of IF-THEN rules

Rule K: IF $x_1 = a_{1K}$ AND $x_2 = a_{2K}$ THEN $\underline{y} = \underline{d}_K$ AND $\overline{y} = \overline{d}_K$, $K = \overline{1,Z}$. (12)

Here $\underline{r}_{IJ}(\overline{r}_{IJ})$ are the lower (upper) bounds of the fuzzy relations r_{IJ} in the sound field energy distribution; $\underline{d}_K(\overline{d}_K) \in \{E_1, \ldots, E_M\}$ are the decision classes for estimating the variables $\underline{y}(\overline{y})$ in the rule K for the lower (upper) acoustic surfaces.

The null solution $\boldsymbol{R}_0 = [r_{IJ}^0]$, $I = \overline{1,N}$, $J = \overline{1,M}$, of the optimization problem (9) allows to parallelize the search for upper and lower bounds of the intervals (11) for each sound energy level E_J, where

$$\overline{r}_{IJ} \in [r_{IJ}^0, 1], \underline{r}_{IJ} \in [0, r_{IJ}^0].$$

Following [8, 30–33], restoration of the acoustic image is accomplished by way of multiple solving the optimization problem (10). If $\boldsymbol{R}(t) = [r_{IJ}(t)]$ is some t-th solution of the optimization problem (10), then $F(\boldsymbol{R}(t)) = F(\boldsymbol{R}_0)$. When forming the intervals (11), the search space is restricted by the intervals $r_{IJ}(t) \in [r_{IJ}(t-1), 1]$ for the upper bounds; $r_{IJ}(t) \in [0, r_{IJ}(t-1)]$ for the lower bounds. The search for the intervals (11) will go on until $r_{IJ}(t) \neq r_{IJ}(t-1)$. If $r_{IJ}(t) = r_{IJ}(t-1)$), then $\overline{r}_{IJ}$ ($\underline{r}_{IJ}$)=$r_{IJ}(t)$.

The genetic-gradient approach is proposed for solving the optimization problems (9), (10) [30–33]. When searching for the null distribution, the chromosome is defined as a string of binary codes of the fuzzy locations parameters β^{C_I}, σ^{C_I} and the fuzzy relations r_{IJ}, $I = \overline{1,N}$, $J = \overline{1,M}$. When searching for the reconstruction set, the chromosome is separated for each sound energy level E_J, where the parameters r_{IJ} are recoded within the search space [8, 30, 31].

The cross-over operation is performed by exchanging parts of the chromosomes in the vectors of fuzzy locations parameters $\boldsymbol{B}_C$, $\boldsymbol{\Omega}_C$ and the matrix of fuzzy relations **R**. The fitness function is based on the criteria (9), (10). The criterion for stopping the algorithm is the absence of new upper and lower bounds for energy distribution (12) within a given time window of the microphone array [35].

When searching for the null distribution, the recurrent relations

$$r_{IJ}(t+1) = r_{IJ}(t) - \eta \frac{\partial \varepsilon_t^0}{\partial r_{IJ}(t)};$$

$$\beta^{C_I}(t+1) = \beta^{C_I}(t) - \eta \frac{\partial \varepsilon_t^0}{\partial \beta^{C_I}(t)}; \quad \sigma^{C_I}(t+1) = \sigma^{C_I}(t) - \eta \frac{\partial \varepsilon_t^0}{\partial \sigma^{C_I}(t)}, \tag{13}$$

are used; and when searching for the reconstruction set, the recurrent relations

$$r_{IJ}(t+1) = r_{IJ}(t) - \eta \frac{\partial \varepsilon_t}{\partial r_{IJ}(t)}, \tag{14}$$

are used [8, 30, 31], which minimize the criteria

$$\varepsilon_t^0 = \frac{1}{2}(\hat{y}_t - y_t)^2,$$

$$\varepsilon_t = \frac{1}{2}(\hat{\mu}_t^{E_J} - \mu_t^{E_J})^2.$$

Here $\hat{y}_t$, y_t are the observed and the model levels of acoustic energy at the t-th training step; $\hat{\mu}_t^{E_J}$, $\mu_t^{E_J}$ are the observed and the model degrees of membership of field energy levels to the classes E_J at the t-th training step; $r_{IJ}(t)$ are the fuzzy relations at the t-th training step; $\beta^{C_I}(t)$, $\sigma^{C_I}(t)$ are the parameters of membership functions for the fuzzy terms of sources locations at the t-th training step; η is a parameter of training.

For the discrete coordinate space of the microphone array, the partial derivatives included in (13), (14) are obtained using finite differences [31, 33, 35].

5. Results of the Acoustic Surface Reconstruction

Terrain monitoring is carried out in order to detect zones of acoustic activity caused by emission of the sources or their groups belonging to certain power classes.

The output classes, the number of which is limited to $M = 7$, are formed as follows:

$$[\underline{y}, \overline{y}] = \underbrace{[50,\ 57]}_{E_1} \cup \underbrace{[57,\ 64]}_{E_2} \cup \underbrace{[64,\ 70]}_{E_3} \cup \underbrace{[70,\ 78]}_{E_4} \cup \underbrace{[78,\ 85]}_{E_5} \cup \underbrace{[85,\ 92]}_{E_6} \cup \underbrace{[92,\ 100]}_{E_7}.$$

The sound field with coordinates $x_1 = x_2 \in [0,250]$ m is divided into sections with a step of 25 m. In this case, the number of input fuzzy terms is limited to $k_1 = k_2 = 9$, where

$$c_{1,1\div9} = c_{2,1\div9} = near\ 25, 50, 75, 100, 125, 150, 175, 200, 225 \text{ m.}$$

The real acoustic surface (Figure 1) is described using the set of rules presented in Table 1. For the observed data (Figure 2), the solution set of the composite SFRE (8) is presented in Table 2.

Table 1

The rule set that describes the real acoustic surface

x_2	x_1								
	~25 m	~50 m	~75 m	~100 m	~125 m	~150 m	~175 m	~200 m	~225 m
~25 m	$\boldsymbol{E_1}$	E_2	E_2	E_1	E_1	E_1	E_1	E_1	E_1
~50 m	E_2	E_4	E_3	E_2	E_2	E_1	E_4	E_3	E_2
~75 m	E_2	E_4	E_3	E_2	E_2	E_4	E_4	E_3	E_3
~100 m	E_2	E_4	E_4	E_2	E_2	E_5	E_5	E_2	E_2
~125 m	E_2	E_3	E_2	E_3	E_3	E_3	E_4	E_2	E_2
~150 m	E_2	E_4	E_2	E_3	E_3	E_5	E_4	E_2	E_2
~175 m	E_3	E_3	E_2	E_6	E_4	E_6	E_2	E_1	E_2
~200 m	E_3	E_4	E_5	E_6	E_7	E_3	E_2	E_1	E_2
~225 m	E_3	E_2	E_4	E_4	E_4	E_5	E_2	E_1	E_1

Table 2

Solution set of the composite SFRE

IF		THEN y						
		E_1	E_2	E_3	E_4	E_5	E_6	E_7
x_1	c_{11}	[0, 0.80]	0.69	0.44	0.12	0	0	0
	c_{12}	0	0.85	[0.22, 0.67]	0.48	[0, 0.60]	0	0
	c_{13}	[0, 0.73]	[0.50, 1]	[0, 0.46]	0.89	[0.45, 0.71]	0	0
	c_{14}	[0, 0.64]	0.78	0.34	[0.23, 0.69]	0.08	[0.85, 1]	0.72
	c_{15}	[0, 0.60]	0.95	[0.41, 0.56]	0.70	0	0	0.91
	c_{16}	[0, 0.53]	0.84	0.49	[0.15, 0.68]	0.73	[0, 0.67]	0
	c_{17}	[0, 0.79]	0.90	[0.34, 0.53]	0.72	[0.49, 0.65]	0	0
	c_{18}	[0, 0.84]	[0.71, 1]	0.82	[0, 0.67]	0.10	0	0
	c_{19}	[0, 0.75]	0.86	[0.18, 0.61]	0.45	0	0	0
x_2	c_{21}	[0, 0.62]	[0.85, 1]	0.49	0.16	0	0	0
	c_{22}	[0, 0.80]	[0.39, 0.73]	0.45	0.68	[0, 0.62]	0	0
	c_{23}	[0, 0.54]	0.76	[0.21, 0.60]	[0, 0.53]	0.82	0	0
	c_{24}	[0, 0.46]	[0.27, 0.69]	0.62	0.78	[0.36, 0.74]	0	0
	c_{25}	0	0.57	0.77	[0.43, 0.64]	0.25	0	0
	c_{26}	0	[0.54, 1]	0.41	0.55	[0.19, 0.68]	0.84	0
	c_{27}	0	0.78	[0, 0.49]	[0.25, 1]	0.81	0.65	0

c_{28}	[0, 0.58]	[0.35, 0.60]	0.39	[0, 0.64]	[0.75, 1]	0.92	0.86
c_{29}	[0, 0.80]	0.64	0.45	[0.38, 1]	0.81	0	0

The solution set for the relational matrix corresponds to the rule set that defines the variants of acoustic field reconstruction presented in Table 3. Due to incomplete data, the interval rule is considered correct if the actual acoustic level E_J in Table 1 is embedded within the lower and upper acoustic levels in Table 3. The rule is considered incorrect if a different acoustic level is reconstructed instead of the acoustic level E_J. Incorrect rules are divided into rules from the contiguous and remote power classes. In Table 3, the contiguous (remote) incorrect rules are marked with * (**).

In Table 1, the total number of rules $Z = 81$ is distributed according to the sound energy levels as follows:

$$z_1 = 12; z_2 = 29; z_3 = 16; z_4 = 15; z_5 = 5; z_6 = 3; z_7 = 1.$$

The risk of incorrect reconstruction of the acoustic levels is presented in Table 4, where $P_J^{0(c)}(P_J^{0(r)})$ – is the risk of reconstruction of the contiguous (remote) acoustic levels instead of the acoustic level E_J.

The probability of correct reconstruction of the acoustic surface is $P^1 = 69/81 = 0.85$. The risk of incorrect reconstruction is $P^0 = 12/81 = 0.15$, which is distributed to the risks of reconstruction of the contiguous (remote) acoustic levels $P^{0(c)} = 10/81 = 0.12$ $(P^{0(r)} = 2/81 = 0.03)$.

The obtained solutions provide the reconstruction of the acoustic field in the form of the lower and upper surfaces, which are shown in Figure 3 together with the real acoustic image.

Table 3

The rule set that describes the lower and upper reconstructed acoustic surfaces

x_2	x_1								
	~25 m	~50 m	~75 m	~100 m	~125 m	~150 m	~175 m	~200 m	~225 m
~25 m	$\boldsymbol{E_{1-2}}$	E_{2-3}	E_{1-2}	E_{1-2}	E_{1-2}	E_{1-2}	E_{1-2}	E_{1-2}	E_{1-2}
~50 m	E_2	E_{3-4}	E_2^*	E_{3-4}^*	E_{1-2}	E_2^*	E_{4-5}	E_{3-4}	E_4^{**}
~75 m	E_{2-3}	E_{4-5}	E_3	E_{2-3}	E_{1-2}	E_{4-5}	E_{4-5}	E_{3-4}	E_{2-3}
~100 m	E_{2-3}	E_{3-4}	E_{4-5}	E_2	E_{1-2}	E_{4-5}	E_{4-5}	E_4^{**}	E_2
~125 m	E_2	E_2^*	E_2	E_3	E_{2-3}	E_{3-4}	E_{2-3}^*	E_2	E_2
~150 m	E_2	E_{4-5}	E_2	E_3	E_{2-3}	E_{5-6}	E_{2-3}^*	E_2	E_2
~175 m	E_{2-3}	E_{3-4}	E_2	E_6	E_{3-4}	E_{5-6}	E_3^*	E_2^*	E_2
~200 m	E_{2-3}	E_4	E_{4-5}	E_{6-7}	E_7	E_3	E_3^*	E_{1-2}	E_{1-2}
~225 m	E_{2-3}	E_{2-3}	E_3^*	E_{3-4}	E_{3-4}	E_5	E_{2-3}	E_{1-2}	E_{1-2}

Table 4
Quality indicators of the reconstruction of acoustic levels

Indicator	Acoustic levels						
	E_1	E_2	E_3	E_4	E_5	E_6	E_7
P_J^1	10/12=0.83	24/29=0.83	14/16=0.88	12/15=0.80	1	1	1
$P_J^{0(c)}$	2/12=0.17	3/29=0.10	2/16=0.12	3/15=0.20	0	0	0
$P_J^{0(r)}$	0	2/29=0.07	0	0	0	0	0

To minimize processing time, the number of microphones is limited, provided that the risk of reconstruction remains acceptable. For the testing set of 170 acoustic images, the risk of reconstruction of the remote acoustic levels does not exceed $P^{0(r)} = 0.05$, which is permissible for the reconstruction of complex acoustic scenes on the terrain.

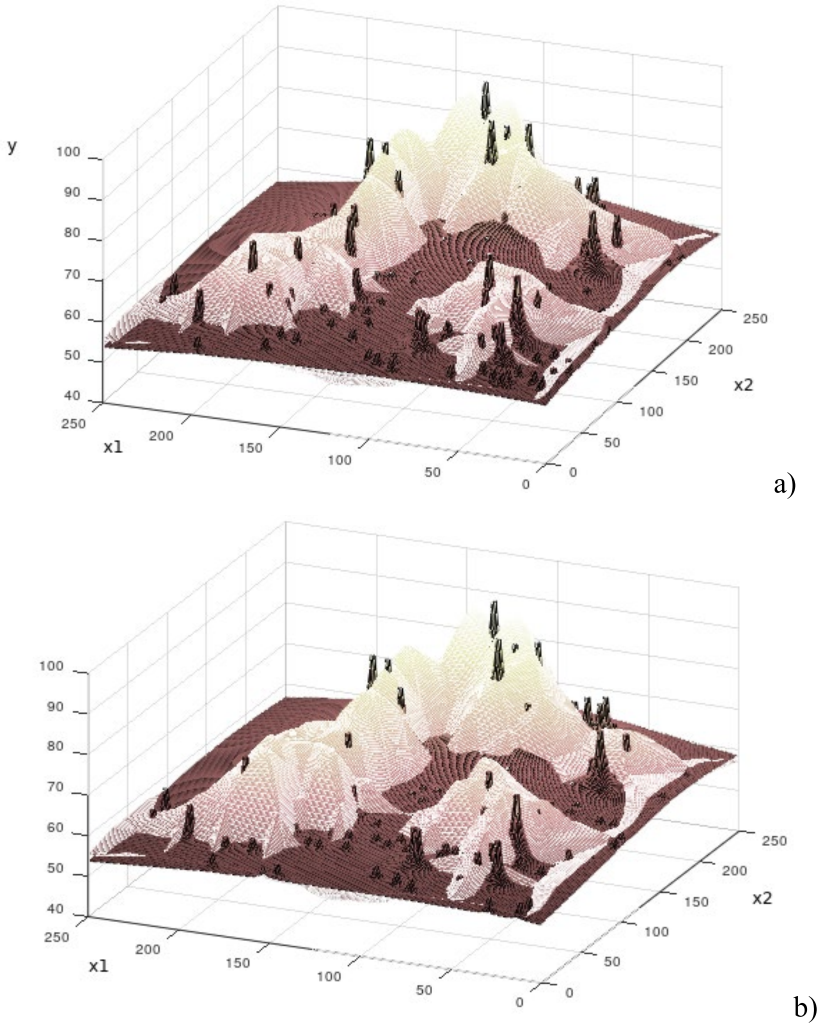

Figure 3: A comparison of the real and reconstructed image in the form of the lower (a) and upper (b) acoustic surfaces

6. Discussion of the Results of Effectiveness Estimation for Reconstruction of Acoustic Surfaces

The experiment was conducted for equipment with the classical method of beamforming, which eliminates the multiple initialization for the location of sources or their groups. The comparison of the proposed method was carried out with the methods of acoustic field reconstruction [26–28]. In [26–28] under similar measurement conditions, the contribution of each source (group of sources) to the total field energy is estimated on the basis of the genetic selection of the relational data model. Each variant of field reconstruction is described by the relational matrix, the search for which requires restarting the genetic algorithm. The number of sources is not limited. Instead, groups of sources are considered in some virtual acoustic volume, and the dimension of the relational matrix is determined by the number of such groups.

The principal difference of the given method is the possibility of simultaneous search for the lower and upper bounds of fuzzy relations for each power class of sound sources, that allows reducing the computational complexity.

Implementation of the models [26–28] with adjustment of the relational matrix requires solving the sequence of V optimization problems with NM parameters, where V is the number of variants of the field reconstruction. Reconstruction of the acoustic surface in the form of solutions of the composite SFRE requires solving the sequence of $2VM$ optimization problems with N parameters for the lower and upper bounds of fuzzy relations. Generation of the null distribution additionally requires solving the optimization problem with $NM+2N$ variables for two-parameter membership functions.

The reduction in computational complexity allows to obtain the following time estimates. The time of acoustic field reconstruction was estimated for the maximum number of input terms $k_1 = k_2 = 9$ according to the given size of the controlled area. For detailed reconstruction of the acoustic surface, the method can be applied to individual areas of the terrain. The time of generation for the lower and upper bounds of solutions using the principles of parallel computing does not exceed 3 s, which provides on-line reconstruction of the acoustic data stream (Intel Core i5-7400 3.0 Ghz processor). Reconstruction of the acoustic surface by the methods [26–28] is carried out with the delay of 7–8 s. Thus, the proposed method allows to halve the time window of the microphone array, i.e. double the frequency of reconstruction, that increases the reliability of terrain monitoring without attracting additional computing resources.

7. Conclusions

For the acoustic surface generated by many sources, the model based on fuzzy rules and relations is proposed. The number of sources in the sound field is not limited. Instead, the number of input terms is limited by the size of the controlled area. For the available measurement data, the problem of acoustic surface reconstruction is reduced to the problem of identifying the matrix of fuzzy relations. In fuzzy relational calculus [9], this problem belongs to the class of inverse problems and requires solving the composite SFRE. Properties of the solution set allow avoiding the generation and selection of the source distribution parameters. The solution set is interpreted in the form of the set of if-then rules “fuzzy location – sound energy level”.

For reconstructing the acoustic surface from incomplete data, the method based on solving the composite SFRE is proposed. The method provides the linguistic approximation of the acoustic image in the form of the lower and upper surfaces, where the number of reconstruction variants is determined by the set of solutions for the relational matrix. To solve the inverse reconstruction problem, the genetic-gradient algorithm is used. Simplification of the reconstruction process is achieved due to the simultaneous search for the lower and upper bounds of solutions for each power class, that allows to increase the frequency of reconstruction when processing acoustic data streams. The method provides the minimum processing time while preserving the permissible risk of incorrect reconstruction of the acoustic field. For the testing set of acoustic images, the risk of incorrect reconstruction is evaluated by the comparison of the extracted rules and the rules which describe the real acoustic surface. The risk of incorrect reconstruction of the acoustic level is defined as the ratio of the number of rules from the contiguous and remote power classes to the total number of rules in the actual power class. The risk of incorrect reconstruction of the acoustic surface is defined as the average risk of incorrect reconstruction over all sound energy levels.

A further area of research is the development of a method for intelligent focusing of acoustic images by optimizing the fuzzy knowledge base that describes the acoustic surface. The problem is to choose the number of input terms, output classes and rules that provide the necessary or extreme levels of accuracy and reconstruction time.

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