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# СИНЕРГІЯ МАТЕМАТИЧНОГО АПАРАТУ ТА ШТУЧНОГО ІНТЕЛЕКТУ

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**Анотація**

*У роботі надано перелік математичних методів, що застосовуються при розробці штучного інтелекту. Описано основні їхні концепти та шляхи їх застосування.*

**Ключові слова:** штучний інтелект, математика, лінійна алгебра, машинне навчання, математичний аналіз, теорія ймовірностей, статистика, теорія графів.

**Abstract**

*The work provides a list of mathematical methods used in the development of artificial intelligence. The main concepts and ways of their application are described.*

**Keywords:** artificial intelligence, mathematics, linear algebra, machine learning, calculus, probability theory, statistics, graph theory.

## Introduction

Nowadays, artificial intelligence (AI) has become essential, since we live in an era, where it is applied more and more in different aspects of life, from medicine to automobiles and factories. Companies tend to replace people with artificial intelligence in tasks, where great precision is needed or human work is more expensive than artificial intelligence. To implement AI in different spheres of life, a complete description of human behavior, language, and way of thinking is needed. Humans can act differently according to the situation, and so must AI. In order to meet these requirements, artificial intelligence is based on a combination of mathematical, statistical, and computer science techniques. Mathematics plays a crucial role in the development of AI, providing the theoretical foundation for machine learning, natural language processing, and other AI techniques. The **purpose** of this work is to analyze the mathematical methods used in the development of artificial intelligence and their perspectives.

## Results of investigation

**Mathematics and Machine Learning**: Machine learning is a subset of AI that focuses on the development of algorithms and models that allow computers to learn from data. The mathematical concepts that underlie machine learning include linear algebra, calculus, probability theory, and statistics. Machine learning plays a key role in AI, as it allows artificial intelligence to dynamically gather information. Unlikely from programming, where computers learn from predefined instructions, in machine learning the information is represented in such a way, that lets machines learn upon the data being presented. [1]

**Linear Algebra**: Linear algebra is used to represent data in a structured way, making it possible to perform operations on large datasets efficiently. For example, matrices and vectors are used to represent images, text, and other types of data in a way that can be manipulated by machine learning algorithms. Vectors are used to represent words for natural language problems. Linear algebra is used in dimensionality reduction to deal with large-dimensional data. Eigenvectors and eigenvalues are the concepts of linear algebra, which allow reducing the number of dimensions of data by transforming multi-dimensional arrays into one-dimensional vectors such that $A\nu = \lambda\nu$, where A is a matrix, $\nu$ is the eigenvector and $\lambda$ is the eigenvalue. Recommendation engines are

based on vector embeddings and matrix factorization. Matrix factorization is basically breaking a matrix into smaller ones.

**Calculus:** Calculus is the study of continuous change. Calculus is a branch of mathematics that deals with the study of rates of change and how things change over time. It involves the analysis of functions, which are mathematical entities that describe how one variable depends on another. Calculus provides a framework for understanding how these functions change, allowing us to make predictions and solve problems in fields such as physics, engineering, economics, and more. Calculus has two main branches: differential calculus and integral calculus. Differential calculus deals with the study of the instantaneous rate of change of a function at a given point, which is known as the derivative. Integral calculus, on the other hand, deals with the study of the accumulation of small changes over an interval of time, which is known as the integral. Together, differential and integral calculus form the foundation of modern mathematical analysis. They are used to solve a wide variety of problems, from predicting the trajectory of a rocket to understanding the behavior of financial markets. Calculus is used to optimize machine learning algorithms by minimizing or maximizing a particular function. This involves taking derivatives and calculating gradients, which are used to update model parameters and improve performance. For example, gradient descent is a popular optimization algorithm used in machine learning. Instead of searching every single layer for an appropriate element, gradient decend is used to optimize the process of learning and finding the answer based on the material learned.

**Probability Theory and Statistics**: The basic concept of probability theory is the probability of an event, which is a number between 0 and 1 that represents how likely an event will occur. An event with a probability of 0 means it will never happen, while an event with a probability of 1 means it is certain to happen. The probability of an event between 0 and 1 represents the degree of uncertainty about whether the event will happen or not. Probability theory has many practical applications in various fields, such as statistics, economics, engineering, and finance. It is used to model and analyze complex systems, to make predictions about future events, and develop statistical methods for data analysis. Probability theory and statistics are used to model uncertainty and measure the reliability of predictions. For example, Bayesian statistics are used to estimate probabilities and make predictions based on incomplete or uncertain data. [2] Bayesian theory states, that probability strongly depends on the degree of belief rather than the frequency of occurrence. For example, spam filters are based on AI, which uses Bayesian statistics to calculate the posterior probability of spam or not spam given the observed data. Bayesian methods are also used for regression tasks, such as predicting a numerical value based on a set of input variables. In this case, Bayesian methods can be used to model the uncertainty in the predictions, which can be useful for decision-making or risk evaluation. We can ask the most popular AI ChatGPT whether Bayesian statistics is used in its development:

Question: “is Bayesian statistics used in developing ChatGPT?”

Answer: “Yes, Bayesian statistics is used in developing ChatGPT, along with other statistical and machine learning techniques.”

**Graph Theory**: Graph theory is a branch of mathematics that deals with the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph consists of a set of vertices (also called nodes) and a set of edges, which are connections between pairs of vertices. Graph theory has a wide range of applications in many fields, including computer science, operations research, social sciences, and more. Some common problems that can be solved using graph theory include finding the shortest path between two points in a network, identifying clusters of related objects, and optimizing the flow of resources through a network. The study of graph theory involves understanding the properties of graphs, such as connectivity, planarity, and colorability, and developing algorithms to solve problems related to these properties. Graph theory has many important applications in real-world systems, including transportation networks, communication networks, and social networks. Graph theory is used in machine learning for data representation and analysis. Graphs are used to represent complex data structures such as social networks, chemical compounds, and biological systems. Machine learning algorithms can then be used to identify patterns and relationships within these graphs.[3]

## Conclusion

Overall, AI relies heavily on mathematical concepts to develop algorithms and models that can learn and make decisions. Artificial intelligence is expected to behave like human beings. To create a such system, we need even more mathematical concepts to describe all the nuances of decision-making, problem-solving, abstraction and learning. As AI continues to advance, it is likely that more and more mathematical concepts will become important in developing more sophisticated and intelligent systems.

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