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TENSOR AND VECTOR APPROACHES TO OBJECTS RECOGNITION BY INVERSE FEATURE FILTERS

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Abstract. *The investigation of the extraction of image objects features by filters based on tensor and vector data presentation is considered. The tensor data is obtained as a sum of rank-one tensors, given by the tensor product of the vector of lexicographic representation of image fragments pixels with itself. The accumulated tensor is approximated by one rank tensor obtained using singular values decomposition. It has been shown that the main vector of the decomposition can be considered as the object feature vector. The vector data is obtained by accumulating analogous vectors of image fragments pixels. The accumulated vector is also considered as an object feature. The filter banks of a set of objects are obtained by regularized inversion of the matrices compiled by object features vectors. Optimized regularization of the inversion is used to expand the regions of object features capture with minimal error. The object fragments and corresponding feature vectors are selected through a training iterative process. The tensor and vector approaches create two channels for recognition. High efficiency of object recognition can be achieved by choosing the filter capture band and creating filter branches according to the given bands. The filters create a convolutional network to recognize a set of objects. It has been shown that the obtained filters have an advantage over known correlation filters when recognizing objects with small fragments.*

Keywords: objects recognition, objects feature, image data tensor, image data vector, inverse filters, optimized regularization

PODEJŚCIE TENSOROWE I WEKTOROWE DO ROZPOZNAWANIA OBIEKTÓW ZA POMOCĄ FILTRÓW CECH ODWROTNYCH

Streszczenie. *Rozważane jest badanie ekstrakcji cech obiektów obrazu przez filtry oparte na prezentacji danych tensorowych i wektorowych. Dane tensorowe uzyskuje się jako sumę tensorów* pierwszego *rzędu,* otrzymanych *przez iloczyn tensorowy wektora leksykograficznej reprezentacji pikseli fragmentów obrazu z samym sobą. Skumulowany tensor jest aproksymowany przez tensor* pierwszego *rzędu uzyskany przy użyciu dekompozycji wartości osobliwych. Wykazano, że główny wektor dekompozycji można uznać za wektor cech obiektu. Dane wektorowe uzyskuje się poprzez akumulację analogicznych wektorów pikseli fragmentów obrazu. Skumulowany wektor jest również uważany za cechę obiektu. Banki filtrów zestawu obiektów są uzyskiwane przez regularyzowaną inwersję macierzy skompilowanych przez wektory cech obiektów. Zoptymalizowana regularyzacja inwersji jest wykorzystywana do rozszerzenia obszarów przechwytywania cech obiektów przy minimalnym błędzie. Fragmenty obiektów i odpowiadające im wektory cech są wybierane w iteracyjnym procesie uczenia. Podejście tensorowe i wektorowe tworzy dwa kanały rozpoznawania. Wysoką skuteczność rozpoznawania obiektów można osiągnąć, wybierając pasmo przechwytywania filtrów i tworząc gałęzie filtrów zgodnie z podanymi pasmami. Filtry tworzą sieć konwolucyjną do rozpoznawania zestawu obiektów. Wykazano, że uzyskane filtry mają przewagę nad znanymi filtrami korelacyjnymi podczas rozpoznawania obiektów z małymi fragmentami.*

Słowa kluczowe: rozpoznawanie obiektów, cechy obiektów, tensor danych obrazu, wektor danych obrazu, filtry odwrotne, zoptymalizowana regularyzacja

Introduction

Tensors are known as multidimensional matrices for coordinate transforms [5]. Recently, they have been used for the description of image object arrays in computer vision and machine learning [3, 6, 10, 13]. Tensors are considered as multidimensional matrices with structural features. These features allow to reduce the dimension or size of a multidimensional data matrix by decomposition into a product of a sequence of tensors of lower dimension and smaller size. The tensors can be approximated by tensors of uncompleted rank to obtain conditions for decomposition. The items of the decomposition can be decomposed in such a way as to create a tensor train [4, 6, 10, 12, 13]. The tensors of incomplete rank are obtained using the tensor or outer product of vectors and matrices [4, 9]. The Singular Value Decomposition (SVD) [20] is used as the tool to decompose matrices and approximate them by the sum of tensor products of orthogonal vectors-columns of two unitary matrices. The number of terms in the sum is defined by the rank of the approximating matrix.

The geometry means of a tensor [5] were modified for binary images as a shape salience detector and a shape descriptor – tensor scale descriptor with influence zones [1, 2]. It is a robust method to compute tensor scale, using a graph-based approach.

The feature of the tensor object can be associated with the core tensor [16] of Tucker decomposition [14, 18]. The decomposition represents the multidimensional matrix as a product of a core matrix and matrices of factors. It allows to reduce a complexity of the problem of objects classification using a least squares approach. Tucker decomposition can be implemented by hierarchical SVD to achieve the required accuracy of the initial tensor approximation [8]. The Canonical Polyadic Decomposition (CPD) [5, 7, 9] represents the tensor as a sum of rank one tensors given by the tensor product of vectors that correspond to each of the dimensions, at least two. One such decomposition is the SVD and its high order extension [3, 13].

The problem of objects recognition is solved by feature filters. The filters create a Convolutional Neural Network (CNN) for recognizing some class of objects. The use of the least squares method for filters creation leads to equations based on the correlation dependencies of the data and the desired results [19]. The simple vector representation of image data is used for this problem.

Multidimensional correlation analysis can be defined for two tensors in a space of some dimensions. The inner product of these dimensions yields the correlation tensor [3, 8]. Correlation tensor analysis, in conjunction with discriminant analysis, is capable of capturing higher order structures in the data patterns by encoding each object as a second- or higher-order tensor. This allows for obtaining a low-dimensional data representation that reflects both class label information and intrinsic manifold structure.

The considered properties of tensors motivate the investigation of the application of the tensor structure of image data in object recognition using correlation filters [7, 11, 19].

1. The aim and objectives of the study

Tensor characteristics of a certain set of objects in the form of images are interesting in the case when each of the objects has its own core feature tensor, and the system of factor tensors is common. The collection of core tensors and factor tensors approximates the image set. Then, by extracting and classifying features from the image, it is possible to recognize the objects contained in it. Such a tensor construction can be created artificially using image data.

The simplest tensor construction in the form of CPD is used for creating a filter bank for object recognition based on Inverse Regularized Feature Filters (IRFF) [11]. Its performance is compared with similar filter banks obtained by the methods considered in [7, 11, 19] using lexicographical vector presentation of object images.

The research method consists of simulating the process of training the filter bank with further filtering of image data in order to recognize objects. Alphabetic and numeric characters with variations in size and perspective were used as a test set for recognizing objects.

The investigation of the influence of the form of data representation – tensor and vector – on the selective properties of the IRFF of image objects is presented.

2. Methods

The problem of object recognition includes the following three main stages: object' feature extraction, feature selection, classifier design. The object' feature characteristics must satisfy such a requirement as stability, it means to be invariant to object variations in order to recognize as many objects of the same type as possible. The fulfilment of this requirement is achieved by training the recognition method using a large number of templates of the target object. The templates arrays of some target objects create multidimensional data structure which can be considered as a tensor. A feature finding and extraction in the form of matrix or vector can be done by tensor decomposition, by lexicographic vectorization over several dimensions and reduction to a matrix form accessible for processing, or by folding dimensions using integration over some dimensions by summation over corresponding indexes [3, 5, 9, 12, 13].

A. Tensor approach

The problem is to recognize a set of image objects. The images are presented by arrays of templates to train for the recognizing. Each of the templates consists of fragments which include desired object features or not. The problem is to select fragments with the object features to create the features filters.

The problem data can be presented as the 7-th order tensor $\mathbf{X}_{i_1,i_2,i_3,i_4,i_5,i_6,i_7}$, where $i_1 = 0....M-1$ is the objects index, $i_2 = 0....N_{T_{i_1}} - 1$ – the templates index of i_1-th object, $i_3, i_4 = 0....N_{F_{i_1,i_2}} - 1$ – i_2-th template of i_1-th object indices, $i_5, i_6 = 0....N-1$ – indices of pixels of fragments of size $N \times N$, $i_7 = 0,\ 1\ ,2$ – index of RGB colors. The solution of the object recognition problem depends on the complexity of the tensor $\mathbf{X}$ data structure. If the tensor includes repeating or cyclic elements, which allows it to be decomposed into outer products of simpler tensors, then this reduces the complexity of the problem and improves recognition accuracy by reducing the number of recognition parameters and their separability by objects. The complexity of the data structure can be characterized by the rank of the tensor. Tensors of incomplete rank lend themselves to tensor decompositions [4, 6, 13]. Tensor $\mathbf{X}$ order can be lowered if to join three last indices by lexicographic representation of image matrices of pixels as a vector, then

$$vec\left(\mathbf{X}_{i_1,i_2,i_3,i_4,i_5,i_6,i_7}\right) \sim \mathbf{X}_{i_1,i_2,i_3,i_4,i=0..3N^2} \tag{1}$$

The target of training process is to obtain a filters bank $F_{i_1,i}$ which classify an image fragments I_i as i_1-th object or not by the next rule:

$$\underset{i_1}{maxarg} \sum_i F_{i_1,i} \cdot I_i = \sigma_{i_1} \begin{cases} \geq threshold : i_1\ true \\ < threshold : i_1\ false \end{cases} \tag{2}$$

The initial filter of k-th object can be found from the equation

$$\sum_i F_{i_1,i} \cdot X_{k,i_2,i_3,i_4,i} = \delta_{i_1,k} 1_{i_2,i_3,i_4}, \tag{3}$$

where $\delta_{i_1,k} = 1 : \ i_1 = k;\ \delta_{i_1,k} = 0 : \ i_1 \neq k =$ 1. The problem statement (3) corresponds to zero-aliasing approach of correlation filters design [7, 19]. The least square solution of the equation (3) is the next:

$$\sum_i F_{i_1,i} \sum_{i_2,i_3,i_4} X_{k,i_2,i_3,i_4,i} X_{k,i_2,i_3,i_4,j} = \delta_{i_1,k} \sum_{i_2,i_3,i_4} X_{k,i_2,i_3,i_4,j} \tag{4}$$

The sum over three indices in the left side of expression (4) is the sum of the matrices of the rank one obtained using the tensor type or the outer product of the image fragment vector by itself. Let denote it as $R_k = \left[R_{k,i,j}\right]_{i,j=0,...,3N^2}$, it has size $3N^2 \times 3N^2$ for k-th object. This sum R_k has rank one if all fragments are mutually collinear, that is, equal up to a factor. Differences in the vectors of fragments form a matrix R_k of higher rank, and with a large number of terms, a matrix of full rank. However, if the problem is to find the feature of each of the set of objects and recognize objects by filtering these features, then, if to assume that each fragment of the object's image templates is the feature with deviations, then this feature should be repeated in the matrix and form a total matrix of rank one. As the matrix R_k is symmetric, its SVD is the follows.

$$\left[R_{k,i,j} = \sum_{m::s_m^{(k)}>0} s_m^{(k)} u_{i,m}^{(k)} u_{j,m}^{(k)}\right]_{i,j=0,...,3N^2} \tag{5}$$

where the unitary matrix $U^{(k)} = \left[u_{i,j}^{(k)}\right]_{i,j=0,...,3N^2}$ of size $3N^2 \times 3N^2$ consists of orthogonal columns, singular values $s_0^{(k)} > s_1^{(k)} > ... \geq 0$. The number of nonzero singular values is equal to the rank of matrix (5). The SVD (5) gives an approximation of the matrix R_k by the rank one matrix as

$$\left[\tilde{R}_{k,i,j} = s_0^{(k)} u_{i,0}^{(k)} u_{j,0}^{(k)}\right]_{i,j=0...3N^2-1} \tag{6}$$

Therefore, the vector

$$\Phi_k = \sqrt{s_0^{(k)}}\, N_{TFk}^{-1} \left[u_{i,0}^{(k)}\right]_{i=0...3N^2-1} \tag{7}$$

where N_{TFk} is total number of fragments of k-th object' templates array, is the feature vector of the object. The equation (3) for feature vector (7) can be written as

$$\sum_i F_{i_1,i} \Phi_{k,i} = \delta_{i_1,k} \tag{8}$$

As it follows from (8), the bank of M filters can be defined as the matrix inverse to matrix compelled by M vectors (7):

$$\left[F_{i_1,i}\right] = \left[\Phi_{i_1,i}\right]^{-1}_{i_1=0...M-1, i=0...3N^2-1} \tag{9}$$

The feature vectors are defined on the base of SVD of the total sum of all fragments in (4). The second step of the filter bank (9) creation is basing on the training process that is the selection by operation (2) that fragments which give true result. Also, the fragments which give false result are selected too. The sum of selected fragments gives two matrices $R_{i_1}^{tr}$ and $R_{i_1}^{fl}$. The SVD of the matrices gives feature vectors (7) $\Phi_{i_1}^{tr}$ and $\Phi_{i_1}^{fl}$ of the extended filters bank

$$\left[F_{i_1,i}^{tr}\ F_{i_1,i}^{fl}\right] = \left[\Phi_{i_1,i}^{tr}\ \Phi_{i_1,i}^{fl}\right]^{-1}_{i_1=0...M-1, i=0...3N^2-1} \tag{10}$$

The using of filters bank (10) gives new arrays of fragments which include features of true and false results in (2). The procedure of features extraction and filters bank (10) creation can be repeated up to the desired result in terms of the ratio of correct and incorrect recognitions results will not be achieved.

B. Vector approach

The convergence of the iterative scheme for determining the filter bank (10) to a given level of recognition errors depends on the structure of the image data, which effects on the ability to approximate matrices $R_{i_1}^{tr}$ and $R_{i_1}^{fl}$ by matrices of unit rank. The feature vector is the result of the projection of the data vectors onto one of the vectors of the vector space of the SVD. Therefore, each of the data vectors can only be partially considered when features are determining.

Let's consider a scheme for generating feature vectors and forming a filter bank in such a way that the data of templates fragments are fully considered. The equation (3) can be rewritten in the manner

$$\sum_i F_{i_1,i} \cdot \frac{1}{N_{TFk}} \sum_{i_2,i_3,i_4} \mathbf{x}_{k,i_2,i_3,i_4,i} = \delta_{i_1,k} \tag{11}$$

where

$$\left\lfloor \mathbf{x}_{k,i_2,i_3,i_4,i} = vec_{i=i_5,i_6,i_7}\left(X_{k,i_2,i_3,i_4,i_5,i_6,i_7}\right)\right\rfloor_{i=0...3N^2-1}$$

i is the index of lexicographic representation of items indexed by i_5, i_6, i_7. The sum over three indices can be denoted as $\overline{\mathbf{x}}_k$. The feature vector is the average image data vectors obtained by lexicographic representation of image frames matrices of pixels in the case of scheme (11), that is,

$$\Psi_{i_1} = N_{TF i_1}^{-1}\left[\overline{x}_{i_1,i}\right]_{i=0...3N^2-1} \tag{12}$$

As it follows from condition (11), it can be obtained the filter bank for feature vectors (12) by the same iterative procedure as the filters bank (10).

$$\left[H_{i_1,i}^{tr}\ H_{i_1,i}^{fl}\right] = \left[\Psi_{i_1,i}^{tr}\ \Psi_{i_1,i}^{fl}\right]^{-1}_{i_1=0...M-1, i=0...3N^2-1} \tag{13}$$

C. Filters bank optimization

The inverse filters like (9) and (13) were investigated in [11]. The experiments have shown that filters on the base of exact inversion are narrow and select only objects which vary from the feature vectors by small values at the level of computational error. The inverse filters' band of selection can be expanded by regularization of the inversion. The SVD of features matrix for inversion in (10) with account regularization parameter λ is the follows.

$$\left\lfloor \Phi_{i_1,i}^{tr}\ \Phi_{i_1,i}^{fl}\right\rfloor_\lambda = \sum_m u_{\Phi i,m}(s_{\Phi m}+\lambda)\left\lfloor v_{\Phi i_i,m}^{tr}\ v_{\Phi i_i,m}^{fl}\right\rfloor \tag{14}$$

The inversion of (14) to define the filters bank (9) yields the next expression.

$$\left[F_{i_1,i}^{tr}\ F_{i_1,i}^{fl}\right]_\lambda = \sum_m \left[v_{\Phi i_i,m}^{tr}\ v_{\Phi i_i,m}^{fl}\right] u_{\Phi i,m}\left(s_{\Phi m}+\lambda\right)^{-1} \tag{15}$$

The regularization in (15) degrades the separation ability of the filter (9). The separation ability will be high if mutual relation of the feature filters

$$\sum_m F_{i,m} \cdot F_{k,m} \approx \delta_{i,k}$$

is minimal as possible at $i \neq k$. This condition demands growth of λ in (15). At the same time, the energy of the regularized feature matrix (14) increases indefinitely under the growth of λ. Therefore, an optimal regularization should balance these two mutually contradictory conditions. The condition of the balance for matrices of $2M$ features vectors (14) and vector of filters (15) may be presented by the functional [11]

$$\mathrm{I}(\lambda) = \underset{\lambda}{argmin}\left\{ \frac{\sum_i \left|\Phi_i\right|_\lambda^2}{\sum_i \left|\Phi_i\right|_0^2} + \frac{\sum_{i,k;i\neq k}\left|\sum_m F_{i,m}F_{k,m}\right|_\lambda^2}{\sum_{i,k;i\neq k}\left|\sum_m F_{i,m}F_{k,m}\right|_0^2} \right\} \tag{16}$$

The matrices in the left side of (14) and (15) are incompatible because they are mutually inverse, therefore the normalization of square norms of the matrices by initial unperturbed norms was used. The condition $\partial \mathrm{I}(\lambda)/\partial\lambda = 0$ yields the following equation in terms of matrix (14) SVD.

$$-\sum_{i;i\neq k}\left|\sum_m \tilde{F}_{i,m}\tilde{F}_{k,m}\right|_0^{-2} \sum_{i,k:i\neq k}\ \sum_{m=0} \frac{v_{i,m}v_{k,m}}{\left(s_m+\lambda\right)^3} + \tag{17}$$

$$+\sum_i \left|\Phi_i\right|_0^{-2} \sum_{i,k}\sum_m v_{i,m}v_{k,m}\left(s_m+\lambda\right) = 0,$$

where $v_{i,m}$ are elements of joined matrices $\left[v_{\Phi i_i,i}^{tr}\ v_{\Phi i_i,i}^{fl}\right]$.

The closest to zero value of the expression (17) and corresponding to it λ_{opt} can be found by iterative way numerically.

The optimization procedure for filters bank (13) is similar. The filters banks (9) and (13) are not identical and therefore can be used as two channels of recognizing. The two channels allow to expand the range of capturing desired objects and increase a number of recognitions, which makes it possible to improve the accuracy of recognition by analysing the distribution statistics of recognized objects and selection of most probable ones.

3. Experimental analysis

Analysis of an effectivity of the IRFF on the base of tensor and vector approaches was made using 2600 images of 35 characters of the size from 13×13 to 41×41 pixels, cropped from car license plate images. Up to 200 thousand fragments with the size 13×13 were used for recognition. Methods of errors elimination considered in [11] were used, such as: spectrums combination; amplitude selection; filters brunching by recognized groups of objects.

A. Errors elimination

Initially, the recognition by spectrum (2) in the basis of filters bank (15) is not effective, the number of errors is slightly less then number of successful recognitions even after completed training by selection of the feature true and false frames.

The true features vectors $\Phi_{i_1}^{tr}$ in (14) can be divided on some parts. The filters (15) subbank can be defined for each part. Since the operation of inversion (15) is ill-posed and regularized its results are ambiguous for matrices compiled by different number of columns $\Phi_{i_1}^{tr}$. Therefore, the filters of the full bank (15) will differ from subbanks' filters. This feature allows to check the correctness of the object spectrum (2) estimation by combining the spectrum of filter bank (15) with the spectrum in the corresponding subbanks. If the spectrum matches according to the assessment of the object, then such the assessment is assumed to be correct. Five subbanks by seven filters and seven subbanks by five filters were used. Thus, the evaluation of the object using expression (2) was carried out by combining the results for the filter bank and two subbanks. This procedure allowed to reduce the number of false recognitions by 20-30%.

Let the fragment $\mathbf{x}_{k,i_2,i_3,i_4,i}$ of k-th object has the spectrum (2)

$$\sum_i F_{i_1,i}^{tr} \cdot \mathbf{x}_{k,i_2,i_3,i_4,i} = \sigma_{i_1,i_2,i_3,i_4} \tag{18}$$

which is recognized as j-th object in the training process. Such mistake was caused by the form of the spectrum (18) in the basis of filters bank. The spectrum can be accumulated by averaging as the matrix $\left\lfloor \varsigma_{i,j,m}\right\rfloor_{i,j,m=0...M-1}$. The first index relates with the current object, second index with recognized object and third with M spectral samples (2), (18). This matrix fixes the cases when different objects can include similar fragments. Such frames should be eliminated. The true spectrum can be selected by finding the minimal spectral distance. If spectrum (2) points on the i-th object and

$$\underset{k}{minarg} \sum_{m=0}^{M-1} \left| \sigma_m - \varsigma_{i,k,m} \right| = i$$

then object is recognized as true, otherwise as false.

The purpose of the training process is to obtain true recognition with minimal error level. Not all objects templates will be recognized by created filters basis. The objects which are recognized create the branch of the recognizing. Remained templates can be used for obtaining the next branches while all templates have been recognized. Each true object recognition can be in a single branch. This is a criterion to eliminating errors as well.

B. Dynamic of the filters training

The dynamic of the training process for filters banks of Tensor IRFF, TIRFF (9), vector IRFF, VIRFF (13), and joined two channels of IRFF banks (JIRFF) are presented in table 1. The number of selected as features fragments, the number of true and false recognitions are shown by three corresponding rows for some numbers of training steps.

As it can be seen from the table, the training processes are stable and gradually improve the recognition result. However, the accuracy of using the filters (9) is somewhat lower comparing to using of the filters (13) for the same number of training steps. The reason for this is the fact that feature vectors (7) are projections (6) of the total data (5) onto the main SVD vector. The use of several SVD vectors for data approximation in (6) deteriorate the recognition result – the range of captured objects narrows and the rating of correctly recognized objects also decreases. The range of captured objects by a two-channel filters narrows in the process of training, as well as for filters (9) and (13), which shows that sets of selected features by filters (9) and (13) gradually converges.

Table 1. Dynamic of training process for TIRFF, VIRFF, JIRFF

Training step	Feature fragments	True fragments	False fragments	True rating
0	1462 1163 1783	1043 868 1291	419 295 492	0.7134 0.7463 0.7240
20	1026 943 1446	852 845 1228	174 98 218	0.8304 0.8960 0.8492
40	960 864 1267	819 805 1137	141 59 130	0.8531 0.9317 0.8974
60	920 839 1187	810 791 1090	110 48 97	0.8804 0.9427 0.9182
80	841 806 1050	761 768 977	80 38 73	0.9048 0.9528 0.9301
100	774 794 945	713 760 893	61 34 52	0.9211 0.9571 0.9452

Table 2. Comparison of IRFF with Correlation Filters

Filter type	Feature fragments	True fragments	False fragments	True rating
TIRFF	758	727	30	0.9591
VIRFF JIRFF MECF	782 912 1285	759 878 851	23 34 434	0.9705 0.9627 0.6622
ZACF	1064	923	141	0.8674

The results of recognition by TIRFF, VIRFF, TIRFF and joined two channel JIRFF, obtained as the result of 200 optimization steps, are presented in Table 2. The residual error depends on the bandwidth of the objects captured by the filters. It can be eliminated by decreasing the regularization parameter in (15). Its optimal assessment value is $\lambda_{opt} \sim (0.007...0.012) \cdot s_0$. The results of recognition by Minimum Energy Correlation Filter (MECF) [19] and Zero-Aliasing Correlation Filter (ZACF) [7] are shown in Table 2 for comparison. The IRFFs have the highest selectivity, the number of selected fragments is the smallest, this can explain the smallest number of errors of these filters.

4. Conclusions

The tensor (3) and vector (11) forms of the problem to define features and filter object sets are considered. The tensor form is based on the approximation of the image's template set data matrix by a matrix of unit rank. The data matrix is obtained as the sum of matrices of unit rank given by the tensor product of the vectors of lexicographic representation of image pixels with itself. The vector form is obtained by the accumulation of the analogous vectors (12). The tensor matrix is symmetrical and can be presented as the tensor product of vectors (7). These vectors are the feature vectors. They allow the creation of filters and the recognition of the image object.

The regularized inversion (15) of the matrix compiled by feature vectors as columns in (9) yields the feature filter bank for recognizing the set of objects with given features. The regularization level defines the selectivity property of the filters – the range of objects captured and the level of error.

The feature vectors are selected by a training iterative process. The two approaches considered create two channels of recognition because they are not identical in their selectivity properties.

The inverse regularized feature filters based on tensor and vector forms allow for the selection of image fragments corresponding to desired objects with regulated accuracy. It is possible to achieve high efficiency in object recognition by choosing a capture range for the filter and creating filter branches according to the given ranges.

The known correlation filters [7, 19] have high recognition ability when the full object is captured and foreign objects are absent. We considered an approach to recognize objects by fragments or by some feature fragments. The IRFF can be used in CNN for recognizing complex objects by filtering some branches of their fragments.

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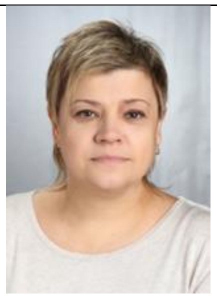

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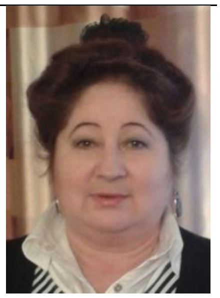

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