Технические науки / 12. Автоматизированные системы управления на производстве

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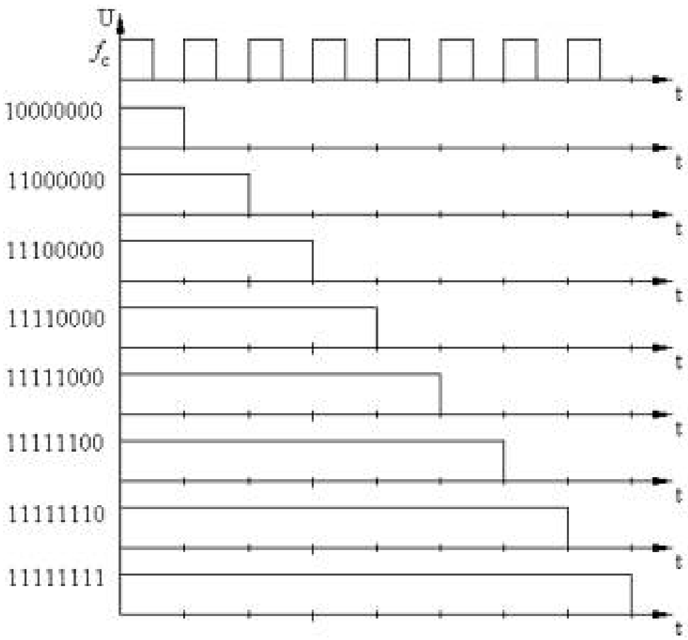
Comparative analysis of the spectrum signals in different bases

Fig. 1 – Possible formation of pulses of different duration



Because of the generalized characteristics include bandwidth, which is a signal, the important role played by the analysis of process harmonic structure of transmission signal. In the optical carrier spectrum practically does not affect the process of transmission, but use on entrance and exit of various electro converters (modulators, demodulators, etc.) requires the analysis frequency band signal. Similar conclusions can be drawn for the case of use of radio channels in computer networks. In most literature considered range of frequencies that occur when single pulse of different forms [1]. However, the lines are superposition of harmonic components of different pulses, so you need to consider spectrum code combinations [2].

In all cases informative code pattern is transmitted continuously and thus can be formed eight different pulse durations, when the code pattern containing one, two, ..., eight units, located near (fig. 1).

Depending on the specific code combinations can displacement pulses by time axis may have their order, but these eight durations are the base for a particular transmission speed, and they determine the bandwidth occupied by the signal. Critical among them are: 10111111V (BFH) and 11111101B (FDH), in which the difference between the pulse duration is maximal.

The duration of these pulses can be determined by the frequency synchronization of the serial port, which is directly connected with the speed of information transfer

$$\tau = \frac{1}{f_c} \cdot i = \frac{1}{k \cdot v} \cdot i, \qquad (1)$$

where $f_c = 1/T_c$ requency sync serial port that corresponds to the speed of information transmission;

i – number of units consistently located in the code pattern;

v – speed of information transfer, bits/s;

k_m – scaling factor that determines the transmission line between speed and frequency synchronization, mostly it is 1 Hz/s.

Frequency spectrum, defined in the Fourier basis functions for the first code combinations represents the expression

$$W_2(\omega) = \frac{1}{\pi}\int_0^\infty \frac{4h^2}{\omega^2}\Big((\sin 8\omega T_c - \sin 7\omega T_c)^2 + 2\sin 6\omega T_c(\sin 8\omega T_c - \sin 7\omega T_c) + \sin^2 6\omega T_c\Big)d\omega = \frac{4h^2 T_c^2}{\pi}\int_0^\infty \frac{1}{\omega^2 T_c^2}\Big(\sin^2 8\omega T_c + \sin^2 7\omega T_c + \sin^2 6\omega T_c - 2\cos\omega T_c + \cos 2\omega T_c + \cos 13\omega T_c - \cos 14\omega T_c + \cos 15\omega T_c\Big)d\omega. \quad (2)$$

for bipolar signals. For unipolar signal this expression takes the form

$$W_1(\omega) = \frac{4T_c^2}{\pi}\int_0^\infty \frac{1}{\omega^2 T_c^2}\Big(h_1^2\sin^2 8\omega T_c + (h_1 - h_0)^2\sin^2 7\omega T_c + (h_1 - h_0)^2\sin^2 6\omega T_c - (h_1 - h_0)(2h_1 - h_0)\cos\omega T_c + h_1(h_1 - h_0)\cos 2\omega T_c + (h_1 - h_0)^2\cos 13\omega T_c - h_1(h_1 - h_0)\cos 14\omega T_c + h_1(h_1 - h_0)\cos 15\omega T_c\Big)d\omega.$$

(3)

When harmonic (sine and cosine) excitation, fluctuations retain its shape during their passage through any linear system, output fluctuations may vary from the front while only the amplitude and phase. These frequency-based research methods associated with the definition of energy spectrum signals.

However, Fourier transform inherent disadvantage, which deprived the Walsh and Haar transform [3]. In the field of communications, and other industries transform often implemented in real time is important to minimize the time machine operations. For Fourier transforms, a significant positive step was the development of various fast transform algorithms, but still kept a large number of multiplication operations, which occupy most of the time machine cultivation data. Multiplications are carried out one after another during the expansion of functions in Fourier series and by performing a Fourier integral.

Decomposition algorithm functions in the Fourier series is to determine the coefficients

$$a_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cdot \cos k\omega_1 t \cdot dt \qquad (4)$$

Thus each of the many values of a function should be multiplied by the value of specific value for *t.* Determine the sum of all multiplication results, which determined the value of the integral and then the value a_k. This procedure is repeated for all *k.* Similarly defined functions and sine coefficients b_k.

It should be noted that although for some functions quite satisfactory approximation turns out to have them in determining a small number of Fourier coefficients, but for most real signals require fast match series is done.

For formula (4) in determining the coefficients of cosine function instead of (or sinus in determining the coefficients b_k) is necessary to put a constant value. If the value of basic functions on the intervals have meaning plus one, minus one or zero, it is generally no need to perform operations of multiplication. Thus, the procedure for determining the coefficients in the series comes down to adding operations that are far from simple multiplication. No need to also calculate the values of sine and cosine, which is also quite simple. Primarily it provides a wide use of Walsh functions in different views and Haar for building and testing both hardware and software

Walsh functions that look like, presented in Fig. describes the difference equation

$$wal(2j+p,\theta) = (-1)^{\mathrm{int}\left(j/2\right)+p}\left(wal\left(j, 2\left(\theta+\frac{1}{4}\right)\right) + (-1)^{j+p}\, wal\left(j, 2\left(\theta-\frac{1}{4}\right)\right)\right), \qquad (5)$$

where $\mathrm{int}\left(j/2\right)$ he greatest integer less than or equal $j/2$

p = 0 or 1;

j = 0,1,2, ...;

$wal\,(0;\ \theta) = 1$ for $-1/2 \le \theta \le 1/2$;

$wal\,(0;\ \theta) = 0$ for $\theta < -1/2,\ \theta > 1/2$.

For comparison, the spectra of frequencies appropriate to consider the above code combination. Since the eight bit code pattern, then the maximum range will contain eight harmonics.

Similarly spectrum, obtained for the Fourier series expansion of functions or code combination on Walsh functions carried out in accordance with the formula

$$S(\theta) = \frac{1}{\theta_{\max}} \cdot \sum_{i=0}^{\theta_{\max}} \sum_{j=0}^{\theta_{\max}} f(j) \cdot wal(i,\theta) \qquad (6)$$

According to this algorithm, the range of code combinations for 11111101 unipolar signal will be

$$S_1(\theta) = \frac{1}{8}\left(7wal(0,\theta) + wal(1,\theta) + wal(2,\theta) - wal(3,\theta) - wal(4,\theta) + wal(5,\theta) + \right.$$
$$\left. + wal(6,\theta) - wal(7,\theta)\right).$$
(7)

For bipolar mode, the range will describe the expression

$$S_2(\theta) = \frac{1}{8}\left(6wal(0,\theta) + 2wal(1,\theta) + 2wal(2,\theta) - 2wal(3,\theta) - 2wal(4,\theta) + \right.$$
$$\left. + 2wal(5,\theta) + 2wal(6,\theta) - 2wal(7,\theta)\right). \quad (8)$$

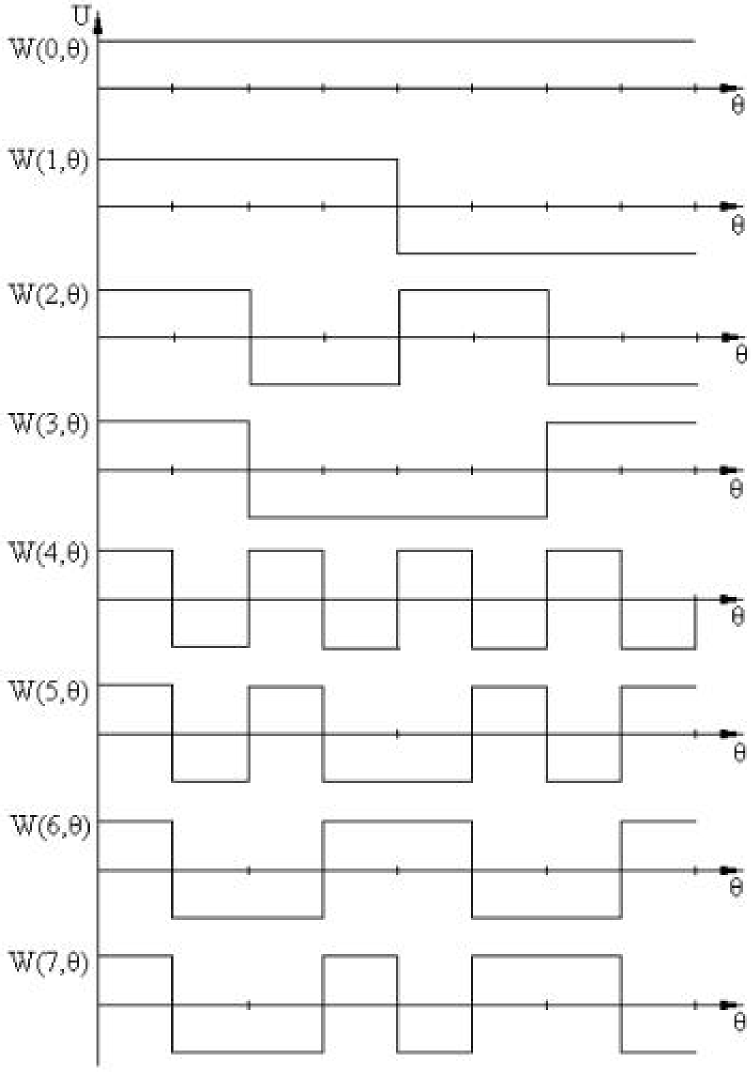


Fig. 2 – The first eight Walsh functions

Thus for the expansion and kept only the difference in the amplitude of harmonics, and not in their stock. Difference from the classical spectrum, built on the Fourier series is that the recovery does not contain a combination of elementary errors. It is obvious, since the first case approximated by rectangular pulses and sine and cosine, and full identity waveform can not achieve in principle, in the second case, the principle of similarity and that no methodological error.

Another option is to analyze the spectrum of a signal decomposition by Haar functions that look like, presented in fig. 3. For the formation of Haar features using the formula [3]

$$H_l^n(\theta)=\begin{cases}2^{l/2}, & \frac{n-1}{2^l}\le\theta<\frac{n-1/2}{2^l}\\ -2^{l/2}, & \frac{n-1/2}{2^l}\le\theta<\frac{n}{2^l}\\ 0, \text{інші} & \theta\end{cases} \qquad (9)$$

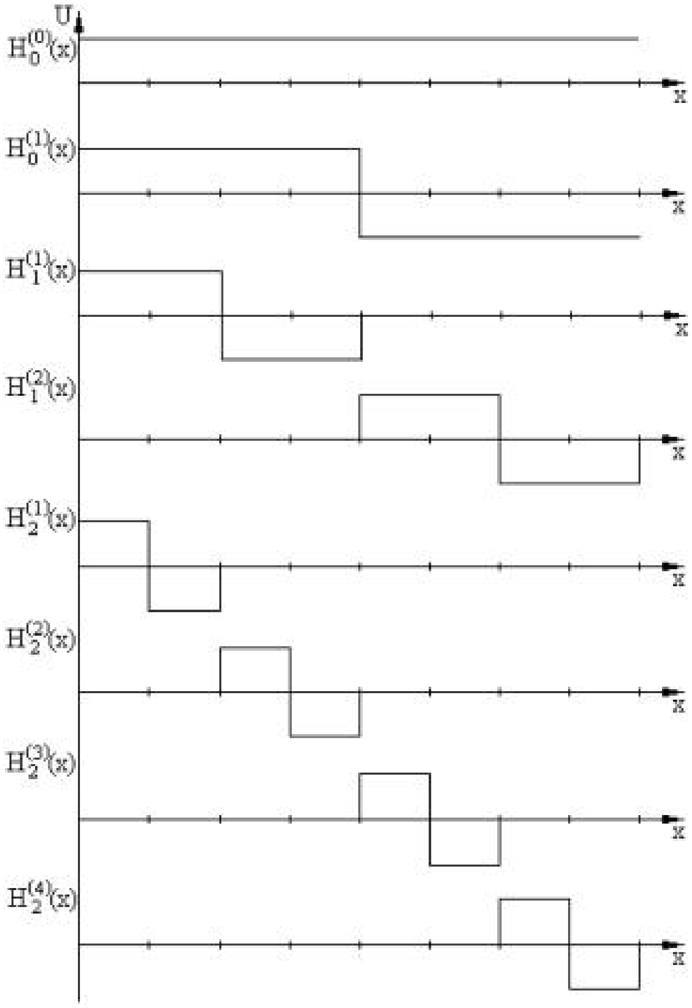


Fig. 3 – The first eight Haar functions

where

$0 \le l < \log_2 N$

N – number of molded functions;

$1 \le n \le 2^l$.

When expansion function in a number of Haar coefficients of members determined in accordance with the formula

$$C_l^{(n)} = 2^{-\log_2 N+l}\cdot\sum_{x=0}^{N-1}\varphi(x)\cdot H_l^{(n)}(x) \qquad (10)$$

For clarity of perception appropriate to the calculation of coefficients in tabular form.

Table 1 – Calculation of coefficients of expansion in the number of Haar unipolar combination code 11111101

Function	Bits								$2^{-\log_2 N+l}$	$C_l^{(n)}$
	7	6	5	4	3	2	1	0		
$\varphi(x)$	1	1	1	1	1	1	0	1		
$H_0^{(0)}$	1	1	1	1	1	1	1	1		
$H_0^{(0)}\cdot\varphi(x)$	1	1	1	1	1	1	0	1	1/8	7/8
$H_0^{(1)}$	1	1	1	1	-1	-1	-1	-1		
$H_0^{(1)}\cdot\varphi(x)$	1	1	1	1	-1	-1	0	-1	1/8	1/8
$H_1^{(1)}$	1	1	-1	-1	0	0	0	0		

$H_1^{(1)} \cdot \varphi(x)$	1	1	-1	-1	0	0	0	0	1/4	0
$H_1^{(2)}$	0	0	0	0	1	1	-1	-1		
$H_1^{(2)} \cdot \varphi(x)$	0	0	0	0	1	1	0	-1	1/4	1/4
$H_2^{(1)}$	1	-1	0	0	0	0	0	0		
$H_2^{(1)} \cdot \varphi(x)$	1	-1	0	0	0	0	0	0	1/2	0
$H_2^{(2)}$	0	0	1	-1	0	0	0	0		
$H_2^{(2)} \cdot \varphi(x)$	0	0	1	-1	0	0	0	0	1/2	0
$H_2^{(3)}$	0	0	0	0	1	-1	0	0		
$H_2^{(3)} \cdot \varphi(x)$	0	0	0	0	1	-1	0	0	1/2	0
$H_2^{(4)}$	0	0	0	0	0	0	1	-1		
$H_2^{(4)} \cdot \varphi(x)$	0	0	0	0	0	0	0	-1	1/2	-1/2

In accordance with the drawn table function can be approximated expression

$$\varphi_1(x) = \frac{7}{8} H_0^{(0)} + \frac{1}{8} H_0^{(1)} + \frac{1}{4} H_1^{(2)} - \frac{1}{2} H_2^{(4)} \tag{11}$$

Similarly, you can create Haar spectrum and bipolar signal. The results are listed in the tab. 2.

Table 2 – Calculation of coefficients of expansion in the number of Haar bipolar combination code 11111101

Функція	Біти								$2^{-\log N+l}$	$C_l^{(n)}$
	7	6	5	4	3	2	1	0		
$\varphi(x)$	1	1	1	1	1	1	-1	1		
$H_0^{(0)}$	1	1	1	1	1	1	1	1		
$H_0^{(0)} \cdot \varphi(x)$	1	1	1	1	1	1	-1	1	1/8	6/8
$H_0^{(1)}$	1	1	1	1	-1	-1	-1	-1		
$H_0^{(1)} \cdot \varphi(x)$	1	1	1	1	-1	-1	1	-1	1/8	2/8
$H_1^{(1)}$	1	1	-1	-1	0	0	0	0		
$H_1^{(1)} \cdot \varphi(x)$	1	1	-1	-1	0	0	0	0	1/4	0
$H_1^{(2)}$	0	0	0	0	1	1	-1	-1		
$H_1^{(2)} \cdot \varphi(x)$	0	0	0	0	1	1	1	-1	1/4	2/4
$H_2^{(1)}$	1	-1	0	0	0	0	0	0		
$H_2^{(1)} \cdot \varphi(x)$	1	-1	0	0	0	0	0	0	1/2	0
$H_2^{(2)}$	0	0	1	-1	0	0	0	0		
$H_2^{(2)} \cdot \varphi(x)$	0	0	1	-1	0	0	0	0	1/2	0
$H_2^{(3)}$	0	0	0	0	1	-1	0	0		
$H_2^{(3)} \cdot \varphi(x)$	0	0	0	0	1	-1	0	0	1/2	0
$H_2^{(4)}$	0	0	0	0	0	0	1	-1		

$H_2^{(4)} \cdot \varphi(x)$	0	0	0	0	0	0	-1	-1	$\frac{1}{2}$	$-\frac{2}{2}$

In this case the function can describe the formula

$$\varphi_2(x) = \frac{6}{8} H_0^{(0)} + \frac{1}{4} H_0^{(1)} + \frac{1}{2} H_1^{(2)} - H_2^{(4)} \quad . \qquad (12)$$

Comparing expressions (11) and (12) shows that again when you change the type of signal (unipolar / bipolar) changing only the amplitude of harmonics and frequency remain unchanged themselves.

Based on expressions can make some conclusions:

- Ø basis regardless of the type of signal only changes the amplitude of harmonics rather than their composition;
- Ø shows that one and the same combination has the longest range in the Fourier basis functions (up to 15th harmonics included), and in basis functions orthogonal rectangular range is considerably reduced and not exceed eight harmonics;
- Ø if the test is done in a certain basis functions, then all the equipment (and especially filters) to build in the same basis.

REFERENCES:

1. Шварцман В.О., Емельянов Г.А. Теория передачи дискретной информации. – М.: Связь, 1979 – 424 с.
2. Кулик А.Я., Компанец Н.Н., Кривогубченко Д.С. Энергетический спектр сигналов при передаче информации оптическими линиями связи // Вимірювальна та обчислювальна техніка в технологічних процесах. – 2000. – № 1. – С. 50 – 51.
3. Залманзон Л.А. Преобразования Фурье, Уолша, Хаара и их применение в управлении, связи и других областях. – М.: Наука, 1989. – 496 с.