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# Functional Dependability of Distributed Control of Multi-zone Objects under Failures Conditions

**Volodymyr Dubovoi [1], Senior Member, IEEE, Maria Yukhimchuk [1], Viacheslav Kovtun [1], Krzysztof Grochla [2]**

[1]Vinnytsia National Technical University, Khmelnitske Shose, 95, Vinnytsia, 21000, Ukraine

[2]Institute of Theoretical and Applied Informatics Polish Academy of Sciences, Bałtycka 5, 44-100 Gliwice, Poland

Corresponding author: Viacheslav Kovtun, kovtun_v_v@vntu.edu.ua

**ABSTRACT** The significance of the functional dependability problem of distributed control systems is constantly increasing due to the spread of inexpensive controllers and sensors and the creation of Internet of Things (IoT) systems. In the case of distributed control of multi-zone objects with the mutual influence of zones, the problem is complicated by changes in the operating modes of local control systems (LCS) due to the failures of elements of the systems of neighbouring zones. This, in turn, leads to a change in their failure rates. In this work, the functional dependability model of the decentralized distributed control system for multi-zone objects was developed and investigated. The model is obtained in the form of an inhomogeneous Markov chain of the system states caused by failures of elements of LCS. The probabilities of transitions between states are described by the Chapman-Kolmogorov equations and additional equations that describe the relationship between the parameters of neighbouring zones and the corresponding LCS. The impact of Data Transmission System (DTS) overload, controller failures, sensor failures, and DTS failures on the probability of the zones' state exceeding the permissible limits was studied. The functional safety assessment model of decentralized control of multi-zone objects, proposed in the previous works of the author was used for the theoretical analysis. Verification of the theoretical model is achieved through simulation, leveraging a library of modules developed by the authors on the Scilab/Xcos platform. The simulation showed that the "cascade failures" process is probable for multi-zone objects with coordinated control according to a global criterion and a strong connection between the parameters of the zones, which leads to the rapid loss of functional dependability.

**INDEX TERMS** functional dependability, multi-zone object, decentralized coordination, distributed control system.

## I. INTRODUCTION

The dependability of distributed control systems (DCS) is the subject of many studies. The large number of components of such systems leads to a high probability of failures of some of them, especially those under the direct influence of the external environment – sensors and regulators. However, in the case of distributed control of multi-zone objects, the failure of one or even several local control systems does not necessarily lead to an instant complete failure of the system as a whole. Local control systems (LCS) of zones surrounding a failed LCS zone can partially compensate for such a failure. At the same time, indicators of the quality of DCS functioning deteriorate, and such deterioration can gradually lead to a loss of functional dependability. In [1], the functional dependability of a system is defined as a set of its properties that determine the ability of the system to correctly perform tasks with an acceptable level of accuracy, while the output results are within acceptable limits. This definition corresponds to the standard [2].

The relevance of the problem of functional dependability of DCS is constantly increasing due to the spread of inexpensive controllers and sensors and the creation of Internet of Things systems. In the case of distributed control of multi-zone objects with the mutual influence of zones, the problem is complicated by changes in the operating modes of local control systems due to the failure of elements of the systems of neighbouring zones. This, in turn, leads to a change in their failure rates. As a result, the phenomenon of "cascade failures" may occur, which quickly leads to a loss of functional dependability. The study of relationships between the parameters of multi-zone objects and decentralized DCS, which can lead to a cascade of failures and loss of functional dependability, is an urgent scientific task.

### A. STATE-OF-THE-ART

One of the main parameters of any technical system is its reliability (or dependability). Dependability is the ability of the system to be ready to perform its functions, in particular, to continue working during failures of its constituent parts.

The problem of the dependability of technical systems is complex and it is considered in various aspects. The classical theory of reliability was developed to ensure the operability of technical systems [3, 4]. It used the concept of "structural reliability" to define that part of the general theory of reliability, which is aimed at researching the processes of failure and restoration of technical objects. The standard [2] expands the interpretation of reliability. It uses the complex concept of "dependability". It is a measure of a system's availability, reliability, maintainability, durability, safety and security [5]. In many sources, you can find both studies of the main aspects of dependability: classical, fundamental and modern, applied:

– reliability – absence of failures, continuity of functioning [6, 7];
– availability – readiness for correct functioning [7];
– survivability – the ability to minimize the loss of quality and maintain the ability to perform functions in the event of failures caused by internal and external causes [14];
– safety – absence of catastrophic consequences for the user(s) and the environment [8, 9, 10];
– integrity – absence of incorrect system changes [15];
– confidentiality – the absence of unauthorized disclosure of information [6];
– high confidence – the possibility of correctly assessing the quality of services, i.e. determining the level of trust in the service [6];
– maintainability – the ability to undergo refinement and repair [7]. For a maintainable system, dependability analysis should take into account the result of component maintenance. For a maintenance-free system, the analysis and calculation of dependability is simpler, since component maintenance can be neglected [12];
– security – protection against unauthorized access, use, disclosure, violation, modification or destruction [14].

The analysis of the mentioned works regarding the content and consequences of the deterioration of these indicators shows that most of them are mutually related, therefore, the violation or improvement of one indicator leads to a change in the entire system of indicators and dependability in general.

Different models and tools are used to study phenomena related to functional dependability. The dependability flowchart tool is designed to quantify dependability risks in large systems, in particular, in energy [13]. These works are mainly focused on the dependability analysis and assessment of the availability of electricity, automatic control systems, etc. Markov models are the main mathematical apparatus for studying the influence of random factors on dependability [17, 29].

Distributed control systems [16] as a separate class of systems have been studied relatively recently. For such systems, the concept of "functional dependability" is used more often [11]. All the indicators mentioned above are also aspects of functional dependability, however, in DCS, the deterioration of a certain indicator can lead to other consequences. In particular, depending on the structure of the system, the deterioration of one of the indicators of one of the RSC subsystems can either be compensated by other subsystems, i.e. preserve functional dependability, or lead to an exponential deterioration of all indicators - the phenomenon of "cascade failures". Various methods of preventing cascade failures caused by the increase in the failure rate are being studied, in particular, various methods of introducing redundancy and diagnostics [39, 40].

The performed analysis showed that the study of the problem of functional dependability of distributed control systems to find ways to increase it and prevent the negative phenomenon of cascade failures is important and far from being a problem completely solved.

### B. RELATED WORKS

Problems of dependability of distributed technical systems have been the focus of attention of many researchers in recent years. Most of the works, for example [19, 20] are devoted to the dependability of large life support systems: energy, water and heat supply, transport, etc. The peculiarity of such systems is noted, which is determined by their ability to function in conditions of partial failures. In case of a partial failure caused by the failure of one of the system components, the normal functionality of some of its elements is disrupted. Nevertheless, the system continues to perform its functions. Actually, in such large systems, the probability of complete absence of failures of all its components approaches zero. The recovery process is continuous and during this process the distributed system must continue to operate, providing fault tolerance and maintaining a certain level of functionality.

A sharp increase in the flow of research on the dependability problem of spatially distributed systems and networks occurred after the blackout in the USA in 2003 [21], where the server software failure was the cause of the global system failure.

In work [22], a thorough review of the works for 10 years after the blackout on the analysis and forecasting of the dependability of distributed software systems and their impact on large-scale networks was carried out. Research on the dependability of distributed software systems, in particular, taking into account server virtualization, continues and develops even now [24]. In [27], the authors propose an approach to ensuring the dependability, availability, safety and security of distributed systems using virtualization and replication of services.

In work [23], a review and generalization of achievements in the analysis of the dynamics of the so-called "cascade failures", which lead to the loss of functionality of a large system due to individual failures of its elements, was performed. Cascade failures in networks with flow redistribution were studied in [25].

The authors of the study [26] analyzed the possibilities of preventing cascade failures in the presence of redundancy in large networks.

Although, as mentioned above, there are many works on the study and provision of dependability of distributed systems, however, unlike the study of spatially distributed networks, there are relatively few works on the dependability of distributed control systems of technological objects. The authors of this work investigated the functional safety of decentralized coordination of distributed cyber-physical objects [30]. However, the article [30] considered only the influence of random factors on a multi-zone technological object with a decentralized system of coordination of the DCS. The impact of possible failures of system elements was not considered.

It is possible to pick out several reasons for the violation of functional dependability of DCS [18]:

- network overload (depending on the data exchange algorithm);
- refusal of DTS;
- failure of the regulator (depending on the state at the time of failure);
- sensor failure;
- exceeding the rate of change of the state of the zones of the object of the calculated value;
- exceeding the variance of the specified state of the zones of the object of the calculated value;
- external intervention in the operation of the control system

etc.

Several studies are devoted to the dependability of DCS based on the Internet of Things [31], the reliability of sensors [32], etc. The authors [28] investigate the ways to increase the dependability of the DCS in conditions of possible controller failures and propose a software definition of the system topology by changing the structure of connections and redistributing the load in the event of a controller failure.

In most of the works on the dependability of the DCS, the performance of applying a model-based approach to the analysis and ensuring the dependability of the DCS is emphasized [33]. The authors of this work in study [30] also used a model-based approach regarding the functional safety of decentralized coordination of distributed multi-zonal objects. The main components of this approach are formulated in the article [34]. In this study, we will also use models from work [34] to develop research [30] and obtain dependability estimates of distributed control of multi-zone objects in the conditions of failures, identifying problems and ways to increase functional dependability.

### *C. OBJECTIVES AND PROBLEMS*

Let us formulate the main provisions of our research.

The object of the study is the functional dependability of distributed decentralized control of multi-zone objects (MZO) under conditions of probable failures of control system elements.

Taking into account the above-mentioned features of the use of the concept of “dependability” to characterize the control of multi-zonal objects, we formulate the goal of the study as the creation of a theoretical basis for analyzing the factors of violation of the functional dependability of the distributed control of multi-zonal objects.

In this paper, we propose a new model for studying and assessing the functional dependability of decentralized distributed control of MZO, based on the analysis of the mutual influence of MZO zones and the coordination of local control systems.

The main contribution of the study is the methodology for studying the functional dependability of MZO distributed control under multiple failure conditions, taking into account its topology and control system structure.

The main points of the study are as follows:

1) A methodology is proposed for studying the functional dependability of distributed control of MZO under conditions of multiple failures, taking into account its topology and the structure of the control system.

2) A formal model has been developed to study and evaluate the functional dependability of decentralized distributed control of MZO.

3) A system of basic elements has been developed for the simulation of distributed MZO control systems under conditions of element failures.

4) The results of the simulation are analyzed using the example of a distributed biotechnological facility.

## II. MATERIALS AND METHODS

### *A. STATEMENT OF THE RESEARCH*

### *A. STATEMENT OF THE RESEARCH*

Let's consider a multi-zone spatially distributed object in which a given state of zones must be maintained. As an example, let us consider a facility for the production of biological products - a multi-zone greenhouse [37]. Each zone is controlled by a local control system. Control is carried out taking into account the compromise between local criteria of optimality and global criterion $E$. Coordination of local systems is carried out for this purpose. This study considers a system with decentralized coordination [34], which assumes the presence of coordinators in each LCS. LCS coordinators of individual zones exchange information on the status and parameters of zones, carry out decentralized coordination and set the optimal setpoint for the LCS regulator.

Let the $n$ is the number of zones of a multi-zone object; $v_i$ is the state of the $i$-th zone; $f_i$ is the given state of the $i$-th zone; $v_{0i}$ is the optimal setpoint for the regulator. Then the actual and set states of the MZO form the vectors $\mathbf{V}$ and $\mathbf{F}$, for which the Manhattan metric L1 is defined. The state vector $\mathbf{V}$ depends on the vector of settings $\mathbf{V}_0$ and MZO parameters according to the systems model. The coordinators calculate the optimal setting based on the criterion

$$E = \min_{v_{0i}} \sum_{j \in \Omega_\varepsilon} \rho_j \left| f_j - v_j \right| \qquad (1)$$

where $\Omega_\varepsilon$ is the set of nearby zones ($\varepsilon$-area [34]); $\rho_j$ are compromise coefficients. Here and further the index $\varepsilon$ means the subset of elements belonging to the $\varepsilon$-area $\Omega_\varepsilon$.

The control of the state of the zone is carried out by providing it with a certain resource $p_i$ and then $\frac{dv_i}{dt} = \frac{1}{C_i} p_i(t)$, where $C_i$ is the parameter of the zone (resource intensity). The resources accumulated in the zone can be spent on productive processes in the zone according to the production function $y(v)$ or flow to neighbouring zones and the surroundings. Figure shows the approach to determining the functional dependability of the system under investigation. Fig. 1a shows a typical production function of a biotechnological object [35, 36], which has a critical state value $v_{cr}$ the excess of which leads to the collapse of the system. Fig. 1b shows the probability distribution of the state of the object $\varphi(v)$, which arises as a result of the action of the external environment, errors of the control system, etc. In multi-zone objects, the mathematical expectation $v_0$ of this distribution is determined by the coordinator and may differ from the given one due $f$ to the global optimization of the system.

The probability of a violation of functional dependability is determined by the area of the shaded part of the distribution. Following the purpose of this work, we will investigate the impact of element failures of LCS on distribution $\varphi(v)$ and, accordingly, on functional dependability.

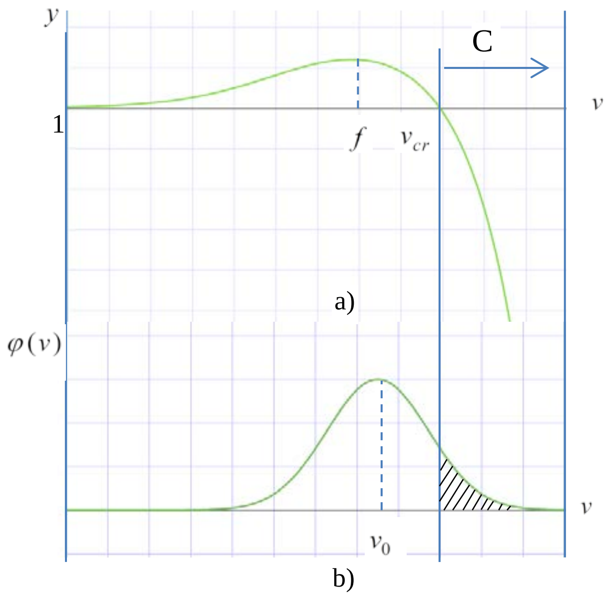



Figure 1. Definition of functional dependability

### B. MATHEMATICAL MODEL OF THE STUDIED PROCESS

Based on the statement of the problem, we will estimate the functional dependability of the MZO distributed control as the probability of the state of all MZO zones being within the permissible limits

$$P_{fr} = \int_{\mathbf{V} \in \Omega_{fr}} \varphi_{\mathbf{V}}\left(v_1, \ldots v_i, \ldots v_n\right) dv_1 \ldots dv_i \ldots dv_n \qquad (2)$$

where $\mathbf{V} = \{v_1, \ldots v_i, \ldots v_n\}$ is the vector of states of zones of a multi-zone object; $n$ is the number of zones of MZO; $\varphi_{\mathbf{V}}(v_1, \ldots v_i, \ldots v_n)$ is the n-dimensional probability distribution of MZO zone states; $\Omega_{fr} = \left\{ v_i \mid v_i \in \left( v_{i\,\min}, v_{i\,\max} \right), i = 1 \ldots n \right\}$ - the range of values of the state vector of the MZO zones, where the operation of the MZO is considered reliable.

We have analyzed the influence of four uncertainty factors of coordination control on functional safety (Table 1) in the work [30]. In the MZO system with $n$ zones controlled with LCS, the number of uncertainty factors is equal to $4n$. Failures of major LCS elements create additional factors of uncertainty. A large number of influential factors gives grounds for the hypothesis about the Gaussian nature of the multidimensional distribution $\varphi_{\mathbf{V}}(v_1, \ldots v_i, \ldots v_n)$.

Let's investigate the effects on probability (2) under the condition $\varphi_i\left(v_i / \left[\mathbf{V} \setminus v_i\right]\right) = G\left(v_i, m_{vi}, \sigma_{vi}\right)$ is the conditional Gaussian distribution of the total error of $i$-th coordinate of vector $\mathbf{V}$ (in further presentation, we will omit the index '$i$' and the condition $\left[\mathbf{V} \setminus v_i\right]$ in the designation of the conditional probability distribution to simplify the notation); $\sigma_v = \sqrt{\sigma^2_{v/\lambda} + \sigma^2_{v/u} + \delta^2_{v/d_{kj}} + \sigma^2_{v/v_0} + \sigma^2_{v/m}}$ ), where $m_v = v_0$ is the mathematical expectation of the probability distribution; $\sigma^2_{v/m}$ is the additional deviation of the state of zones caused by failures of LCS elements.

TABLE 1
ESTIMATES OF THE IMPACT OF COORDINATION CONTROL UNCERTAINTY FACTORS ON DISTRIBUTION VARIANCE

| | | |
|---|---|---|
| Random fluctuations in the propagation parameter $\lambda$ | $\sigma^2_{v_j/\lambda} = \left(\frac{\partial v_j}{\partial k_{ij}} \sigma_{k_{ij}}\right)^2$ | $\sigma^2_{k_{ij}}$ is the dispersion of propagation parameter fluctuations; $k_{ij}$ is the coefficient of mutual influence of the zones |
| Random effects of the external environment on the state of the elements $u$ | $\sigma^2_{v_j/u} = \left(\frac{\partial v_j}{\partial u} \sigma_u\right)^2$ | $\sigma^2_u$ - deviation of influences. |
| A systematic error caused by the spatial discretion of affecting a distributed object | $\delta_{v/d_{kj}}\left(d_{ij}, t\right) = \frac{2}{d_{kj}} \int_0^{d_{kj}/2} \left[v_i\left(d_{ij}, t\right) - v_j(t)\right] d\left(d_{ij}\right)$ | In case of $k_{ij}(d_{ij}) = const$ where $d_{ij}$ is integration variable that is the distance from j-th affecting point to i-th point of object; $d_{kj}$ is the distance from k-th to j-th elements. |
| Dynamic errors due to control impact in case of wave coordination algorithm | $\sigma_{v/v_0} = \int_0^{\infty} \frac{2\overline{\Delta v_0}}{\omega T} \cdot \left\lvert \frac{W_r(j\omega) W_{ob}(j\omega)}{1 + W_r(j\omega) W_{ob}(j\omega)} \right\rvert d\omega$ | $\overline{\Delta v_0}$ is the average correction of the coordinator; $T_{co}$ is the period of the coordination wave; $W_r(j\omega)$ is the transfer function of a regulator |

### *C. THE MODEL OF THE INFLUENCE OF ELEMENT FAILURES ON THE DEVIATION OF THE STATE OF ZONES*

*Phenomenological description*

The structural and logical scheme of the study is shown in Fig. 2. The diagram represents the conditions and consequences of failures of LCS elements.

The MZO decentralized distributed control system with $n$ zones consists of $n$ LCS and each one contains a controller, sensor and coordinator. The work of the coordinator is based on the exchange of information with other coordinators and the optimization of the setpoints for the regulator according to the local-global criterion [34].

Considering failures and halting of LCS elements without taking into account recovery, we will highlight the following processes:

- Permanent (regulator failure, sensor failure, communication failure);
- Temporary (communication halting).

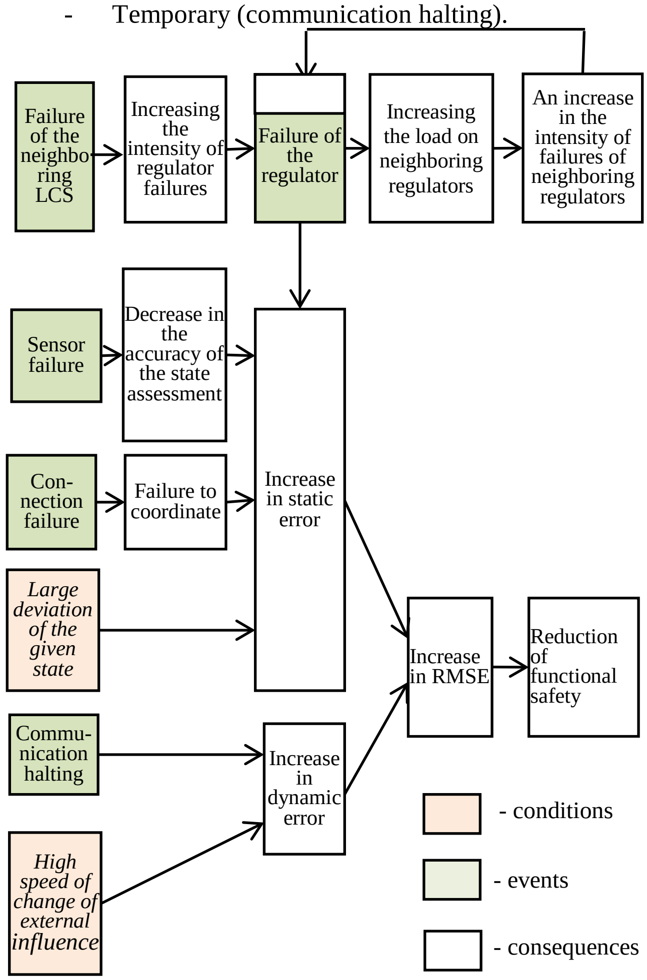



Figure 2. Structural and logical diagram of the failures impact study

The peculiarity of MZO control systems is the mutual influence of zones and the influence of failures of LCS elements of one zone on the state and dependability characteristics of other zones accordingly. In the structural-logical scheme, this influence is reflected by the dependence of the failure rate of the regulator on the load on them, which, in turn, depends on the performance of the LCS of neighbouring zones.

The states of an individual LCS form a Markov chain since failure events of LCS elements do not depend on the previous history. However, in the MZO, this chain becomes heterogeneous, since the transition probabilities between states can change depending on the LCS states of neighbouring zones.

Let's investigate the heterogeneous Markov chain of LCS states under the conditions of failure of the main elements.

Each LCS of the distributed control system can be in the $n = 2^4 = 16$ following states $S_i\left\{s_{i1}, s_{i2}, s_{i3}, s_{i4}\right\}$: $S_0$ means all elements are working; $S_1$ - $S_4$ mean one of the possible processes of failures or failures has occurred; $S_5$ - $S_{15}$ mean are their combination. All elements of the LCS do not work in the final state $S_{11}$. The corresponding graph of transitions between states is shown in Fig. 3. A certain asymmetry of the graph is caused by the fact that a communication halting can be observed only with a functioning communication device. As a result of failure processes and communication halting/restorations, 40 transitions between states are possible, which have different consequences from the point of view of the functional dependability of the system. We will describe these processes by a matrix of transition probabilities between states $\mathbf{P}_{SS}[16;16]$, which has 40 nonzero elements $P_{ij}$. The set of possible LCS states forms a vector $\mathbf{P}_S[1;16] = \left\{P_{s_0}, P_{s_1}, \ldots, P_{s_{15}}\right\}$.

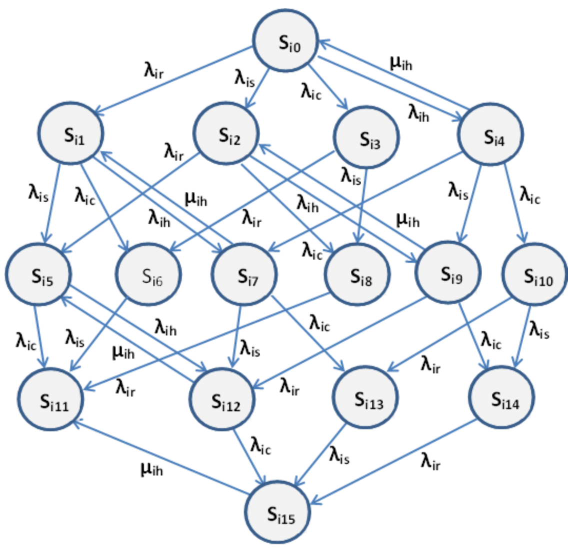



Figure 3. Tree of LCS states and transitions

Analysis of the state graph shows that the longest sequence of failure processes leading to a deadlock state or one of the previous states consists of 5 steps. Therefore, the Markov chain of a separate LCS contains 5 transitions. We characterize the state of the $i$-th LCS by a vector of probabilities $P_{S_i}\left[p_{ir}, p_{is}, p_{ic}, p_{ih}\right]$, where $p_{ir}$ is the probability of the regulator working; $p_{is}$ is the probability of sensor performance; $p_{ic}$ is the probability of serviceability of communication; $p_{ih}$ and is the probability of no communication halting.

The initial state of the LCS $\mathbf{P}_S(t=0)$ characterized by probabilities $P_{S_0} = 1$, $\forall P_{S_{i\neq 0}} = 0$.

Let's create a system of Chapman-Kolmogorov equations for a distributed control system. It will contain $n+1$ blocks of equations, where $n$ is the number of zones: $n$ blocks of 16 equations of changes in LCS states of each zone and 1 block of equations of mutual influence of zones. The system of equations for the $i$-th zone, where $i = 1 \ldots n$, corresponds to the graph of transitions in Fig. 3:

$$
\begin{aligned}
\frac{dP_{s_{i0}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_s + \lambda_c + \lambda_h\right) P_{s_{i0}}(t) + \mu_h P_{s_{i4}}(t) \\
\frac{dP_{s_{i1}}(t)}{dt} &= -\left(\lambda_s + \lambda_c + \lambda_h\right) P_{s_{i1}}(t) + \lambda_{ir}(v) P_{s_{i0}}(t) + \mu_h P_{s_{i7}}(t) \\
\frac{dP_{s_{i2}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_c + \lambda_h\right) P_{s_{i2}}(t) + \lambda_s P_{s_{i0}}(t) + \mu_h P_{s_{i9}}(t) \\
\frac{dP_{s_{i3}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_s\right) P_{s_{i3}}(t) + \lambda_c P_{s_{i0}}(t) \\
\frac{dP_{s_{i4}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_s + \lambda_c + \mu_h\right) P_{s_{i4}}(t) + \lambda_h P_{s_{i0}}(t) \\
\frac{dP_{s_{i5}}(t)}{dt} &= -\left(\lambda_c + \lambda_h\right) P_{s_{i5}}(t) + \lambda_s P_{s_{i1}}(t) + \lambda_{ir}(v) P_{s_{i2}}(t) + \mu_h P_{s_{i12}}(t) \\
\frac{dP_{s_{i6}}(t)}{dt} &= -\lambda_s P_{s_{i6}}(t) + \lambda_c P_{s_{i1}}(t) + \lambda_{ir}(v) P_{s_{i3}}(t) \\
\frac{dP_{s_{i7}}(t)}{dt} &= -\left(\lambda_s + \lambda_c + \mu_h\right) P_{s_{i7}}(t) + \lambda_h P_{s_{i1}}(t) + \lambda_{ir}(v) P_{s_{i4}}(t) \\
\frac{dP_{s_{i8}}(t)}{dt} &= -\lambda_{ir}(v) P_{s_{i8}}(t) + \lambda_c P_{s_{i2}}(t) + \lambda_s P_{s_{i3}}(t) \\
\frac{dP_{s_{i9}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_c + \mu_h\right) P_{s_{i9}}(t) + \lambda_h P_{s_{i2}}(t) + \lambda_s P_{s_{i4}}(t) \\
\frac{dP_{s_{i10}}(t)}{dt} &= -\left(\lambda_{ir}(v) + \lambda_s\right) P_{s_{i10}}(t) + \lambda_c P_{s_{i4}}(t) \\
\frac{dP_{s_{i11}}(t)}{dt} &= \lambda_c P_{s_{i5}}(t) + \lambda_s P_{s_{i6}}(t) + \lambda_{ir}(v) P_{s_{i8}}(t) + \mu_c P_{s_{i15}}(t) \\
\frac{dP_{s_{i12}}(t)}{dt} &= -\left(\lambda_c + \mu_h\right) P_{s_{i12}}(t) + \lambda_h P_{s_{i5}}(t) + \lambda_s P_{s_{i7}}(t) + \lambda_{ir}(v) P_{s_{i9}}(t) \\
\frac{dP_{s_{i13}}(t)}{dt} &= -\lambda_s P_{s_{i13}}(t) + \lambda_c P_{s_{i7}}(t) + \lambda_{ir}(v) P_{s_{i10}}(t) \\
\frac{dP_{s_{i14}}(t)}{dt} &= -\lambda_{ir}(v) P_{s_{i14}}(t) + \lambda_c P_{s_{i9}}(t) + \lambda_s P_{s_{i10}}(t) \\
\frac{dP_{s_{i15}}(t)}{dt} &= -\mu_h P_{s_{i15}}(t) + \lambda_c P_{s_{i12}}(t) + \lambda_s P_{s_{i13}}(t) + \lambda_{ir}(v) P_{s_{i14}}(t)
\end{aligned}
\qquad (3)
$$

where $\lambda_{ir}(v)$ is the failure rate of the controller as a function of state; $\lambda_s$ is the failure rate of the sensor; $\lambda_c$ is the failure rate of communication; $\lambda_h$ is the intensity of communication haltings; $\mu_h$ is the ability to restore connections after haltings.

Each equation of system (3) describes the change in the probability of being in a certain state: from the moment of transition to this state to the moment of leaving it. On this interval, the functions $P_{S_{ik}}(t)$ are continuous and differentiable. However, during the stay of the *i*-th object in the state $S_{ik}$, changes in the state of other zones of the distributed system are possible, which, due to the mutual influence of the elements, lead to a change in the coefficients of the corresponding Chapman-Kolmogorov equation. Thus, for the problem under consideration, at least a part of the Chapman-Kolmogorov equations is nonstationary. When solving the problem numerically, this is not an obstacle, since stationarity is continuing during intervals of constant states of all elements of the DCS.

Let's analyze the processes of mutual influence of zones, which determine an additional block of equations.

To take into account the mutual influence of the zones of a multi-zone object on the failure rate, we write down the linear dependence of the failure rate of the regulator on the load [38]

$$\lambda_r(v) = k_p \lambda_{r0}, \tag{4}$$

where $\lambda_{r0}$ is the failures rate of regulator in idle mode; $k_p$ is the load factor, which is a linear function of the load $p$ at a low load

$$k_p = 1 + k_{p0} \cdot p = 1 + k_{p0} \left| v - v_0 \right|. \tag{5}$$

For a distributed system in general, equations (4) and (5) will be written in vector form

$$\mathbf{\Lambda}_r(v) = \left(\mathbf{1} + \mathbf{K}_{p0}\mathbf{p}\right)\mathbf{\Lambda}_{r0}. \tag{6}$$

Equation (6) is one of the equations of the additional block of the Chapman-Kolmogorov system.

Let's define the topological matrix $\mathbf{K}$ with dimension $[n,n]$. The elements $k_{ij}$ of the $i$-th row of the state matrix are the coefficients of the influence of the state of the zone $j$ on the state of the zone $i$ (equation (19) in [30]). According to the principle of short-range action [30], $k_{ij} = 0$ for all zones for which the condition $\frac{k_{ij} C_i \left| v_j - v_i \right|}{v_i} \geq \varepsilon$ is not true, where $\varepsilon$ is the significance criterion.

The deviation $\Delta_i = v_i - v_{0i}$ depends on the states of the surrounding zones $v_{j, j \in n_{\varepsilon i}}$. We will use the structural diagram of the LCS in Fig. 4 to obtain the influence equation $\Delta_i\left(v_{j, j \in n_{\varepsilon i}}\right)$.

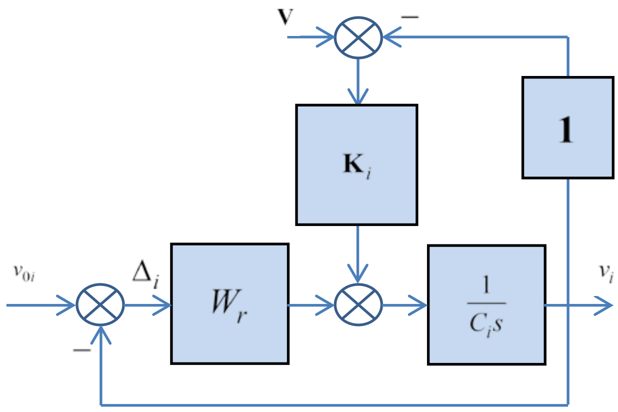



**Figure 5. Structural diagram of LCS**

We use the assigns on the diagram: $W_r$ is the transfer function of the LCS regulator; $\mathbf{K}_i$ is the $i$-th column vector of the topological matrix; $1/C_i s$ is the transfer function of the $i$-th MZO zone; $\mathbf{V}$ is the row vector of zone states; $v_{0i}$ is the state of the zone given by the coordinator $i$-th zone; $v_i$ is the actual state of the $i$-th zone; $\mathbf{1}$ is a unit vector. According to the schematic diagram $\Delta_i$ the Laplace deviation image.

$$\Delta_i(s) = \frac{v_{0i}\left(C_i s + \mathbf{1} \cdot \mathbf{K}_i\right) - \mathbf{V} \cdot \mathbf{K}_i}{C_i s + \mathbf{1} \cdot \mathbf{K}_i + W_r}.$$

Then we have a stationary mode

$$\Delta_i(s) = v_{0i} \left| \frac{1}{1 + \dfrac{W_r}{C_i s + \mathbf{1} \cdot \mathbf{K}_i}} \right| - \mathbf{V} \cdot \mathbf{K}_i \left| \frac{1}{C_i s + \mathbf{1} \cdot \mathbf{K}_i + W_r} \right|.$$

Let's define two matrices with dimensions $[n,n]$:

$\mathbf{M}_0$ is the diagonal matrix, $M_{0ii} = \left| 1 + \dfrac{W_r}{\mathbf{1} \cdot \mathbf{K}_i + C_i s} \right|^{-1}$, and $\mathbf{M}_\mathbf{V}$ is the matrix with diagonal elements $M_{\mathbf{V}ii} = \left| \mathbf{1} \cdot \mathbf{K}_i + C_i s + W_r \right|^{-1}$. Then the vector of deviations will be determined by the formula

$$\mathbf{\Delta} = \mathbf{V}_0 \cdot \mathbf{M}_0 - \mathbf{V} \cdot \mathbf{K} \cdot \mathbf{M}_\mathbf{V}, \tag{7}$$

where $\mathbf{V}_0$ is a row vector of corresponding states.

Accordingly, the vector equation for the intensities of failures of DCS regulators should be another part of the additional block of equations of the Kolmogorov-Chapman system:

$$\mathbf{\Lambda}_r = \left(\mathbf{1} + k_{p0}\left|\mathbf{V}_0 \cdot \mathbf{M}_0 - \mathbf{V} \cdot \mathbf{K} \cdot \mathbf{M}_\mathbf{V}\right|\right)\mathbf{\Lambda}_{r0}, \qquad (8)$$

where the operation "module" and multiplication by a coefficient are performed as element by element.

*The influence of the deviation of the MZO state on the load of the regulator*

Communication between coordinators is necessary for global optimization in distributed control systems. Disruption of the communication of the coordinator of one LCS leads to a change in the control algorithm: neighbouring zones become part of the surrounding environment, as well as to a change in the result of the search for the optimal setting for the LCS in all coordinators in the $\varepsilon$-area.

A typical object production function $y(v)$ is shown in Fig. 5a. This work is based on the production function of biotechnological systems, which shows the dependence of the productivity of organisms on the temperature of the environment [35, 36]. Since the processes of reproduction and death are exponential, the production function is well approximated by logistic curves

$$y(v) = \frac{e^{a(v-v_m)}}{1+e^{a(v-v_m)}} - e^{b(v-v_{cr})}$$

where $v_m$ is the point of maximum productivity; $v_{cr}$ is the critical value after which recovery is impossible; $a, b$ are approximation coefficients.

The optimal productivity of the zone is determined by the minimum costs per unit of production

$$f_{opt} = \arg\min_{v} \frac{v}{y(v)}$$

In Fig. 5b, the dependence $E = \dfrac{v}{y(v)}$ is shown by curve 1.

As mentioned above, the LCS coordinators in each zone determine the optimal setting for the regulators according to criterion (1). As a result, the settings $\mathbf{V}_0$ differ $\mathbf{F}_{opt}$. Let's compare criterion (1) with model (7) and write down the spread of the vector components $\mathbf{V}_0$ relative to the corresponding vector components $\mathbf{F}_{opt}$

$$\sigma_{\mathbf{V}_0-\mathbf{F}} = \left|\mathbf{V}_0 - \mathbf{F}_{opt}\right| = \left|\mathbf{\Delta}\right| = \left|\mathbf{V}_0\mathbf{M}_0 - \mathbf{VKM}_\mathbf{V}\right|. \quad (9)$$

Considering the probability distribution $\varphi(v_0)$ to be Gaussian, we will show it in Fig. 5b with curve 2.

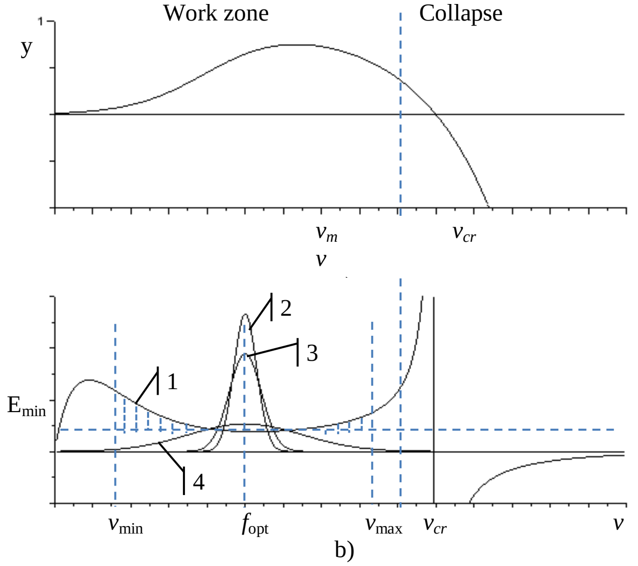



Figure 5. Evaluation of the influence of the control error on the load of the regulator

The actual state of the zone differs from the one set by the coordinator $v_0$ by an error $\Delta$. Taking into account the spread $v_0$, the general spread of the state of the zone $v$ $\sigma_v = \Delta\sqrt{2}$. The corresponding distribution of probabilities is shown in Fig. 5b by curve 3.

An increase in the spread of the distribution leads to a corresponding increase in the average load of regulators per unit of production, which is shown in Fig. 5b by the shaded area, and an increase in the load factor of the failure rate of the regulator $k_p$ in equation (6). For a Gaussian probability distribution

$$\Delta p = \int_{v_{\min}}^{v_{\max}} \left[\frac{v\left(1+e^{a(v-v_m)}\right)}{e^{a(v-v_m)} - \left(1+e^{a(v-v_m)}\right)e^{b(v-v_{cr})}} - E_{\min}\right] \times \frac{1}{\sigma_{\left(\mathbf{V'}_0-\mathbf{F}_{opt}\right)}\sqrt{2\pi}} e^{-\frac{\left(v-f_{opt}\right)}{2\sigma^2_{\left(\mathbf{V'}_0-\mathbf{F}_{opt}\right)}}} dv \qquad (10)$$

This is another additional equation to the Chapman-Kolmogorov system.

*The Impact of coordination interruption due to failures/haltings/recoveries of communication*

In the event of a communication failure or halting of the coordinator of the $i$-th zone, it sets the setpoint by criterion (1) $v_{0i} = f_{opt\,i}$. For the rest of the coordinators, the $i$-th zone is considered an uncontrolled one, since no information is received from it. Accordingly, all coordinators of the $\varepsilon$-area of $i$-th zone increase the components of the setpoints vector $\mathbf{V'}_{0\varepsilon}$ to compensate for the missing adjustment so that

$$\sum_{j\in\Omega_\varepsilon} k_{ji}\cdot dv_{0j}=v_{0i}$$

This leads to an increase in the deviation of the state probability distribution

$$\sigma_{\left(\mathbf{V}'_0-\mathbf{F}_{opt}\right)}=\left|\left(\mathbf{V}_0+d\mathbf{V}_0\right)\mathbf{M}_0-\mathbf{VKM}_{\mathbf{V}}\right| \qquad (11)$$

The corresponding distribution of probabilities is shown in Fig. 5b by curve 4. Then the load on the regulators (10) increases. Equation (11) is another equation of the Kolmogorov-Chapman system.

*Effect of LCS regulator failure on model parameters and functional dependability*

In case of failure of the regulator of the $i$-th zone, we will assume that the influence of the controller on the state of the zone of the MZO is $r_{i0}=0$. However, the influence of the external state $u$ of the environment and neighboring zones $R_i=r_{iu}+\sum_{j\in\Omega_\varepsilon} r_{ij}$, where $r_{ij}=k_{ij}\left(v_j-v_i\right)$; $r_{iu}=k_{iu}\left(u-v_i\right)$ remains. The state of the zone approaches the average value between the surrounding environment and neighbouring zones as a result of regulator failure according to the equation

$$\frac{1}{C_i}\cdot\frac{dv_i}{dt}=k_{iu}\left(u-v_i\right)+\sum_{j\in\Omega_\varepsilon} k_{ij}\left(v_j-v_i\right)$$

or

$$\frac{1}{C_i}\cdot\frac{dv_i}{dt}+v_i\left(k_{iu}+\sum_{j\in\Omega_\varepsilon} k_{ij}\right)-\left(k_{iu}u+\sum_{j\in\Omega_\varepsilon} k_{ij}v_j\right)=0\,.$$

According to the change in the state of the $i$ - h zone, the coordinators of the neighbouring zones of the set $\Omega_\varepsilon$ will change the coordination parameters due to the results of solving the new optimization problem according to criterion (1) in vector form

$$\widetilde{\mathbf{V}}_{0\varepsilon}=\arg\min_{\widetilde{\mathbf{V}}_{0\varepsilon}}\left|\boldsymbol{\rho}_\varepsilon\left(\mathbf{F}_\varepsilon-\widetilde{\mathbf{V}}_{0\varepsilon}-\boldsymbol{\Delta}\right)^T\right| \qquad (12)$$

with an additional limitation

$$\begin{cases}\tilde{v}_{0i}=0\\ \tilde{v}_i=k_{iu}u+\sum_{j\in\Omega_\varepsilon} k_{ij}\tilde{v}_{0j}\end{cases},$$

where $\tilde{v}_0$ is the coordination parameter after failure; $T$ is the vector transposition sign. A corresponding increase in the load of the regulator of the adjacent zone is $\Delta p_j=\left|v_{oj}-\tilde{v}_{0j}\right|$, which leads to a corresponding change in the load factor. So the system of equations

$$\begin{cases}\widetilde{\mathbf{V}}_{0\varepsilon}=\arg\min_{\widetilde{\mathbf{V}}_{0\varepsilon}}\left|\boldsymbol{\rho}_\varepsilon\left(\mathbf{F}_\varepsilon-\widetilde{\mathbf{V}}_{0\varepsilon}-\boldsymbol{\Delta}\right)^T\right|\\ \tilde{v}_{0i}=0\\ \tilde{v}_i=k_{iu}u+\sum_{j\in\Omega_\varepsilon} k_{ij}\tilde{v}_{0j}\\ \Delta p_j=\left|v_{oj}-\tilde{v}_{0j}\right|\end{cases} \qquad (13)$$

is also part of the Kolmogorov-Chapman system.

*Effect of sensor failure*

In distributed control systems of MZO, the state of the zone is determined by the algorithm of optimal estimation using the Kalman filter, and the failure of one sensor leads to an increase in the error of the estimation of the state of the zone. As a result, the LCS does not fail due to sensor failure, but the deviation of the state from the set value increases. The dynamic errors of the LCS with a non-working sensor also increase, since the state of the controlled zone is considered constant between two consecutive executions of the evaluation procedure. As noted above, an increase in error leads to an increase in the load on the regulators and their failure rate.

Let's estimate the error of determining the state of the zones.

We denote the components of the system state vector that are measured by $\tilde{\mathbf{V}}_\varepsilon$. In general, the relationship between model parameters $\mathbf{V}_\varepsilon$ and the vector of measured data $\tilde{\mathbf{V}}_\varepsilon$ has the form

$$\tilde{\mathbf{V}}_\varepsilon=\mathbf{V}_\varepsilon+\boldsymbol{\xi}_{\tilde{V}},$$

where $\tilde{\mathbf{V}}$ has dimension $l=n_\varepsilon-1$; $\boldsymbol{\xi}_{\tilde{V}}$ is the vector of measurement errors with dimensionality $l$.

Let's turn to Fig. 4 again and write down the system model in the form

$$v_i=v_0\frac{W}{1Cs+W+\mathbf{1K}}+\mathbf{V}\frac{\mathbf{K}}{Cs+W+\mathbf{1K}},$$

and for the entire $\varepsilon$-area

$$\mathbf{V}_0\frac{W}{1Cs+W+\mathbf{1K}}+\mathbf{V}\frac{\mathbf{K}}{Cs+W+\mathbf{1K}}=\mathbf{V}_\varepsilon \qquad (14)$$

The assessment task consists of finding such values of the components $\mathbf{K}_\varepsilon$ of the topological matrix $\mathbf{K}$ and the components $\mathbf{V}_\varepsilon$ of the state vector $\mathbf{V}$ which ensure the maximum proximity of the calculated parameters of the state $\mathbf{V}_\varepsilon$ to the measured values $\tilde{\mathbf{V}}_\varepsilon$:

$$\min\delta=\sum_{j\in\Omega_\varepsilon}^{l}\left(\tilde{v}_j-v_j\right)^2.$$

The residual error value of each estimated value with Kalman filtering is

$$\sigma_{v_i} = \sqrt{1 - \sum_{j\in\Omega_\varepsilon} R_{ij}}$$

where $R_{ij}$ are the elements of the correlation matrix of estimated and measured parameters of MZO [34]. The rows of the matrix correspond to the estimated parameters, and the columns correspond to the measured ones. When one sensor fails or halts, the number of non-zero values in the sum of correlation coefficients decreases, and the estimation error increases. The states $\mathbf{V}$ of the zones are measured in LCS and we are not interested in all filtering errors, but only in the residual errors of the zone state estimation after sensor failure. So for linear MZO

$$R_{ij} = \sigma_{sj}^2 \frac{\partial v_i}{\partial v_j},$$

where $\sigma_{sj}$ is the standard error of the $j$-th sensor; $\frac{\partial v_i}{\partial v_j}$ is the derivation from the dependence $v_i(v_j)$ according to model (3) in [30]. Thus, the equation

$$\sigma_{v_i} = \sqrt{1 - \sum_{j\in\Omega_\varepsilon} \sigma_{sj}^2 \frac{\partial v_i(v_j)}{\partial v_j}}, \quad i = 1\ldots n; \quad j = 1\ldots(n-1) \qquad (15)$$

together with the model

$$\frac{dv_j(t)}{dt} = p_{0j}(t) + \sum_{k=1}^{n}\left\{\frac{p_{0k}(t) + v_k(t) - v_j(t)}{8\left[\pi k_{kj}(t-t_k)\right]^{3/2}} e^{-\frac{d_{kj}^2}{4k_{kj}t}} \left[1 + \left(\frac{d_{kj}^2}{k_{kj}(t-t_k)} - 6\right)\cdot\frac{r_{0k}^2}{40k_{kj}t}\right]\right\} \qquad (16)$$

(where the notation was described in [30]) are the part of Chapman-Kolmogorov system.

*Generalized assessment of functional dependability*

The received estimates of the influence of LCS element failures on the control error and on the failure rate accordingly, allow to obtain a generalized assessment of functional dependability

$$P_f = \prod_{i=1}^{n} \int_{v_{\min i}}^{v_{\max i}} G(v_i, m_{vi}, \sigma_{vi})\, dv_i \qquad (17)$$

where $G(v, m_v, \sigma_v)$ is the Gaussian distribution of the total error;

$\sigma_v = \sqrt{\sigma_{v/\lambda}^2 + \sigma_{v/u}^2 + \delta_{v/d_{kj}}^2 + \sigma_{v/v_0}^2 + \sigma_{v/m}^2}$);

$m_v = v_0(P_c)$;

$\sigma_{v/m}^2$ is additional deviation n of the condition of zones caused by failures of LCS elements.

Additional deviations for all zones caused by element failures will be calculated in vector form

$$\sigma_{v/m}^2 = \boldsymbol{\sigma}_m \cdot \mathbf{S} \cdot (\mathbf{1} - \mathbf{P}_S), \qquad (18)$$

where $\boldsymbol{\sigma}_m = \{\sigma_r^2, \sigma_s^2, \sigma_c^2, \sigma_h^2\}$ is the vector of additional variances caused by the failure of individual elements; $\mathbf{S}[4\times16]$ is the matrix of states of LCS elements, for each of the 16 states of the system, according to Table 2.

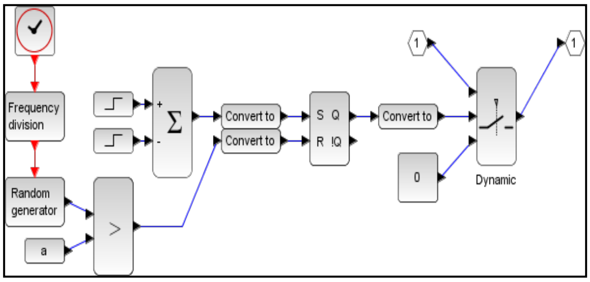
TABLE 2. MATRIX OF STATES OF LCS ELEMENTS (0 – OPERATIONAL; 1 – INOPERABLE)

| | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ | $S_8$ | $S_9$ | $S_{10}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{14}$ | $S_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Regulator | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| Sensor | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| Communication failure | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Communication failure | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

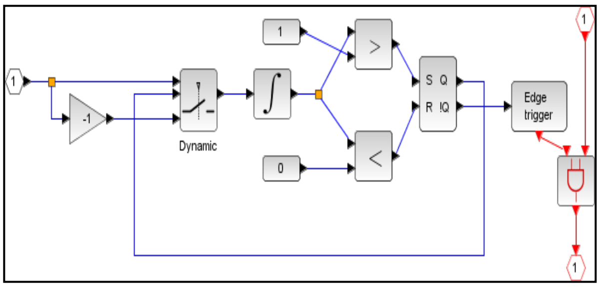
## IV. NUMERICAL EXPERIMENTS AND RESULTS

Observing failure processes in real conditions takes a lot of time, and to obtain reliable statistical estimates, it is necessary to have many realizations of the random process of changing the state of the system. Therefore, to obtain preliminary estimates, we focused on the simulation of these processes. In work [34], an approach based on simulation in the Scilab/Xcos system using the library of typical elements developed by the authors, in particular, models of controlled and uncontrolled elements of a distributed object was proposed for the study of distributed object control systems. The models are modified to take into account element failures for our study. The failure simulation module is shown in Fig. 6a. With a uniform distribution of the probability of values at the output of the random number generator, the probability of failure is $Q = \frac{r_{\max} - a}{r_{\max} - r_{\min}}$, and the failure rate $\lambda_r = \frac{f}{n_f}\cdot\ln(1-Q)$ where $n_f$ is the divisor of the frequency $f$ of random number generation events; $[r_{\min}; r_{\max}]$ is the range of uniform distribution; $a$ is a setting parameter. Fig. 6b shows the frequency-controlled generator module (the "Convert to" data type matching blocks are not shown here and in the diagrams below). The module in Fig. 6c simulates failures with a rate that depends on the input signal arriving at input “2”. The module in Fig. 6d simulates temporary haltings and interruptions of communication that may occur due to network overload, exceeding the interference level of the calculated value, etc.

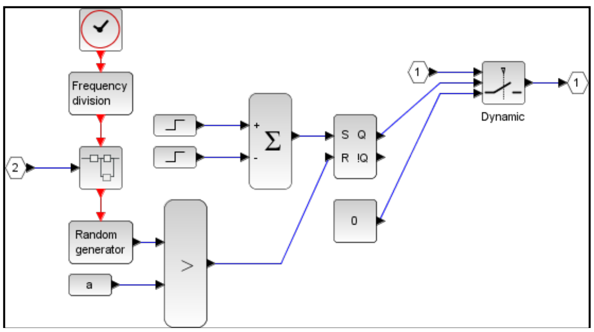
Fig. 7 shows the use of failure modelling modules in LCS models of zones of a multi-zone facility. Fig. 7a shows the use of the failure module with a variable rate (Fig. 6c) for simulating regulator failures (the failure rate depends on the load of the regulator). Fig. 7b shows the use of the failure module with a constant rate (Fig. 6a) for modelling LCS feedback sensor failures.



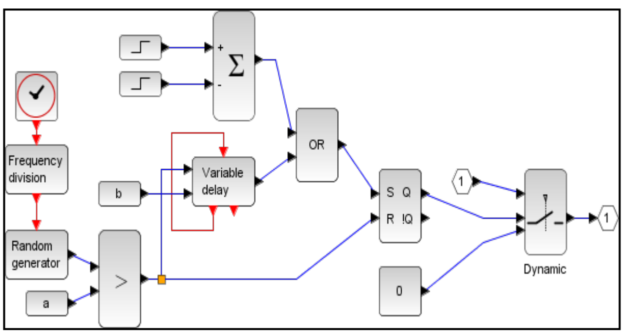

a)




b)



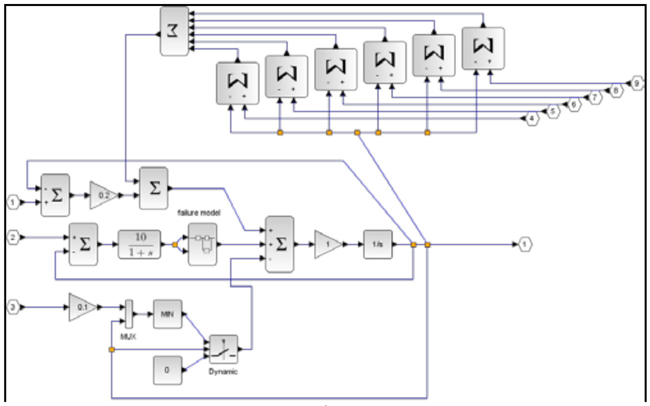

c)



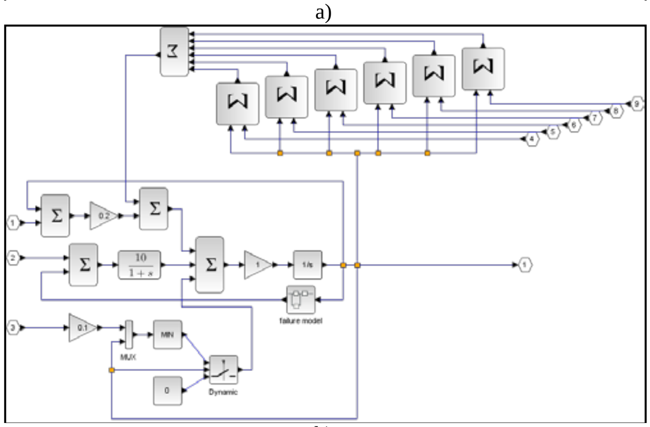

d)

Figure 6. Simulation of random failure processes

The graphs in Fig. 8 show the result of the simulation of the distributed control system of a multi-zone object with 3 zones. The graph in Fig. 8a shows the moment of failure of the regulator of one of the zones ("zone 2"). Fig. 8c shows changes in the state of this zone, and Fig. 8b and 8d show the state of neighbouring zones (the ε-area of zone 2 is zones 1 and 3 respectively). Fig. 8b-8d also shows the setpoint state for each zone and the deviation limits, which exceeding is considered as a violation of functional dependability.



a)



b)

Figure 7 Simulation of failures in LCS control of a distributed object

The graph in Fig. 8c shows that after the failure of the regulator, the state of zone 2 changes and the state goes beyond the permissible limit. After that, the LCS coordinators of neighbouring zones begin to compensate for the deviation of zone 2 using changes in the state of zones 1 and 3, taking into account their impact on zone 2. As a result, the state of zone 2 returns to acceptable limits, but the deviation of the state of zones 1 and 3 from the set value increases. At the same time, the presence of random influences can lead to short-term violations of functional dependability in these zones. Periods of functional dependability violations in the zones are shown in Fig. 8 with a translucent background.

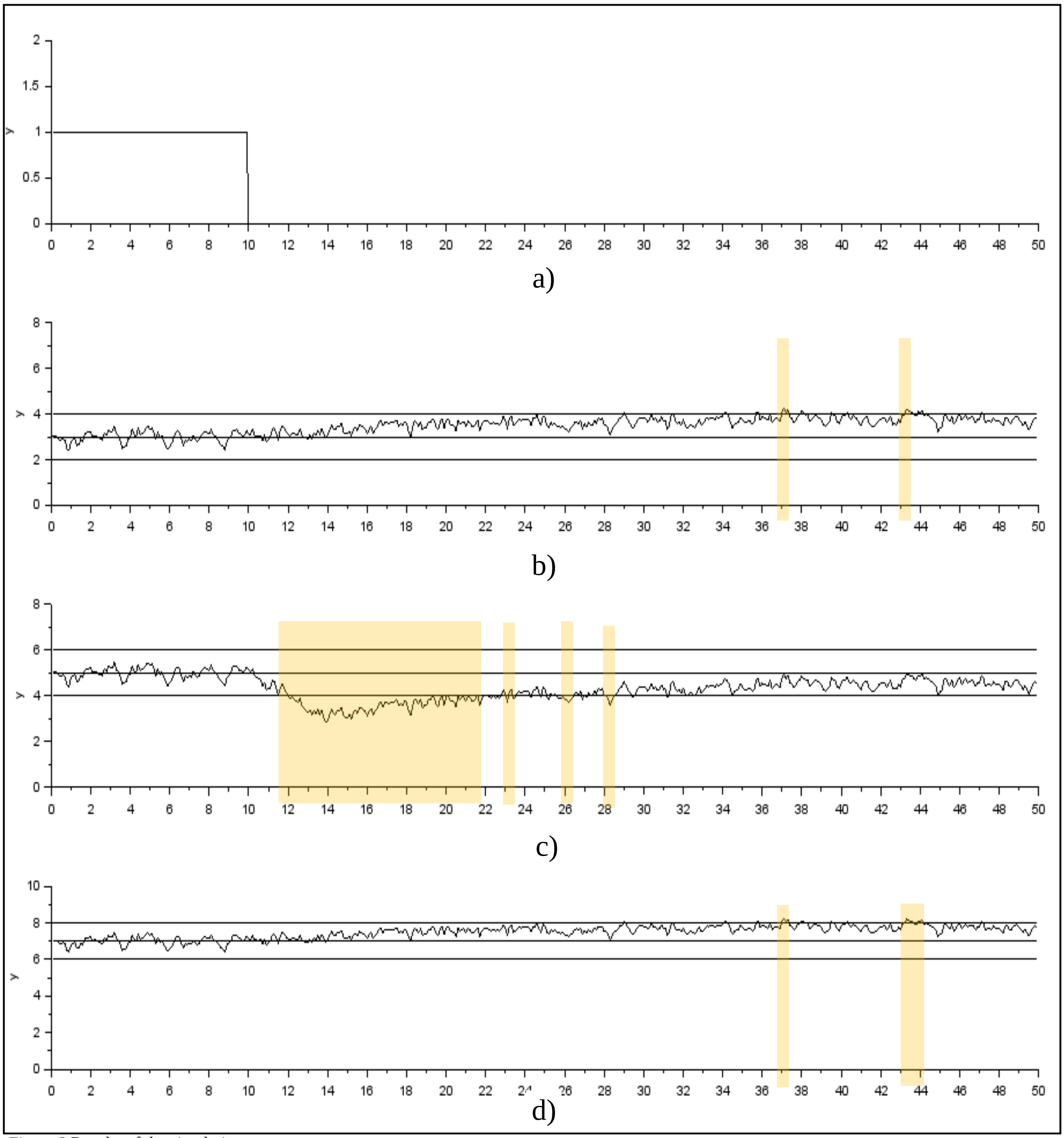



Figure 8 Results of the simulation

Let's investigate the distribution of failure events of the MZO distributed control system over time. For this task, the model of the distributed control system with 9 MZO zones was created based on the LCS models with failures shown in Fig. 7. Failure events in the simulation, i.e. moments of system state change over time, are shown in Fig. 9. The time scale is compressed 100 times for the convenience of the image.

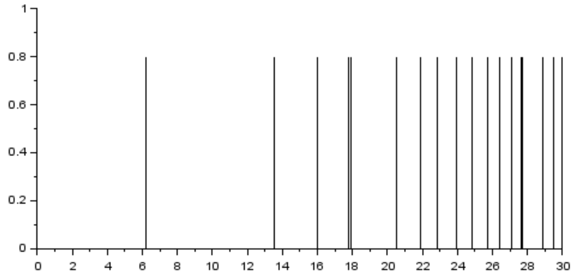
A study on a simulation showed that the flow of failures in DCS increases over time due to an increase in the load on the regulators when one of them fails. The diagram in Fig. 10 shows the failure rate calculated based on the results of simulation experiments. The diagram shows 4 dependences of the failure rate on time: at the maximum coefficient of mutual influence of zones $k_{ij} = 0.1$ and $k_{ij} = 0.2$ and at the number of controlled zones n=9 (symmetrical arrangement 3×3) and n=16 (symmetrical arrangement 4×4).

It can be seen from the diagram that the failure rate increases over time according to a close-to-exponential law ("cascade failures"). The growth rate depends on the coefficient of mutual influence of the zones $k_{ij}$:

$$\lambda(t) = \lambda_0 \left(1 + k_{p0} e^{\eta \tilde{k} t}\right), \tag{19}$$

where $\eta = \frac{m}{n^2}$ is the topological matrix density; $\tilde{k}$ - the average indicator of the mutual influence of zones $k_{ij}$.

An increase in the number of controlled zones at the initial stage leads to an increase in the system's resistance to failures, because the failure of an individual element is compensated by a large number of neighboring zones and, accordingly, to a smaller increase in their load.


a)

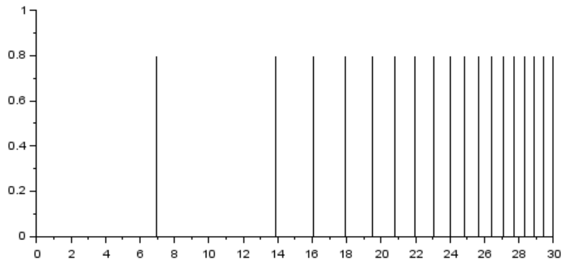

b)

Figure 9 - Failure events: a is one implementation; b is the average values of failure moments

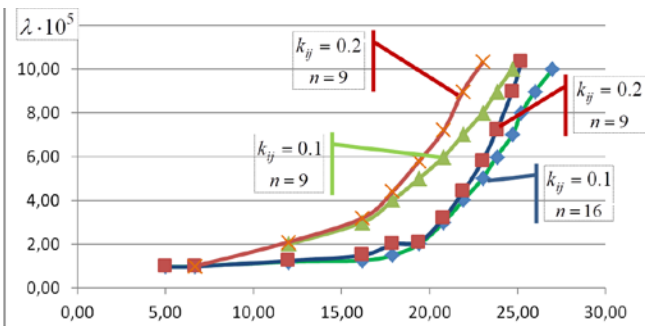



Figure 10 Change in failure rate over time

However, failures of elements lead to a decrease in the number of controlled zones and the initial dependence for n=16 approaches the dependence n=9, but with a time delay. Moreover, the dependence for the system n=16 (4×4) after failures of 7 elements does not quite coincide with the characteristics of the system n=9 (3×3) because the location of the failed elements is random and this leads to differences in the distribution of loads on the regulators.

## V. DISCUSSION

Both modelling approaches used in this work: formal modeling using the Chapman-Kolmogorov equations and the equations of the mutual influence of zones and local control systems and simulation show qualitatively convergent results, namely an exponential increase in the failure rate (19) and a corresponding decrease in systems functional dependability. This phenomenon is caused by the system's features: the mutual influence of zones and the presence of a global coordination criterion. As a result, the failure of the control elements of one zone tries to be compensated by the coordinators of neighbouring zones. This leads to an increase in the load on neighbouring zones and a corresponding decrease in their dependability.

More precisely, a quantitative comparison of methods is complicated by the peculiarities of the formal model of the system. In addition to the large dimension (16 Chapman-Kolmogorov equations, 7 vector equations of mutual influence for each zone, so the total number of equations $16 \cdot n + 7 \cdot n$. For a system of 9 zones which was studied in the simulation, this is 207 equations), the important fact is that it is necessary to calculate the integral (10) and look for the minimum of the function (11) in the process of solving such a system of equations. This makes it difficult to apply precise Gaussian, Kramer and so on methods. However, the application of iterative methods requires careful selection of the initial approximation to ensure convergence, which is a non-trivial task for such a large system.

The dependencies obtained as a result of simulation in Fig. 10, which demonstrate the exponential nature of the increase in the failure rate (19), should be considered only near the initial value $\lambda_0$, because when the failure rate increases, the system quickly exceeds the limits of functional dependability. Taking this into account, it is possible to propose an approximation of the dependence of the probabilistic indicator of functional dependability on time and system parameters

$$P_f = \exp\left(-\tilde{k}\eta\left[\max_i \frac{\sigma_i}{v_{\max i} - v_{\min i}}\right]\left(\lambda_r t^{1+k_p} + \left(\lambda_s + \lambda_c + \lambda_h - \mu_h\right)t\right)\right). \quad (20)$$

The accuracy of the estimate (20) remains to be verified for different topologies of distributed systems and coordination algorithms.

## VII. CONCLUSION

The relevance of the problem of functional dependability of distributed control systems is constantly increasing due to the spread of cheap controllers and sensors and the creation of IoT systems. In the case of multi-zone objects distributed control with the mutual influence of zones, the problem is complicated by the changes in the operating modes of local control systems due to the failure of systems elements of the neighbouring zones. This, in turn, leads to a change in their failure rates.

In this work, a functional dependability model of a decentralized distributed control system for multi-zone objects is developed and investigated. The model is obtained in the form of a heterogeneous Markov chain of system states caused by failures of elements of local control systems. The probabilities of transitions between states are described by the system of Chapman-Kolmogorov equations and additional equations that describe the relationship between the parameters of neighbouring zones and the corresponding local control systems.

The resulting system of equations is characterized by both large dimensions and nonlinearity since it includes the optimization procedure (13). The presence of such nonlinearity prevents the use of exact difference methods for solving the system, and approximate iterative methods are prone to instability, especially near the boundaries of the functional dependability area. Therefore, simulation on the Scilab platform was used to study the system using the module library developed by the authors for modelling distributed control systems, which were developed for modelling element failure flows.

The simulation showed that for multi-zone objects with coordinated control according to the global criterion and a strong connection between the parameters of the zones, the "cascade failures" process is likely, which quickly leads to a loss of functional dependability.

Further research is planned to study the ways to combat the increase in the failure rate and increase functional dependability by optimizing the topology of the distributed control system, the coordination control algorithm and the recovery strategy after failures.

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