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Optimal Decomposition of Control of Distributed Cyber-physical System

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***Abstract*—The article proposes the use of a formal approach to the decomposition of the architecture of a decentralized coordination control system of distributed cyber-physical systems using microservices, which minimizes the cost of its creation and maintenance. This approach is based on the convertion of system gipergraph to optimal bipartite graph by the criterium of minimize interconnection flows and modification of Stoer-Wagner algorithm.**

***Keywords*—decomposition; distributed cyber-physical system; coordination; control; microservices**

I. Introduction

The problem of distributed technological objects controlling is becoming increasingly important due to the spread of the Internet of Things and the corresponding increase in the number of local sensors and actuators on technological objects. As a result, complicated sets of multi-element (multi-zone) technological objects, software and hardware for measurement and control, which are called distributed cyber-physical systems (DCPS) are formed. In the process of developing such systems, there is a need to solve the problem of synthesis of the optimal control structure of DCPS, in particular, the problem of decomposition of controls on the local part, which is performed directly on the technological object, and the general part, which can be implemented in the form of cloud microservices.

II. Actuality

Recently, methods of analysis and synthesis of complex engineering objects, which are called methods of decomposition, division of the areas, etc., have been developing quite intensively. Their main idea is as follows: the main problem to be solved is quite complex, but it can be divided into sub-areas, and for each of them can be solved the subtask by an appropriate method. This significantly increases the speed and efficiency of the calculation of large complex systems [1, 2].

Researches [3 - 5] consider the problem of formalizing relationships in distributed control systems in terms of creating an automated control system at certain levels of control. Solving the decomposition problem allows you to optimize the process of control such systems by arranging connections, as well as by reducing the complexity of these connections.

The authors of [6] consider it appropriate to decompose distributed systems into subsystems in the areas of automation. Such subsystems particularly well reflect the advantage of system decomposition in several crossrelated directions, as it helps to cover all that is common in different subsystems for automated information acquisition, processing, etc. Decomposition of the system in a large number of crossrelated directions can create several difficulties, however, leads to the coordination of all subsystems and ensures their successful merger after implementation into a single system.

Research [7] offers other directions of decomposition of distributed systems for a specific purpose:

- structural decomposition: allows you to get a relatively independent subsystem or set of tasks for each unit or group within units;
- functional decomposition: gives the chance to create subsystems or groups of tasks of the executed functions of the control of process;
- human-machine approach: gives the chance to allocate the human factors and a hardware part.

The proposed by [8] method is to modify the composition of the variables of the control problem due to mathematical models and certain limitations at the time of control decisions. This modification is carried out to maximize the simplification of the problem by minimizing the number of variables, transformations of the target function and the conditions of the problem. The result of the modification should be the ability to effectively solve the transformed problem using nonlinear programming methods. Based on the analysis of the current situation, the composition of the essential variables of the control problem, the corresponding structure of the target function, mathematical model and constraints are determined. All insignificant variables are discarded, the target function, models and constraints are simplified, which simplifies the problem in general.

In work [9] the decomposition of control problems is carried out using matrices "tasks-information elements" and in [10] the matrix approach is used for the decomposition of production systems.

Despite the significant amount of work on the problem of complex control systems decomposition, the task of developing mathematical methods for optimal decomposition of controls on the local part, which is performed directly on the process object, and the general part, which can be implemented in the form of cloud microservices solution was not solved so far.

III. Aim of the Research

The purpose of this work is to develop mathematical methods for optimal decomposition of DCPS controls on the local part, which is performed directly on the technological object, and the common part, which can be implemented in the form of cloud microservices by improvement of the partitioning method of a graph of system with a variable structure.

IV. Solution

Consider the problem of decentralized coordination control of a distributed cyber-physical system [11]. We present DCPS with decentralized coordination of local control systems of a continuous technological object in the form of a hypergraph G (Fig. 1).

Each element of the hypergraph is a graph that reflects the structure of information links between the main subsystems (element of the technological object (E), local control system (LCS), coordinator (C)). The connections between the elements of the hypergraph reflect the mutual physical influence of the elements of the technological object of the DCPS and the information interaction of the coordinators.

Coordinators are also represented by a graph of module connections (Fig. 2): object model C_1; estimation of uncertain parameters C_2; clustering C_3; forecasting C_4; evaluation of the step of optimal coordination C_5; calculation of coordination criterion C_6.

Thus, to solve the problem of decomposition of the system into local subsystems and common microservices based on a three-level representation of the system architecture, we characterize the hypergraph by matrices of connection intensities: $\mathbf{G}\{\mathbf{S},\mathbf{I},\mathbf{A},\mathbf{C}\}$, where $\mathbf{S}$ is the topology of the technological object obtained by object clustering to the areas of mutual physical influence of elements; $\mathbf{I}$ is matrix of information flows between coordinators; $\mathbf{A}$ is matrix of information flows between the subsystems of DCPS; $\mathbf{C}$ is the matrix of information flows between the coordinator modules.

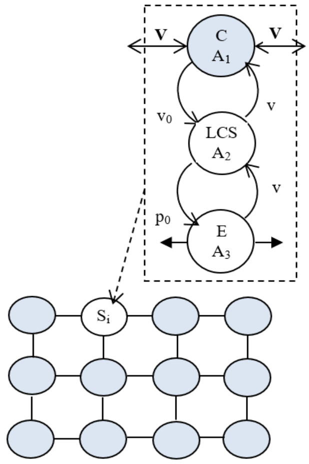

Figure 1. DCPS hypergraph with decentralized coordination of local control systems

Acquisition of the information, you need to coordinate comes at a cost. Especially if staff, experts, etc. are involved in this process. Depending on the nature and methods of information acquisition, you can select different components of costs:

- the cost of material media [12];
- the cost of staff [13];
- cost of energy consumption [14];
- depreciation of equipment [15];

etc.

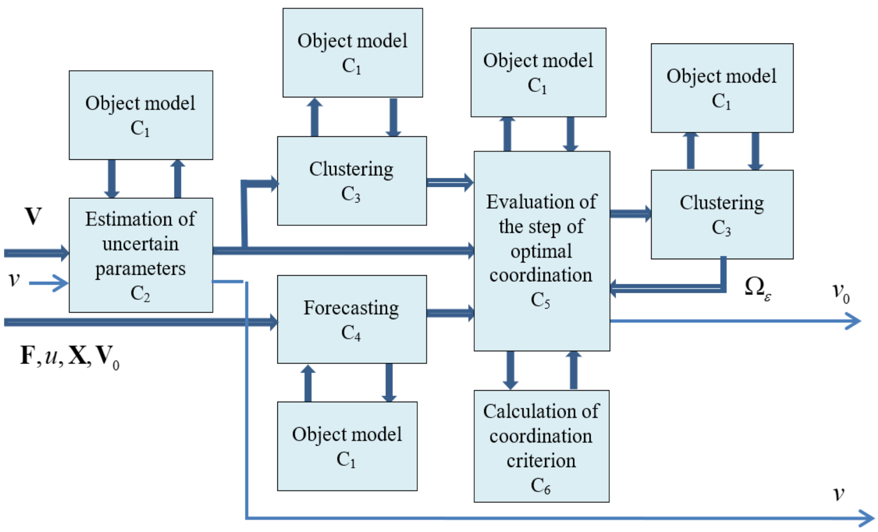

Figure 2. Modules interaction scheme

Most often, a piecewise linear model is used to estimate the cost of technical information.

We form a decomposition criterion based on two characteristics: the average connection intensity $\eta = \frac{\partial I}{\partial t}$ and the uniqueness of the module γ ($\gamma = 0$, if the module is unique; $\gamma = 1$, if the module is common) for a single element DCPS. Then we obtain the general criterion of decomposition using the cost approach:

$$q = {c_\eta \eta} \Big/ {c_\gamma \Delta_\gamma} \tag{1}$$

where c_η, c_γ are the cost coefficients of information exchange and complicated support when combining into one group of modules Δ_γ.

The physical interaction of the elements of the technological object is due to the gradual spread of influences from the control point.

Each element of the object is affected by all controlled elements. In [16] the model of interaction of elements of the distributed object is given. The differential equation of state of the j-th element of the object has the form:

$$\frac{dv_j(t)}{dt} = p_{0j}(t) + \sum_{\substack{k=1 \\ k \neq j}}^{n} \left\{ \frac{p_{0k}(t) + v_k(t) - v_j(t)}{8\left[\pi\lambda(t-t_k)\right]^{3/2}} e^{-\frac{d_k^2}{4\lambda t}} \times \left[1 + \left(\frac{d_k^2}{\lambda(t-t_k)} - 6\right) \cdot \frac{r_{0k}^2}{40\lambda t}\right]\right\}, \tag{2}$$

where j, k are the indexes of elements of the technological object; v is the state of the element; p_0 is control influence; r_0 is the average radius of the area of application of the control impact; d_k is the distance from the k-th to the j-th element of the technological object; λ is a parameter of influence distribution; t_k is the moment of influence of control on the k-th controlled element.

For remote control elements:

$$\lim_{r\to\infty} \Delta v(r,t) = \frac{v_k - v_j}{8(\pi\lambda t)^{3/2}} e^{-\frac{d_k^2}{4\lambda t}} \times \left(\frac{d_k^2}{\lambda t} - 6\right) \cdot \frac{r_0^2}{40\lambda t} = 0 \tag{3}$$

All elements of the object affect each other, but the sensitivity of the element at each input decreases exponentially depending on the spatial distance between the elements.

The limiting distance d_m to the elements, the impact of which is considered significant, satisfies the ratio:

$$\frac{e^{-\frac{d_m^2}{4\lambda\tau_k}}}{8(\pi\lambda\tau_k)^{3/2}} > \varepsilon \tag{4}$$

where ε is a significance indicator.

Thus, with decentralized coordination, the DCPS is divided into ε-areas (clusters) with centers in each controlled element of the distributed technological object. Moreover, the radius of the ε-areas depends on the ratio of the state parameters of neighboring elements. Accordingly, after the implementation of the coordination effect, the boundaries of the clusters change. This necessitates sliding clustering at the rate of the coordination wave.

We describe the topology of ε-areas by the matrix of adjacency of elements $\mathbf{S}$. Each line i of the matrix defines the set of elements in the ε-areas of i-th element. Then in vector-matrix form, the model DCPS (2) will look like:

$$\mathbf{V} = \mathbf{V}_0 + \mathbf{W\Lambda}(\mathbf{V} - \mathbf{S} \times \mathbf{V}) + \mathbf{W}_u u \tag{5}$$

where $\mathbf{W}$ is the diagonal matrix of the transfer functions of the LCS; $\mathbf{\Lambda}$ is matrix of coefficients of mutual influence of elements; $\mathbf{V}_0$ is a vector of element-state values specified by the coordinator.

In the coordination process, the central element of each cluster receives information about the status and parameters of the other elements of the cluster. Let's estimate the intensity of such exchange of information.

Each cluster is characterized by a vector of parameters $\{\mathbf{V}_\varepsilon, \mathbf{\Lambda}_\varepsilon, \mathbf{F}_\varepsilon, u, \mathbf{X}_\varepsilon, \mathbf{V}_{0\varepsilon}\}$, which are measured or set with a certain level of uncertainty.

Initial entropy of operation parameter values:

$$H_{O_i}^{(0)} = \log(D_{V_\varepsilon}) + \log(D_{\Lambda_\varepsilon}) + \log(D_{F_\varepsilon}) + \log(D_u) + \log(D_{X_\varepsilon}) + \log(D_{V_{0\varepsilon}}) = \log(D_{V_\varepsilon} D_{\Lambda_\varepsilon} D_{F_\varepsilon} D_u D_{X_\varepsilon} D_{V_{0\varepsilon}}) \tag{6}$$

where D is the range of parameter values.

If the standard errors of the parameters are respectively $(\sigma_{V_\varepsilon}, \sigma_{\Lambda_\varepsilon}, \sigma_{F_\varepsilon}, \sigma_u, \sigma_{X_\varepsilon}, \sigma_{V_{0\varepsilon}})$, then for the normal distribution of errors, ignoring the correlation between the parameters, we find the residual entropy of the state of the cluster O_i:

$$H_{O_i} = \log\left(\sigma_{V_\varepsilon} \cdot \sigma_{\Lambda_\varepsilon} \cdot \sigma_{F_\varepsilon} \cdot \sigma_u \cdot \sigma_{X_\varepsilon} \cdot \sigma_{V_{0\varepsilon}}\right) + C,$$

where C is a constant that depends on the type of error probability distribution. For the normal distribution $C = \frac{3}{2}\log(2\pi e)$.

Then the amount of information transmitted for coordination:

$$I_{O_i} = H_{O_i}^{(o)} - H_{O_i} = \\ = \log\left(\frac{D_{V_\varepsilon} D_{\Lambda_\varepsilon} D_{F_\varepsilon} D_u D_{X_\varepsilon} D_{V_{0\varepsilon}}}{\sigma_{V_\varepsilon} \sigma_{\Lambda_\varepsilon} \sigma_{F_\varepsilon} \sigma_u \sigma_{X_\varepsilon} \sigma_{V_{0\varepsilon}}}\right) - C \quad (7)$$

The parameter ranges are constants that are determined by the features of the DCPS. Standard errors are determined based on errors in measuring the states of elements $\mathbf{V}$ and the environment u, errors in setting the initial data $\mathbf{F}_\varepsilon$, $\mathbf{X}_\varepsilon$ and the system model. For example, from a model (5) we find:

$$\mathbf{V}_0 = \mathbf{V}\left(\mathbf{1} - \mathbf{W\Lambda} + \mathbf{W\Lambda S}\right) - \mathbf{W}_u u \quad (8)$$

Then:

$$\sigma_{\mathbf{V}_0} = \sigma_{\mathbf{V}}\left(\mathbf{1} + \mathbf{W\Lambda}\left(\mathbf{1} + \mathbf{S}\right)\right) + \mathbf{VW}\sigma_{\Lambda} + \mathbf{W}_u \sigma_u \quad (9)$$

where σ_{Λ} is the standard error of estimating the spreading parameters in module C_2.

To decompose the system into local tasks and microservices based on the intensity of the connection criterion of the hypergraph elements, we modify the Stoer-Wagner algorithm [17]. The algorithm constructs the optimal cut of the graph on the edges with the minimum total weight. The essence of the modification is that the assignment of a particular module to microservices changes the dimension of the hypergraph and the intensity of communication because the local part of the module serves the control system of one element DCPS, and microservice serves all local control systems.

The initial data of the modified algorithm are three matrices: topological matrix $\mathbf{S}$ adjacency of DCPS elements of dimension $[n,n]$, where n is the number of elements of the distributed technological object (level 1), matrix $\mathbf{A}$ of adjacency of LCS blocks (level 2) and adjacency matrix $\mathbf{C}$ coordinator modules (level 3); and the initial communication intensities according to (1). In the simplest case:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} C \\ LCS \\ E \end{pmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The main procedures of the modified algorithm:

1. Input matrices S, A, C.
2. Transformation of a three-level structure (S, A, C) into a one-level G with the dimension $[n \cdot (1+2+6), n \cdot (1+2+6)]$.
   2.1. Construction of the matrix G as a block-diagonal one, each block of which is formed from matrices (A, C) and has dimension $[8,8]$;
   2.2. Arranging relationships between blocks according to the matrix S;
   2.3. Sort rows and columns of the matrix G so that the rows and columns of the same type of modules go sequently.
3. Combine elements E into a subgraph of local elements.
4. Finding the module with the minimum intensity of connections and assigning it to the subgraph of microservices.
5. Collapse the graph: add all rows and columns that correspond to the module related to microservices, and reduce the dimension of the graph by $(n-1)$.
6. While not all modules are connected to any of the two groups, then:
   6.1. Find the next connection with the highest intensity of connections with any element that is already attached to one of the two groups and join it to the group.
   6.2. If this group is microservices, then
      6.2.1. Collapse the graph: add all rows and columns that correspond to the module related to microservices, and reduce the dimension of the graph by $(n-1)$.
7. End.

As a result of the conducted researches, the typical architecture of a system of coordination control of DCPS with the use of microservices is shown on Fig. 3.

The architecture is based on multi-agent technology. Agents implement an algorithm of interaction and decentralized coordination, and part of the functions is determined by the optimal partition of the hypergraph, implemented in the form of microservices.

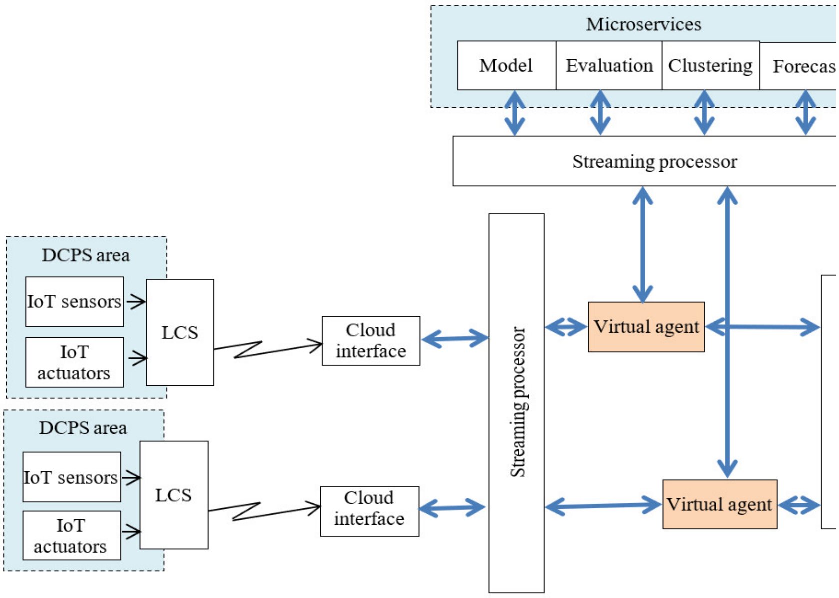

Figure 3. Architecture of coordination system using microservices

V. CONCLUSION

The proposed formal approach to the decomposition of the architecture of the system of decentralized coordination control of the DCPS with the use of microservices allows minimizing the cost of creating and maintaining the system. For this approach, the convertion of system gipergraph to optimal bipartite graph by the criterium of minimize interconnection flows and modification of Stoer-Wagner algorithm were been proposed. The authors see the prospects for further research in improving the information model, taking into account the use of intelligent technologies in the implementation of the main modules of the coordination system.

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