Visual modeling technologies in the mathematical training of engineers in modern conditions of digital transformation of approaches to creating visual content

Vitalii I. Klochko[1], Oksana V. Klochko[2] and Vasyl M. Fedorets[3,4]

[1] *Vinnytsia National Technical University, 95 Khmelnytske Hwy., Vinnytsia, 21021, Ukraine*
[2] *Vinnytsia Mykhailo Kotsiubynskyi State Pedagogical University, 32 Ostrozhskogo Str., Vinnytsia, 21100, Ukraine*
[3] *Municipal Higher Education Institution "Vinnytsia Academy of Continuing Education", 13 Hrushevskoho Str., Vinnytsia, 21050, Ukraine*
[4] *University of Educational Management, 52A Sichovykh Striltsiv Str., Kyiv, 04053, Ukraine*

Abstract

The article researches the problem of designing visualization technology for an advanced mathematics course by students of engineering specialities. The main components of visual learning technology are highlighted: the formation of essential educational elements, models of images of learning objects, improving the quality of students' analytical training, and forming their knowledge, abilities, and skills in the use of scientific and methodological apparatus. A feature of the recommended pedagogical technology is that modern digital technologies are used in its implementation.

Increasing the level of mathematical education of future engineers is an essential component of their quality training. This involves understanding the essence of basic concepts and statements studied in advanced mathematics, their interpretation in various sciences, and the ability to build mathematical models and apply mathematical methods in solving applied problems. The article considers the main approaches to establishing interdisciplinary connections in teaching advanced mathematics to students of engineering specialities, particularly mechanical engineering. The importance of using problems that illustrate the need to introduce basic mathematical concepts, which gives motivation and stimulates the study of mathematics, is emphasized. To implement the professional orientation of the advanced mathematics course, considerable attention is paid to applied problems, which contributes to the development of research skills of future specialists. It is noted that mathematical modelling is essential for establishing interdisciplinary connections and forming a scientific and holistic perception of the world. The following types of student activities related to visual and mathematical modelling were determined: development of visual mathematical models related to a specific task; selection and – if necessary – an adaptation of the existing model to solve this task; use of interpretation to make recommendations and predict the behaviour of a particular procedure, avoiding explicit work with models.

Keywords

Advanced mathematics course, mathematical training, future engineers, mathematical modeling, visualization, computer mathematics systems,

1. Introduction

Our time is characterized by modern digital technologies being actively integrated into engineering education, significantly facilitating the learning process and introducing innovative teaching methods [1, 2]. One of the main applications of digital tools as innovative technologies and tools is modern engineering education, which requires advanced tools to ensure a high level of specialist training [3, 4]. Using Artificial Intelligence, Immersive Technologies, Simulation Platforms, specialized Information Systems, and various interactive software tools enables students to better understand and apply complex engineering concepts in practice.

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vi.klochko.7@gmail.com (V. I. Klochko); klochkoob@gmail.com (O. V. Klochko); bruney333@yahoo.com (V. M. Fedorets)
http://klochkovitaliy.vk.vntu.edu.ua/pub (V. I. Klochko);
https://library.vspu.edu.ua/inform/nauk_profil.htm#klochko_oksana (O. V. Klochko);
https://academia.vinnica.ua/index.php/en/pidrozdili/98-pidrozdili (V. M. Fedorets)
0000-0002-9415-4451 (V. I. Klochko); 0000-0002-6505-9455 (O. V. Klochko); 0000-0001-9936-3458 (V. M. Fedorets)

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In the teaching of mathematics, digital technologies undoubtedly play an essential role, in particular through the creation of visual content, such as graphic images, in particular, interactive and dynamic graphics, virtual models and animations, visualized tasks, virtual laboratories, etc. [5, 6]. This enables teachers to effectively explain abstract concepts and creates conditions for students to understand mathematical ideas better. Visual math content enhances understanding of complex concepts and makes learning material more accessible. For example, dynamic graphs can be used to demonstrate the process of changing the value of a function when parameters are varied, and 3D models provide an opportunity to study three-dimensional figures and spatial relations. Also, the use of virtual mathematical laboratories and mathematical simulation programs enables students to conduct experiments with mathematical models and visualize results in real time, which significantly improves their understanding and assimilation of the material by students.

Therefore, the research aims to create visual content based on mathematical models to effectively present the concepts and content of the advanced mathematics course. This is connected with the development of constructive thinking, creative abilities, ingenuity in the selection of possible variants of the developed equipment, their simplification through partial cases, etc. Of great importance in the process of student education is the planned formation of cognitive activity, the main pedagogical goal of which is the development of student's skills in establishing the desired characteristics of objects and connections and relationships between concepts, objects, and phenomena, partial desired characteristics, as well as memorization skills and reproduction of ideas about objects.

A semantic cluster analysis of modern research by scientists was carried out using the VOSviewer system [7] based on data from the Scopus scientometric database [8] (see figure 1). A search using the keywords "visualization And engineering And learning And mathematics" found 130 scientific papers published during 2019-2024.

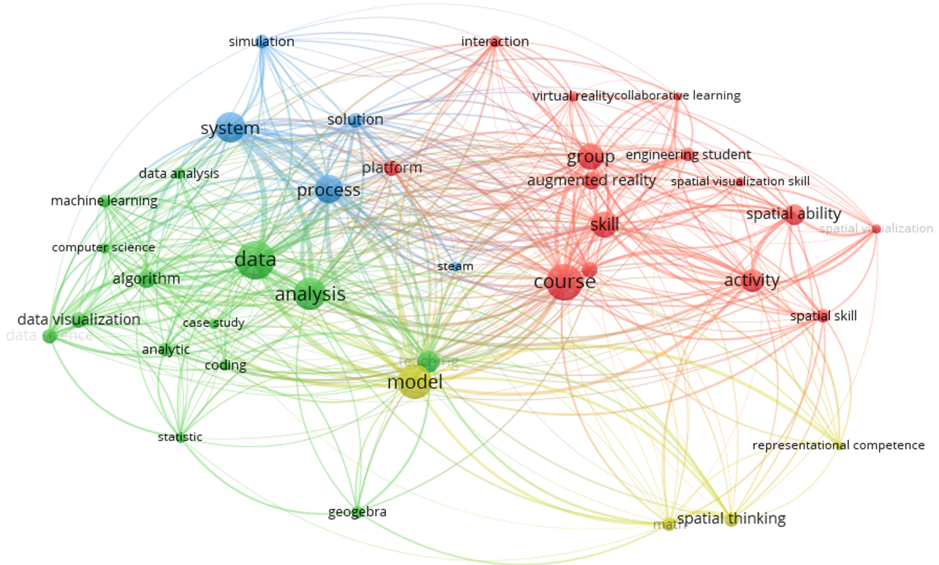


Figure 1: Visual representation of the semantic clustering of the content of 130 research papers on request "visualization And engineering And learning And mathematics" placed in the scientometric database Scopus [8] during 2019-2024. Made using the scientific landscape visualization system VOSviewer [7].

The figure shows clusters, the content of which characterizes the search directions of scientists using the keywords "visualization And engineering And learning And mathematics" (see figure 1). We can highlight the following main areas related to the mathematical training of future engineers using visualization (see figure 1):

1. Block of mathematical modelling: modelling, mathematics, spatial solutions, representational competence.

2. Educational block: courses, skills, engineering students, collaborative learning, virtual reality, augmented reality, interaction, activity, spatial visualization, spatial skills, visualization skills.
3. Block of intellectual analysis and machine learning: data analysis, data visualization, statistics, analytics, algorithmization, machine learning, computer science.
4. Block of information processes and systems: systems, processes, simulation, solutions, STEAM.

The development of the latest technologies is an integral task of mathematical modelling and computer engineering specialists since the rapid development of modern technologies inevitably leads to the moral ageing of various devices and systems [9]. It has been established that the fundamental core of the training of software engineers should ensure that students achieve leading learning outcomes [10]. Suppose the theoretical base is based on the principles of language functioning and instrumental and computing tools for software development. In that case, mathematical modelling methods require more and more new approaches depending on the state of development of computer engineering. It is also reasonable to assume that some engineering technologies, such as 3D printing, simulators, and virtual/remote laboratories, significantly affect the effectiveness of engineering education.

In engineering training programs, the fundamentals of statics and the strength of materials necessary for designing complex mechanisms are offered for study. For example, the Computer Mathematics System MATLAB is used to solve problems with numerical and analytical calculations; the authors and experts of the programs M. Dupac and Dan B. Marghitu [11] consider the static behaviour of engineering objects and their components based on the knowledge of the mechanics of materials. Researchers analyze fundamental concepts, logical conclusions, and interpretations of general patterns. They also consider methods of developing mathematical models and formulating mathematical equations. In the work, the authors give examples of problems solved analytically and numerically with the help of MATLAB [12], while emphasizing that numerical solutions strengthen the visual learning of both students and professionals.

Authors U. Kohut and M. Shyshkina consider the problems of using Computer Mathematics Systems as a tool for providing a fundamental component of education, as well as systems for developing applications that provide a number of ready-made tools for working with databases, analytics, and other aspects [6]. An important place among Computer Mathematics Systems is occupied by the Maple, Maxima, and Mathcad systems, which enable the user to create an intellectual environment for educational and scientific research, provide interactivity, convenience and flexibility of learning, and also contribute to the development of practical skills necessary for professional activity. The use of digital tools in the study of mathematics allows for the creation of individualized approaches to learning, which contributes to the deepening of students' understanding and interest in the subject.

The requirements for engineers are constantly changing due to the development of technology and automation. Traditionally, during the selection and evaluation of engineers, the primary attention is paid to the knowledge of mathematics and physics. However, engineering requires a much more comprehensive range of skills, including creativity, intuition, and abstract thinking. Such a narrow focus on mathematical and physical knowledge can alienate potentially talented engineers who have other strengths. Researchers Ph. Isaac and D. Bergsagel [13] propose to develop a new assessment method that will reveal the creative and intuitive abilities of potential engineers. This method can be useful for selecting students and popularizing engineering among the general public.

N. Kulcsár and É. F. Szakos, address the problem of engineering education, which is characterized by the ever-increasing complexity of engineering problems and possible solutions [14]. Modern engineering problems are becoming increasingly complex, which requires specialists to have a deep understanding of mathematics. However, many engineering students lack the motivation to study this discipline. Scientists N. Kulcsár and E. F. Szakos suggest using visualization as an effective tool to overcome this problem [14]. They believe integrating neurocognitive research and adult learning theories into mathematics teaching can significantly improve student understanding.

Probability and statistics are essential topics in many professional fields, including engineering, where statistical data is used for analysis. The basics of probability are also often applied to decision-making in everyday life. Despite this, many students struggle with the study of probability and statistics. As the study's authors D. Raviv and D. R. Barb notes, math can be both exciting and frustrating, in part due to the lack of clarity and intuitiveness of the learning materials, which are often not connected to real life [15]. This article emphasizes a visual, intuitive, practical, and engaging approach to studying the normal distribution [15]. This approach involves providing students with simple examples that help them better understand the topic and deal with equations and calculations [15]. The article illustrates the concept through examples, visualizations of essential and critical concepts, real-life examples, puzzles, and experiments [15]. The author also used this approach in teaching statics, calculus, algorithmization, "Differential Equations, Control Systems, Digital Signal Processing, Newton's Laws of Motion" [15]. In all these cases, the students highly appreciated the approach and found it effective for learning [15].

Mechanical engineering is a branch closely related to the design and implementation of mechanical, thermal, or energy systems to improve the conditions of human life. As noted by the authors of the article E. Marquez, S. Garcia, and S. Molina, the successful work of an engineer requires good mathematical training and a deep understanding of the physics of processes in order to effectively apply analytical and numerical methods in design [16]. However, traditional teaching methods that emphasize the formal presentation of material can make it difficult for students to understand and apply engineering design principles. To solve this problem, the study proposes improving the educational process by including physical visual applications during lectures on the discipline "Engineering Mechanics" in the first year [16]. In particular, three visual models have been developed [16]: a crane, a four-cylinder engine, and a bridge model in Baltimore. These models differ in that they include a real-time monitoring system that allows students to see how the models work and better relate theoretical concepts to practical applications [16]. Thanks to this, with the help of visualization, students get an idea of calculation requirements, design, possible sources of problems, and ways to reduce costs.

Scientists have made a significant contribution to the development of visualization in teaching mathematics, but this problem remains relevant today.

The *purpose of the research* The purpose of the research is the development of Visual Modeling Technologies in the Mathematical Training of Engineers in Modern Conditions of Digital Transformation of Approaches to Creating Visual Content.

2. The methodological basis of the study

The analysis of scientific literature on the problems of visualization in education shows the variety of publications in scientific journals and monographs that clarify the problems of visualization in different contexts.

The term "visualization" is ambiguously explained in psychological and pedagogical literature. Some consider visualization to be the presentation of numerical and textual information in the form of graphs, diagrams, structural diagrams, etc. Others see visualization as the process of presenting data through video, augmented reality, virtual reality, etc.

Data visualization expert E. Tufte stated, "A good data visualization displays every aspect of the data in the best possible way and allows the viewer to follow the relationships between the data and see patterns in the data" [17].

A visual object is an action object, which helps us get new information about the studied object. The visual object, in turn, is a model of this researched object. Applying the modelling method, the researcher studies the object of interest through its models. So, the model is a connecting link between the researcher and the object of knowledge. In modelling, the researcher studies an auxiliary artificial or natural system that correlates with the researched object at a certain level of abstraction according to the research direction. With the help of such an auxiliary system, which under certain conditions can replace the object in the process of tracking, we examine the simulated object. Such an object model can be presented using various visualization methods.

In the process of the research, a *research hypothesis* was formed that the level of assimilation of the content of the advanced mathematics course by students of engineering and technical specialities will increase if it is accompanied by visual modelling of mathematical educational information.

The analysis of scientific sources related to the creation of didactic means of managing students' mental and mnemonic learning activities using the principle of visualization provides grounds for theoretical and experimental research into the problems of visual modelling of didactic mathematical objects (concepts, means of mathematical language, processes, etc.). To solve the problem, didactic tools are needed, with the help of which the task of combining figurative and symbolic elements of the educational process would be solved. The creation and use of educational models and schemes of didactic objects that have figurative and conceptual features and provide the formation of knowledge and skills, as well as the skills of independent cognitive activity in mathematics classes, are especially relevant. This is due to the fact that the vast majority of processes, concepts, and phenomena, due to the nature of mathematics, cannot be offered in a natural form.

The task of the research is to determine the main components of the visual modelling technology according to the main components: formulation of the research problem and presentation of the description of the research object; formation of essential educational elements (concepts, theorems, problems, algorithms for solving problems, means of evaluating acquired knowledge); building a model of images of learning objects (graphics, texts, animation, video, images of objects); improving the quality of training students in mathematics, forming their knowledge, abilities and skills in the use of research equipment and digital technologies in the process of substantiating and evaluating problem solutions.

3. Results and discussion

Visual modelling as a complex multidimensional cultural and educational phenomenon is based on the "visual-spatial-cognitive nature of man" as a spatial creature - Homo Spatialis (Latin). The essence of a person's visual-spatial-cognitive nature is his ability to "master" three-dimensional space, starting from "elementary" spatial perception and orientation to developed spatial thinking, which in specialists of engineering specialities has its own professional and technological features and integrates with other formats of intellectual activity - analytical-synthetic, creative-analytical, mathematical, technological, etc. That is, such a specialist must see, imagine, and "understand" space and things in this space as a place, an environment and at the same time as a system of tools for intellectual and professional activity and, accordingly, be able to operate with this space as well as with things, as well as be capable of "spatial interpretations", understanding and creation of "new" spatial phenomena.

Along with the need for "mental mastery" of space by the future engineer, a necessary factor in forming his professional competence is mathematical thinking and, at a certain level, mastery of mathematics as a tool of professional activity. Mathematical disciplines that are "ideal-abstract" in nature and improve the analytical-synthetic potential of the intellect in their applied sense are the keys to understanding life and professional phenomena. Accordingly, the professional training of the future engineer includes two complementary areas - mathematics and related to work with spatial phenomena- which are presented in the form of professional competencies in design, analysis of the condition of machines, structures, etc. Therefore, it is expedient in the process of mathematical training of an engineer to actualize the applied aspect, which is aimed at forming the professional thinking of a future specialist with expressive spatial and mathematical analytical-synthetic components.

The need to use visual modelling in teaching mathematical disciplines is also due to the "methodical need" to present mathematics and, above all, geometry in spatial-visual formats. The specified methodical inquiry is also based on the "visual-spatial-cognitive nature of man" in which the defining systemic components of the development of human intelligence are the interaction of a person with the environment and, above all, with three-dimensional space and with time, which is realized by the development and functioning of visual and auditory analyzers. Therefore, the specified methodological request for visualization of mathematical concepts, ideas, and problems as spatial phenomena is natural. Through the educational process, we implement visualization through visual modelling using digital technologies and visual, illustrative material. Based on the integrative application of phenomenological, structural-functional, and hermeneutic approaches to the approach, we distinguish the following main *formats of visual modeling* – visual-spatial representation, visual-spatial illustration, visual-spatial clarification, visual-spatial metaphorization, visual-spatial interpretation, visual-spatial algorithms, and schemes. We will reveal the essence of the above-mentioned visual modelling formats, which we also consider in relation to the *system-organizing principles* based on which they are developed (see figure 2).

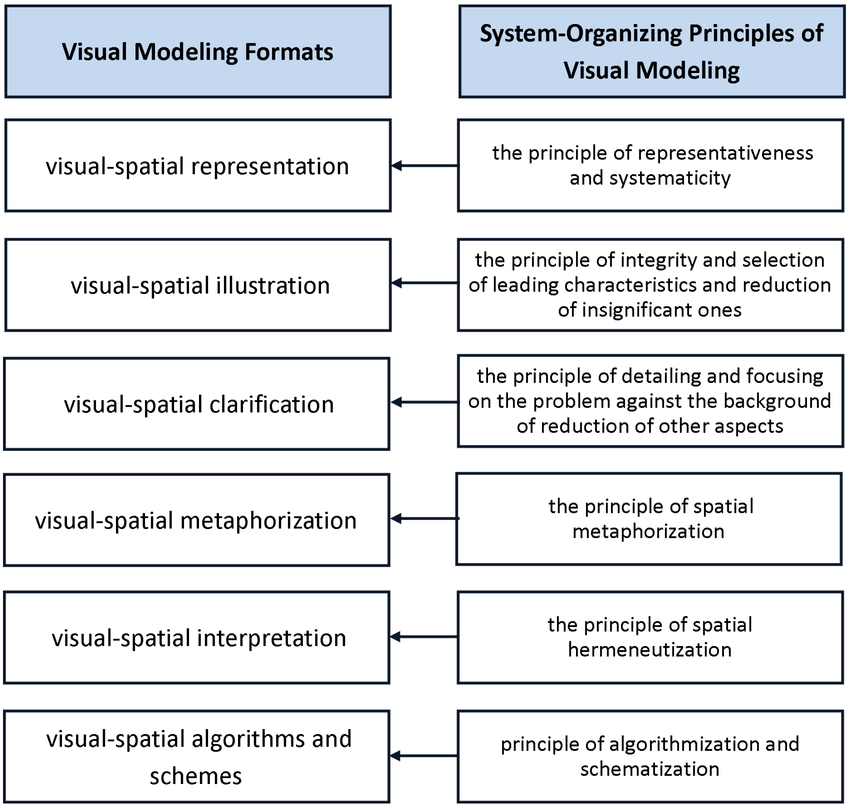


Figure 2: Interrelationship of visual modelling formats and system-organizing principles.

Visual-spatial representation is a sufficiently (“maximum”) complete and comprehensive visual representation of mathematical phenomena, ideas, concepts, and problems using visual modelling. An important criterion is that such a model can represent mathematical phenomena relatively comprehensively, autonomously, and “independently” being quite “independent” of the teacher’s explanations. Such models explain relatively simple mathematical and, above all, geometric phenomena. When applying this format of models, we are also guided by the ideology of narrative and dialogic approaches. This includes considering relevant mathematical issues, using the teacher’s stories (explanations) and dialogic practices to solve specific problems. This is because mathematics is, first of all, a semiotic-symbolic system and visualization from a neuropsychological point of view mainly uses images. Text models are an exception to this plan but also need “revitalization” (updating) through the teacher’s articulation, dialogues, and polylogues. The specified format corresponds to *the principle of representativeness and systematicity*, the essence of which is the presentation of a mathematical phenomenon as a system, as well as in its maximum, and that in “excessive” completeness and expressiveness, which includes examples of practical application.

Visual-spatial illustration is such a visual presentation of mathematical phenomena with the selection of general and most characteristic features against the background of the reduction of other aspects. This format often plays a complementary, illustrative role when considering a specific material. This presentation format is synergistic and complementary to the process of explanation or dialogue on a particular issue. The specified format corresponds to *the principle of integrity and selection of leading characteristics and reduction of insignificant.* This principle is a “visual problematization” of a specific phenomenon.

Visual-spatial refinement is such a visual representation of mathematical phenomena with the selection of private features that are considered significant against the background of the reduction of general and attributive features while preserving the idea of integrity. This format of visual modelling is relevant because it provides an opportunity to expand, deepen, and clarify specific characteristics of mathematical phenomena that may be significant. The specified format corresponds to *the principle of detailing and focusing on the problem against the background of reduction of other aspects*. In essence, this principle is a "visual problematization" of a specific aspect of some mathematical phenomenon as a detail or aspect of the whole. At the same time, such detailing and clarification make it possible to single out a specific detail as a separate phenomenon and determine the ways of working with it and its significance.

Visual-spatial metaphorization is a visual representation of mathematical phenomena based on the use of metaphors that can have a spatial representation (pictures, landscapes, images of objects, etc.). Metaphorization is a significant cognitive technique that determines an emotionally saturated vivid perception of educational material and creates appropriate cultural-emotional and professional-emotional semantic contexts. The specified format corresponds to *the principle of spatial metaphorization*, within which it is expedient to actualize the semantic, hermeneutic, value-symforce, and emotional-semantic potential of metaphors for illustrating mathematical phenomena.

Visual-spatial interpretation is a visual representation of mathematical phenomena using a hermeneutic approach, which includes a "variational-interpretive" representation of the material, changing some conditions and factors of the issue under consideration and semantic contexts. The specified format corresponds to *the principle of spatial hermeneutization*, within the framework of which it is appropriate to actualize the variable presentation of the material and, accordingly, its variable understanding and interpretation.

Visual-spatial algorithms and schemes are a visual representation of mathematical phenomena based on the integrative application of structural-functional, phenomenological, and systemic approaches, including their schematization and algorithmization through images, symbols, and drawings. The specified format corresponds to *the principle of algorithmization and schematization.*

The use of schemes and models of didactic objects and concepts in the mathematical training of students of engineering and technical specialities shows that using different types of visualization is expedient. This makes it possible to take into account students' individual capabilities, provide educational content by varying the levels of difficulty of educational material and tasks, and also support complex mental learning processes.

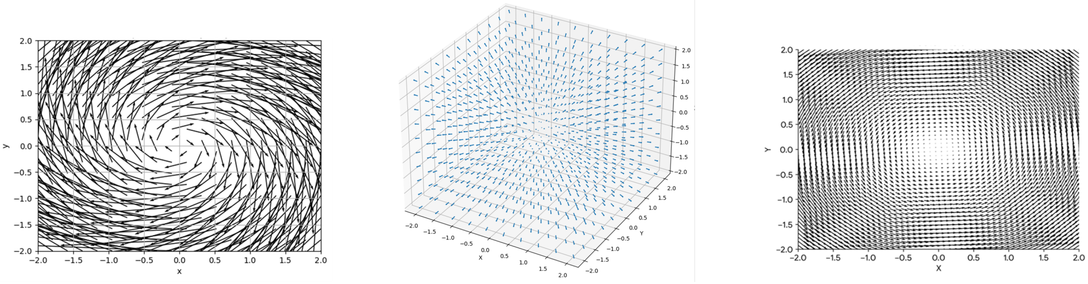
One of the many possible examples of visualization of the presentation of educational material is an example of the theory of ordinary differential equations, which is somehow related to the dynamics section of mechanical, electrical, and radio engineering and other disciplines or, for example, to the oscillatory process in a nonlinear electric circuit.

For example, vehicle dynamics can be considered as applied mechanics and, using as a research tool the theory of ordinary differential equations, reduced to Newton's second law, written as a $\vec{a} = \vec{f}(t, \vec{x}, \vec{v})$, where t is time, and $\vec{x}$, $\vec{v}$ and $\vec{a}$ are, respectively, the vector of the movement of the centre of gravity of the car, the speed and acceleration of the centre of gravity of the car. Not all students recognize, do not connect and, therefore, do not use the properties of Newton's equation studied in the advanced mathematics course in the form $\frac{d^2\vec{x}}{dt^2} = \vec{f}(t, \vec{x}, \frac{d\vec{x}}{dt})$, where t is time and $\vec{x}$ is a spatial function, $\frac{d\vec{x}}{dt}$ and $\frac{d^2\vec{x}}{dt^2}$ are, respectively, the first and second derivatives of the direction of motion of the centre of gravity of the car. Interviewing teachers of technical disciplines showed that the problem is that students do not remember this mathematical knowledge, or perhaps they did not understand the meaning of these mathematical objects during the advanced mathematics course.

So, we will need a deeper understanding of the sciences that underlie the art of engineering, and we will need to know what mathematical skills are required to apply those sciences. Since progress in the use of digital technologies in mathematics education, in particular, Computer Mathematics Systems, has affected engineering analytical methods, production, and management processes, the following questions arise: what is and will be the role of mathematics in engineering education? What math skills do the engineers of tomorrow (engineers 4.0) need, and how and when are they best acquired?

For example, using the application of modern digital technologies, their capabilities to operate with mathematical analogues, in particular, cloud platforms (Google Collaboratory, Processing, Kaggle, GitHub, etc.), artificial intelligence systems (MathAI(GPT)/MathGPT/Math Solver(GPT), Gemini etc.), programming languages with appropriate libraries, such as Python (SymPy, Matplotlib, NumPy, Math, SciPy, Statsmodel, Scikit-learn, etc.) or JavaScript (MathJs), etc.

For example, if you ask “graph the vector field” using the chatbot based on artificial intelligence ChatGPT [18], you will get a detailed explanation of how the 2D and 3D vector field function looks like, as well as a description of the steps of building 2D and 3D vector fields with the corresponding implemented algorithms in the Python programming language. As a result of executing these program codes, we will get the images presented in the figures 3 (a and b).

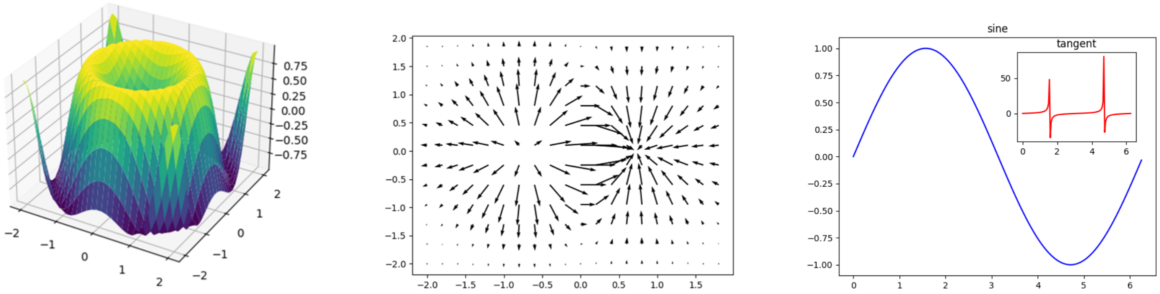


(a) (b) (c)

Figure 3: Graphs that can be obtained with the help of chatbots based on artificial intelligence by the request “graph the vector field”: (a) obtained by request in ChatGPT [18], a 2D graph of the vector field $F(x, y) = (y, -x)$ is constructed based on code provided by the system in the Python [19] programming language in Google Collaboratory [20]; (b) obtained by request in ChatGPT [18], a 2D graph of the vector field $F(x, y, z) = (x, y, z)$ is constructed based on the code provided by the system in the Python [19] programming language in Google Collaboratory [20]; (c) graphic image received on request in Gemini [21].

If we enter the query in the Gemini [21] chat bot “graph the vector field”, we will receive the image illustrated in the figure 3 (c).

For the development of mathematical graphic content, they are provided, for example, by the Matplotlib library in the Python [19] programming language. Figure 4 shows different options for graphs built using the matplotlib library in the Python [19] programming language in the Google Collaboratory [20] system.



(a) (b) (c)

Figure 4: Examples of graphs constructed using the Matplotlib library in the Python [19] programming language in the Google Collaboratory [20] system: (a) Surface plot, using the $plot_surface(x, y, z)$ function; (b) Quiver Plot, $quiver(x, y, u, v)$ function is used; (c) Multiplots, $subplot()$, $sin()$, and $tan()$ functions are used.

The technology of visual modelling during the study of the course of advanced mathematics at technical institutions of higher education will be presented in the following scheme (see figure 5):

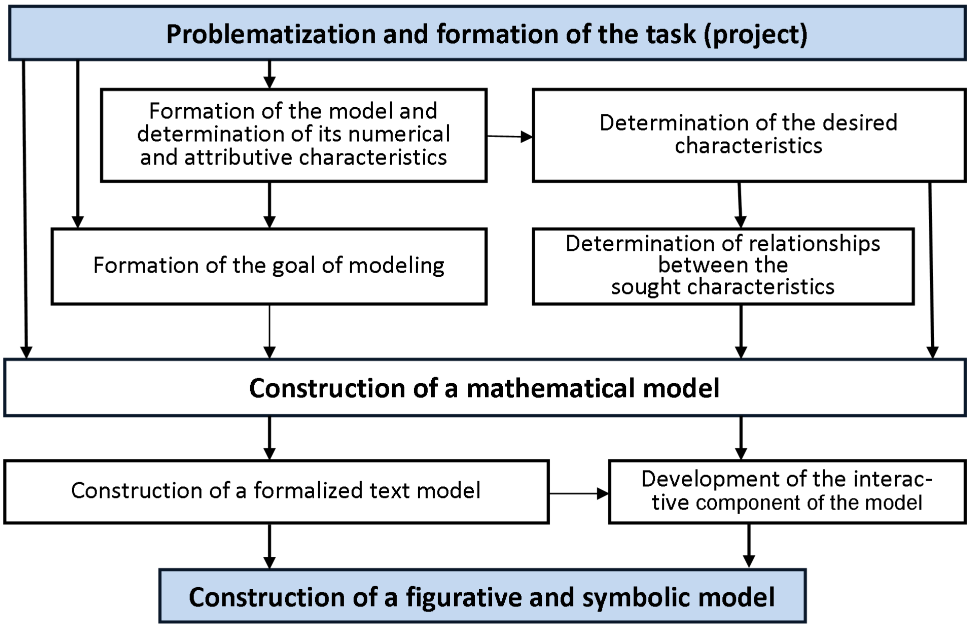


Figure 5: The technology of visual modelling during the study of advanced mathematics course at technical institutions of higher education.

1. *Statement of the problem, formulation of project tasks.* The description of the physical object or task is presented (formulated) in the form of a project, a typical calculation, a topic of a conference report, etc.
2. *Formation of the model and determination of its numerical and attributive characteristics.* Numerical characteristics of the object that are set and obtained based on the results of the task. The model performs the heuristic function of highlighting all the general characteristics of the objects being studied. Suppose visualization reproduces only the external sides of the object. In that case, modelling is a holistic reproduction of separate and general, sensual and logical, internal and external
3. *Building a mathematical model.* Since the task is educational, the formation of the mathematical model is carried out with the participation of the teacher. It can be an equation, a system of equations, etc.
4. *Determination of the desired characteristics.* Estimates of numerical characteristics are found according to the constructed model(s).
5. *Determination of relationships between the sought characteristics.*
6. *Formation of the goal of the modelling process:* to find values (estimates) of numerical characteristics of the model.
7. *Construction of a formalized model.* General principles of building a formalized text model.
8. *Development of the interactive component of the model.* At the same time, it is essential that the visual representations are interactive, in which the researcher could change the initial conditions and process parameters and observe changes in the behaviour of the image of the object being studied. This will make it possible to put forward reliable hypotheses for study, correctly determine the directions of research and make corrections to the constructed model.
9. *Construction of a symbolic model.* General principles of building a symbolic model. While modelling nonlinear circuits (with transistors, diodes, etc.), there is a problem presenting the nonlinear current-current characteristics of the corresponding active devices in an analytical form. ModellingIn some cases, modelling is carried out using a system of two nonlinear differential equations. Computer Mathematics Systems (Mathcad, Maple, Maxima, or others) are used to solve them. If, as a result of simulation, the current and voltage are considered as functions of time in the form of stationary or almost sinusoidal oscillations, then in electronics this determines

that the device performs the functions of a generator of high-frequency oscillations. Principles of finding partial connections. Construction of a mathematical, symbolic model in the form of a system of equations. An example of building the given models for a specific task by recoding information.

Let us consider an example of modelling the mechanical processing of parts using differential equations.

A brief description of the project is as follows: During the implementation of the project task, students should familiarize themselves with the mathematical models that correspond to the elements of the scheme and its possible simplification, as well as the mathematical models that correspond to the simplified schemes of the given process.

Full description of the project: The task of dynamic modelling of the process of surface plastic deformation of a part with a fixed hydraulic damper in the centre during mechanical processing is under consideration (see figure 6).

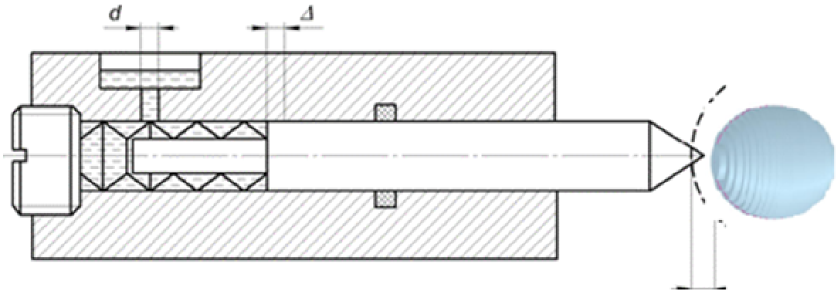


Figure 6: Scheme of dynamic modelling of the surface plastic deformation process with a hydraulic damper [22].

Formulation of the task: determine the model equation, find the solutions, plot the graphs of the solutions and estimate the errors at the points "far" from Ox, solve the equations by numerical methods, use the isocline method to compare the results, conduct computational experiments, prepare for control. Analyze the tension curves at different frequency values ω and conclude the dynamics of the process. This determines the angular velocity at which work is impossible because the contact between the tool and the part is not constant but periodically repeated.

The colour scheme of the physical model can affect the general atmosphere of the visual space and, therefore, the student's well-being. Therefore, the academic chooses colours that evoke positive emotions in students and, at the same time, attract attention or strengthen the verbal part. The content of concepts and theorems should be highlighted in colour. Colours enhance the perception of content, instantly creating associations with physical objects in the student's mind.

The mathematical model is a differential equation that describes the process of surface plastic deformation of the part fixed in the centres with the beating b has the form [22]:

$$mx'' = Cx - k\left(B + x + \sin(\omega t)\right) + ES\left(B_1 - x - b\sin(\omega t)\right) \cdot sign\left(B_1 - x - b\sin(\omega t)\right),$$

$$\frac{dx}{dt} = x_1(t), m\frac{dx_1}{dt} = mx'',$$

where

m — total mass of the tool, kg;

b — the beating of details, mm;

C — damping coefficient, H/mm/s;

k — stiffness of the spring, N/mm;

B – pre-tension of the spring, mm;

ω — angular velocity detail, rad/s;

E — stiffness of the material, H/mm^2;

S – contact spot area, mm^2;

B_1 – previous static tension of the tool in detail, mm.

In the above differential equation, the $sign(x)$ function is used to simulate the force action of the part surface on the tool in the presence or absence of contact. With the help of the Mathcad system, students tried to solve the corresponding differential equation related to nonlinear differential equations. So far, it is impossible to obtain the equation's solution in an analytical form. Therefore, students were familiarized with the corresponding functions of the numerical solution of systems of differential equations and the construction of graphs of solutions in the Mathcad environment (see figure 7 and figure 8). For specific parameter values, the system of differential equations (at $t > 0$) takes the form:

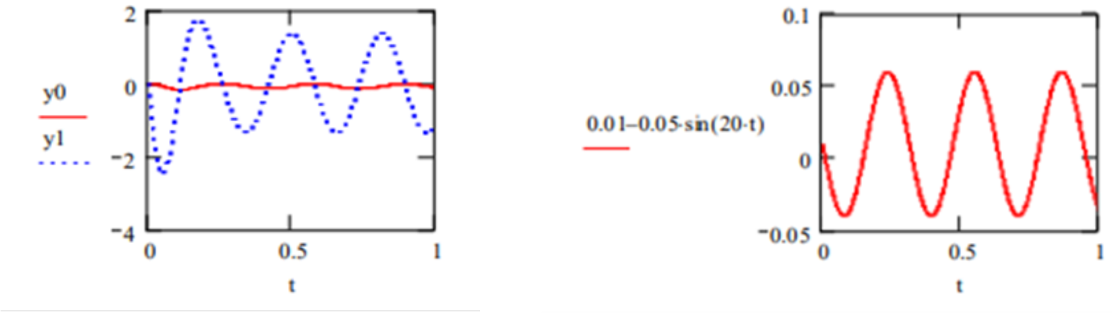
$$\frac{dx}{dt} = x_1(t),$$

$$m\frac{dx_1}{dt} = [10 \cdot (0.01 - x - 0.05 \cdot \sin(20 \cdot t)) \cdot \sin(0.01 - x_1 - 0.05 \cdot \sin(20 \cdot t))] - 20x_1 -$$

$$-100 \cdot (1 + x + 0.05 \cdot \sin(20 \cdot t))$$

$$x(0) = 0, \quad x_1(0) = 0.$$

With the help of Computer Mathematics Systems, dynamic processes are visualized. If you create a series of graphic drawings, they can be animated. For example, several figures for the movement of the impactor (figure 6) have been constructed, which are solutions of differential equations (see figure 7). Immobile graphs are placed one under the other as animation frames; then, with the help of appropriate commands (implemented in many Computer Mathematics Systems), the motion graph is implemented on the computer screen. With the help of appropriate commands in Computer Mathematics Systems, you can change the direction of the object's movement, speed it up or slow it down, or pause it.



(a) (b)

Figure 7: Graphs of solutions of differential equations built in the Mathcad environment.

$$S = k_1 \left(1 + B_1 - x - b \cdot \sin(\omega t)\right)^3$$

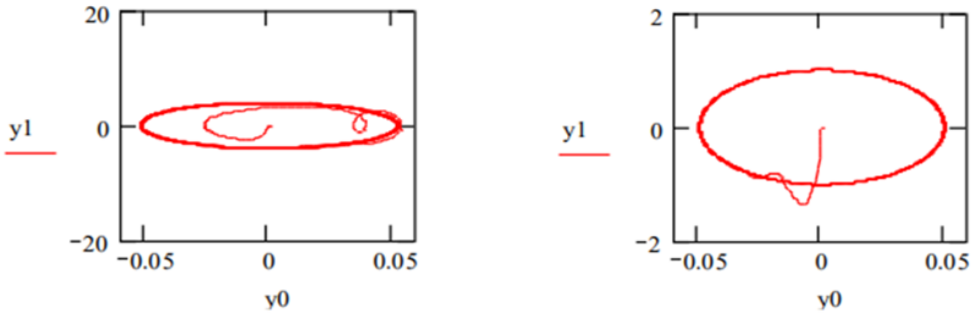
where

$$k_1 = 1, k = 1000, \omega = 100 rad/s, b = 0.01 mm, C = 200 H(mm/s), B_1 = 0.01 mm,$$

$$ES = 100 kH/mm, B = 1 mm.$$

To a certain extent, it can be argued that with the help of constructed graphs, the solution of differential equations can be approximately determined using various approaches.

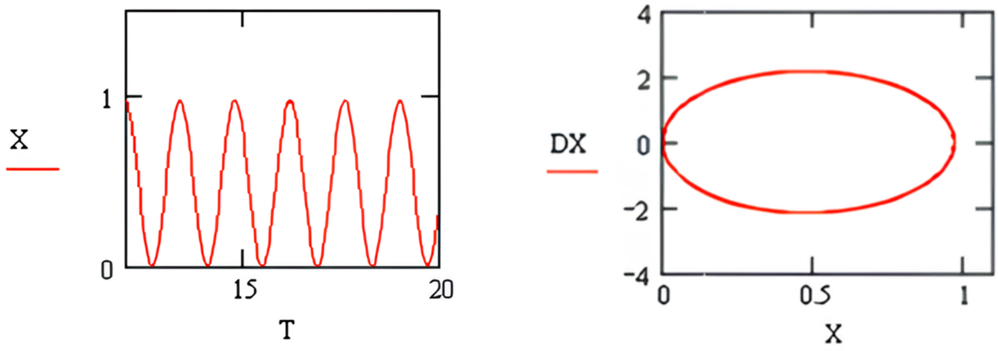
Figure 9 shows an example of the phase portrait of the Cauchy problem, constructed according to the numerical solution, and figure 10 shows the phase portrait of the same problem, constructed



(a) (b)

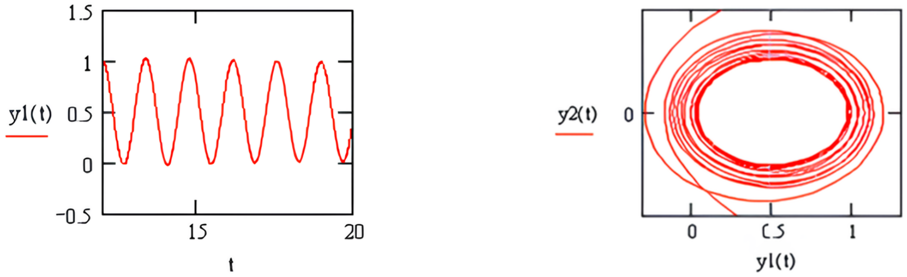
Figure 8: Phase portrait at the critical frequency $\omega = 76.03$ rad/s (a) and at the frequency $\omega = 20$ rad/s (b), built in the Mathcad environment.

according to the approximate solution. The phase portraits in both figures are characterized by stable limit cycles, i.e., for any processes, the phase trajectories collapse to this cycle. The analysis of the obtained approximate solutions allows us to determine the parameters of the investigated process.



(a) (b)

Figure 9: An example of a phase portrait of the Cauchy problem constructed by numerical solution, built in the Mathcad environment.



(a) (b)

Figure 10: An example of a phase portrait of the Cauchy problem constructed by an approximate solution built in the Mathcad environment.

The analysis of the phase trajectories in figures 9 (b) and figures 10 (b) shows that the dynamic system has a stable position of equilibrium under various external actions on the device. This is a limited cycle. The initial conditions are in the middle of the cycle, and the phase trajectory is a spiral unfolding towards the cycle. The limit cycle determines the presence of stable periodic oscillations in the system.

Thus, the given technology of visual modelling from the use of technical and software tools of Computer Mathematics Systems when performing typical calculations, preparing a report for a conference, etc., can be presented as follows: Students independently prepare for the task, perceive the task, study the theoretical material, perform preliminary calculations, obtain the exact solution of the equation. The teacher formulates the statement of the task, the goals of the task, and their motivation. With the help of digital learning technologies, individual tasks are generated, and students' knowledge necessary for simulation is tested.

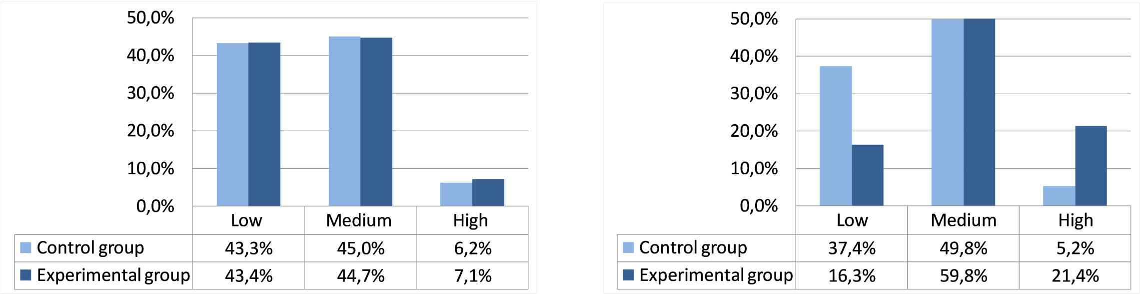
4. The experimental work

According to the set goal and hypothesis of the research, the pedagogical experiment was carried out with engineering and technical specialities students in the second and third academic semesters. The experimental work involved Vinnytsia National Technical University, Lutsk National Technical University, and Pavlo Tychyna Uman State Pedagogical University students. Experimental (47 students) and control groups (53 students) were formed to conduct a pedagogical experiment to assess the level of understanding of mathematical concepts, the ability to apply mathematical methods of visual modelling, technological literacy of the process of mathematical visual modelling, critical thinking, and independence. The experimental group was taught using modern systems of computer mathematics and visualization of mathematical objects, while the control group studied the material using traditional methods.

Both groups were tested before the experiment started to determine their initial level of knowledge and skills in mathematics. The results showed no significant differences.

In the experimental group, training was conducted using modern computer mathematics systems and visualizations of mathematical objects, particularly interactive visualizations and virtual laboratories. Traditional classes were held in the control group.

The experiment used the following pedagogical research methods (see figure11): observation, conversation, questionnaire, and discussion. After the end of training, testing was conducted in the experimental and control groups to assess the level of mastery of the material.



(a) (b)

Figure 11: Distribution of students according to the level of formation (low, medium, high) of skills before (a) and after (b) experimenting, respectively, in the control and experimental groups.

Training effectiveness was determined using the developed criteria for the formation levels of mathematical visual modelling skills among students while studying in the advanced mathematics course. Figure 11 shows the distribution of students according to the levels of formation of the specified skills before and after the experiment, respectively.

Figure 11 shows the differences in the distribution of points in the groups. After the formative stage of the experiment, the number of students in the experimental groups who had a high level of knowledge and skills in information technologies increased by 14.3%. In contrast, in the control groups, it decreased by 1%. The increase in the number of students in the experimental groups with a medium level of formation of mathematical visual modelling skills is 15.1%, and in the control groups, it is 4.8%. The decrease in the number of students in whom the formation of knowledge and skills at the low level in the experimental groups is 27.1%, and in the control groups – 5.6%.

Thus, the results of the statistical processing provide a basis for the conclusion that the use of the proposed technology improved the results of learning advanced mathematics. Accordingly, the question arises: Did this factor affect the success of studying advanced mathematics during the semester? To solve this issue, a non-parametric method of testing statistical hypotheses was used—the $X2$ method.

According to experimental data, the matching criterion $X^2_{\text{exper}} = 14.86$. We will use the X^2 criterion table to test the null hypothesis. In our case, the number of degrees of freedom is $n = 3$. According to the table, we find that for three degrees of freedom at the confidence level of 0.99 $X^2_{\text{contr}} = 11.35$, that is, $X^2_{\text{exper}} > X^2_{\text{contr}}$.

Therefore, the statistical processing results give grounds for the conclusion that the use of the proposed technology improved the results of learning advanced mathematics.

The results of the formative stage of the experiment show that the purposeful use of visual modelling and the means of its implementation in the process of learning the advanced mathematics course has a positive effect on the process of forming students' mathematical educational and research skills. The application of the proposed components of the advanced mathematics course technology contributes to the formation of students' abilities and skills in the use of modern mathematical content visualization systems; abilities and the desire to adapt to the rapidly changing information environment of activity increases the practical significance of the results of mathematics education.

During the experimental research, based on the analysis of the characteristics and results of the student's training, it is possible to draw a conclusion about positive professionally oriented changes in the cognitive, need-motivational, emotional-volitional, and behavioural spheres of the respondents. Accordingly, as empirical evidence of the effectiveness of the use of visualization technologies in mathematics to improve learning outcomes, we consider the changes observed in the professional and psychological dimension of the individual, namely:

- increased interest and motivation for learning;
- formation of a holistic vision of tasks and problems;
- actualization of the ability to problematize and its visual presentation;
- the ability to quickly and professionally conceptualize the task with the subsequent formation of a concept presented in a visual format (details, geometric figures, block diagrams, etc.);
- increasing speed of operation with mathematical algorithms;
- cognitive skills to quickly find inconsistencies and errors and effectively correct them;
- professionally oriented orientation and specification of mathematical knowledge, which determines their inclusion in the composition of professional competence and subsequent application as a tool of the engineer's applied activity;
- arbitrary manipulation of the dimensions of spatial objects;
- activation of spatial, logical-discursive, and mathematical thinking as tools of professional activity;
- integrative application of mathematical and spatial thinking as tools of professional activity;
- development of professionally oriented value orientations;
- mastering professional discourse.

The ability to arbitrarily concentrate attention on the subject of study for a long time, emotional harmonization and balance, actualization of emotional intelligence, and enthusiasm for activities are also observed.

Based on the analysis of experimental research results, scientific literature data, and our application of visualization technologies, including the main visual modeling formats highlighted above, we have developed practically oriented recommendations aimed at the practical implementation of visualization technologies in the practice of teaching mathematics for specialists in engineering specialities:

1. *Selection of visualization formats.* Integratively and individually apply the different formats of visual modeling highlighted above (visual-spatial representation, visual-spatial illustration, visual-spatial refinement, visual-spatial metaphorization, visual-spatial interpretation, visual-spatial algorithms, and schemes).
2. *Selection of software tools.* Software tools are selected according to educational requirements and innovative orientation.
3. *Professional and educational motivation.* Promote the development of a professional motivational factor when using visual modelling.
4. *Intellectualization.* To stimulate the development of spatial, logical-discursive, and mathematical thinking as tools of professional activity in applying visual modelling; integrative application of mathematical and spatial thinking as tools of professional activity; and the ability to arbitrarily operate with the dimensions of spatial objects.
5. *Creative focus.* Develop visualization tools as an illustration of mathematical phenomena and as a factor in activating professional creativity.
6. *Harmonization of emotions and development of emotional intelligence.* Contribute to the development of a positive emotional background, good mood, harmonization of emotions, and development of emotional intelligence in the process of visualization.
7. *Aesthetics of professional activity and ergonomics.* Update the aesthetic aspect when applying visual modelling.
8. *Development of professional visions and outlook.* Application of visualization for the formation of visions and worldviews.
9. *Active use of professional narratives in the visualization process.*
10. *Orientation to professional discourse.* Activation of professional discourse using visualization.
11. *Taking into account the psychophysiological and psychological characteristics of a person in the visualization process.*
12. *Taking into account professional and cultural traditions in the visualization process.*
13. *Application of the scientific principle and didactic approaches.*
14. *Actualization of innovative potential.*

5. Conclusions

According to the results of theoretical and empirical research, the following conditions were determined as factors of visual content design: individual features of information perception by students, which guide the provision of variability of visual means; types of perception of visual information, allowing to choose visual means, taking into account the peculiarities of their perception by students; the specifics of the advanced mathematics course, which consists in the presence or absence of opportunities to create visual educational content of various types.

Visual familiarization with mathematical concepts and visual perception of their properties, connections, and relationships between them allow students to quickly and visually unfold individual fragments of the theory, form and spread a generalized algorithm of operations, apply the acquired knowledge and skills to learning the content of other sections of the advanced mathematics course and fields of knowledge.

Studies have shown that although students actively used their intellectual potential, in some cases, quantitative models could make the work more effective. Generating mathematically expected results that can be predicted using visual models can also facilitate the process of predicting possible outcomes, identifying non-standard situations (such as breakdown situations), and identifying directions for overcoming them.

Students were provided with a visual model of the proposed tasks. They had to find out the content of the problem within the framework of this model and then direct actions to find a solution to this problem. When performing modelling using this technology, the level of knowledge in mathematics, particularly in applied mathematics, which students master, increases. They also acquire the skills of comparing experimentally obtained formalized dependencies with known mathematical models for the relevant problem. A feature of the use of Computer Mathematics Systems, in particular, Mathcad, is that students can obtain results using different methods, compare them, and estimate errors, which enables the student to answer a problematic question.

After analyzing scientific and methodological literature, my own research, and achievements in the field of digital technologies, it was established that the formation of visual modelling skills, as one of the components of modern mathematical training of engineers and their research skills, is effective in teaching advanced mathematics.

In the research process, separate components were developed (means of searching for model characteristics and their evaluation, connections between the sought characteristics, general principles of building a formalized text model, general principles of building a visual model), technologies for the formation of visual, mathematical content as one of the means of mathematical training of students; a system of tasks from relevant sections of the advanced mathematics course was developed for students' independent works, covering the first and third semesters, containing examples and training tasks.

The dynamism and interactivity of built visual objects made in mathematical content visualization environments, such as Mathcad, in the process of studying the advanced mathematics course allows students to effectively form knowledge, skills, and abilities on the relevant topic, promoting the mastery of various problem-solving methods. One of the crucial advantages of a dynamic visualized object in studying advanced mathematics is that it provides an opportunity to present the step-by-step process of its development and to immediately conduct research on the existence of solutions, their number and properties, etc. Dynamic drawings contribute to the student's development of spatial imagination, spatial logic, research thinking, and spatial vision.

Analysis of the pedagogical experiment results confirms the proposed technology's effectiveness. The results of the formative experiment show that the purposeful use of visual modelling and its implementation with the help of Computer Mathematics Systems in teaching an advanced mathematics course has a positive effect on forming students' mathematical educational and research skills. The application of the proposed components of Visual Modeling Technologies in the course of advanced mathematics contributes to the formation of students' abilities and skills in using modern digital technologies, abilities and the desire to adapt to the rapidly changing information environment of activity. Also, it increases the practical significance of the results of mathematics education.

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