ISSN 1064-2307, Journal of Computer and Systems Sciences International, 2021, Vol. 60, No. 5, pp. 793–801. © Pleiades Publishing, Ltd., 2021.
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SYSTEMS ANALYSIS
AND OPERATIONS RESEARCH

# Reliability Analysis: from Fault Tree to Catastrophe Tree

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Received February 6, 2021; revised March 25, 2021; accepted May 31, 2021

**Abstract**—In this paper, we propose a method for analyzing a catastrophe tree that is an analog of the classical method for analyzing a fault tree, which does not require knowledge of the probabilities of events. The concept of the *possibility of bifurcation*, which is estimated by a number in the interval [0,1] using catastrophe theory and corresponds to the membership function of a fuzzy set, is used instead of the concept of the *probability of failure*. The possibility of bifurcation (or catastrophe) of the system is calculated using specially introduced rules for aggregating the possibilities of bifurcations for various logical operations of the fault tree. The catastrophe analysis method is illustrated using the example of a fault tree for a road accident at a T-junction.

**DOI:** 10.1134/S1064230721050117

## INTRODUCTION

The fault tree analysis (FTA) method proposed in 1962 and detailed in [1] is still one of the main methods for analyzing reliability, safety, and risk not only in technical but also in socio-economic, military, and other systems [2, 3].

The FTA probabilistic models provide for a binary concept of system failures and its elements: 1, no failure; 0, failure. Therefore, a failure event can be interpreted as a bifurcation, i.e., a jump from one steady state (no failure) to another steady state (failure). An approach to modeling and optimizing reliability based on bifurcation forks of the chaos theory is considered in [4, 5].

A special mathematical apparatus for modeling bifurcations is the theory of catastrophes [6–8], which is a development of the theory of nonlinear oscillations [9]. Despite the widespread use of the theory of catastrophes in various fields, there are no studies on its application in reliability models.

In this paper, we propose a catastrophe tree analysis (CTA) method, which is an analog of the classical FTA method, which does not require knowledge of the probabilities of events. The concept of the possibility of bifurcation, which is estimated by a number in the interval [0, 1] using the catastrophe theory and corresponds to the membership function of a fuzzy set, is used instead of the concept of the probability of failure [10–12]. The possibility of bifurcation (or catastrophe) of the system is calculated using specially introduced rules for aggregating the possibilities of bifurcations for various logical operations of the fault tree.

The paper is organized as follows. Section 1 provides the concept of a characteristic function and a membership function necessary to understand the difference between the probability and possibility of failure, which are calculated based on the structural function of reliability. In Section 2, an example of describing a bifurcation using the membership function of a fuzzy set is considered. Section 3 contains the basic relations of the catastrophe theory that are used in the catastrophe tree analysis method. In Section 4, rules for the aggregation of bifurcation levels for logical operations of the catastrophe tree are proposed. Section 5 illustrates the disaster tree method using the example of a road accident.

**Table 3.** Possibility of an accident for various scenarios

| Scenarios | $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | $\pi_5$ | $\pi_6$ | $\pi_7$ | $\pi_{1-7}$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | (1) | (2) |
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.5 | 0.00 | 0.00 |
| 2 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.32 | 0.42 |
| 3 | 0.0 | 0.1 | 0.0 | 0.0 | 0.01 | 0.0 | 0.2 | 0.45 | 0.51 |
| 4 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.3 | 0.55 | 0.57 |
| 5 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.4 | 0.63 | 0.63 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.5 | 0.71 | 0.68 |
| 7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.6 | 0.77 | 0.71 |

It is necessary to clarify Table 3: column 1 corresponds to aggregation using min and max, and column 2 corresponds to the use of the geometric mean and arithmetic mean. As can be seen in Table 3, if the car is moving along the main road $(\pi_7 \neq 0)$, the absence of an accident $(\pi_{1-7} = 0)$ is possible only at $\pi_1 = \pi_2 = \ldots = \pi_6 \approx 0$; i.e., when there are no violations related to the driver, visibility, road, brakes, and speed. The presence of at least one of these violations leads to a sharp increase in the possibility of an accident.

## CONCLUSIONS

This paper is the first study in which the catastrophe theory is applied to simulate the possibility of undesirable events based on the method of fault trees known in reliability and safety problems.

Refusal is considered as a bifurcation, i.e., jump between steady states, and is estimated by normalized formulas of the catastrophe theory. The possibility of failure concept is estimated by a number in the interval [0, 1] and is modeled according to the rules of fuzzy logic.

The result of the study is a catastrophe tree analysis method that corresponds to a fault tree. The proposed method consists in the formulated rules for the aggregation of bifurcation possibilities for various logical nodes of the tree.

The catastrophe analysis method is illustrated using the example of a fault tree simulating a traffic accident at a T-junction.

The catastrophe tree method does not replace the fault tree method, but supplements it with the following new qualities:

1. There is no need to carry out time-consuming experiments related to obtaining the probabilities of primary events that affect the probability of system failure. The possibilities of primary events, the levels of which are estimated expertly or based on measured parameters and the corresponding membership functions, are used instead of probabilities.

2. Monitoring the dynamics of changes in input parameters allows switching to online monitoring of the level of the possibility of system failure.

3. An application of catastrophe theory models allows observing nonlinear effects related to a sharp increase in the possibility of failure at minor changes in the input parameters.

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*Translated by A. Ivanov*

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