# Image compression by Ramer–Douglas–Peucker approximation algorithm

**Melnyk R.A.** [1]**,, Viazovskyy P.V.** [2]

[1]Prof. of software department, Lviv Polytechnic National University, S. Bandery Str., 12, Lviv, 79013, UKRAINE, E-mail: ramelnyk@polynet.lviv.ua,

[2]Master of software department, Lviv Polytechnic National University, S. Bandery Str., 12, Lviv, 79013, UKRAINE, E-mail:pviazik@gmail.com

***Abstract* —Algorithm of Ramer–Douglas–Peucker for piecewise approximation was used for image compression. To decrease an approximation error the algorithm was modified. The compression method was investigated for different tolerances and steps in the algorithm application for pixel rows and columns..**

***Key words – image compression, approximation, coding, piecewise-linear functions***

# Компресія зображень алгоритмом апроксимації Рамера-Дугласа-Пекера

**Мельник Р.А**[1]**.,. Вязовський П.В**[2]

[1]Проф. кафедри програмного забезпечення Національного університету "Львівська політехніка", Львів, вул. С. Бандери 12, e-mail: ramelnyk@polynet.lviv.ua
[2] Магістр. кафедри програмного забезпечення Національного університету "Львівська політехніка", Львів, вул. С. Бандери 12, e-mail: pviazik@gmail.com

***Анотація — Реалізовано метод стиснення зображення з використанням алгоритму Рамера-Дугласа-Пекера кусково-лінійної апроксимації. Для зменшення похибки апроксимації алгоритм модифіковано. Досліджено характеристики компресії та точності апроксимованих зображень в залежності від точності наближення та кроку застосування алгоритму до матриці пікселів.***

***Ключові слова с зображення, апроксимація, кодування, стиснення, кусково-лінійні функції***

.

## INTRODUCTION

Popular algorithms of image encoding and compression include: JPEG, which uses discrete cosine transform and Huffman algorithm [1]; LZW, which replaces the original set of bytes in file with reference to a previous occurrence of the same set (GIF, TIFF formats) [2] .

Compression is achieved by replacing the set of pixel values using special functions: wavelets, fractals, polynomials etc. Implementation of these functions requires considerable computational costs. In this paper a simplified approach is proposed, namely the piecewise-linear approximation. This mathematical tool is being constantly improved [5-7]. Some examples of its application for cartography purposes are given in [8,9].

## APPROXIMATION OF BRIGHTNESS FUNCTIONS

To obtain image function by color intensity colored image is converted to grayscale. Each of image pixels can have values from the range 0-255, which corresponds in color from black to white.

For piecewise-linear approximation of brightness functions Ramer–Douglas–Peucker algorithm [3,4] was taken as basic and developed its modification to reduce the mean square error.

### 1) Ramer–Douglas–Peucker algorithm.

The essence of the algorithm is approximating initial curve represented by the set of points $P_i(x_i; y_i) \in P$, $i = \overline{1..n}$, with piecewise-linear function represented with points $P_j(x_j; y_j) \subseteq P$, $j = \overline{1..m}$, $m \le n$. The algorithm defines maximum distance (tolerance) between the original and the approximating functions. This distance must be less than preassigned maximum allowable tolerance, which determines desired accuracy of the approximation.

The initial curve is an ordered set of points or lines within $\varepsilon > 0$. The process of the curve approximation is shown in Fig. 1.

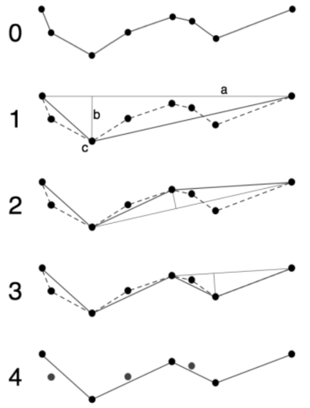


Fig.1. Curve approximation by Ramer–Douglas–Peucker algorithm.

The algorithm recursively divides the line. Input of the algorithm are the coordinates of all points between the first and the last. The first and the last points are kept unchanged. Then the algorithm finds the point farthest from the line drawn through the first and the last. If the point is located at distance less than $\varepsilon$, then all the points that have not yet been signed for storing may be removed from the set and received line will smooth curve with an accuracy not less than $\varepsilon$.

If the distance is more than $\varepsilon$, then the algorithm recursively calls itself on the set of points from first to the current and from the current to the endpoint (which means that current point will be signed for storing).

After all the recursive calls output polygon is based only on those points that were signed for storing.

The results of the brightness function approximation using Ramer–Douglas–Peucker algorithm for the test image for different values of the maximum allowable deviation (tolerance) are presented on Fig. 2.

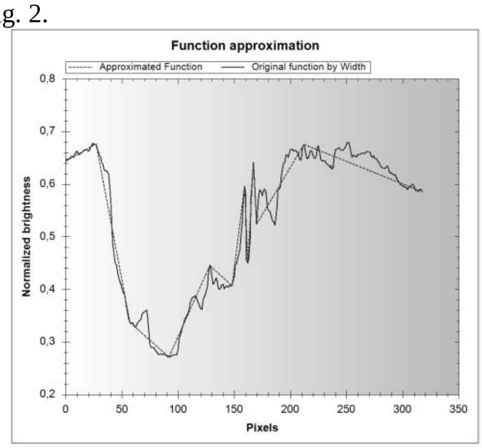


Fig.2. Brightness function approximation by Ramer–Douglas–Peucker algorithm for tolerance = 0,075

**2) Modified Ramer–Douglas–Peucker algorithm for mean square error minimization.**

This method uses original Ramer–Douglas–Peucker algorithm, which results in obtaining approximating piecewise-linear function for the initial $y = f(x)$, presented by $n$ points, where $x \in X$, $y \in Y \in [0;1]$. Approximating function can be represented as:

$$y_A = \begin{cases} k_1 x + b_1, \ 1 \le x < x_i \\ k_2 x + b_2, \ x_i \le x < x_j \\ \dots \\ k_m x + b_m, \ x_k \le x < x_n \end{cases}, \text{ where } \ y_A \subset Y, \ \ x_i, x_j, x_k, x_n \in X. \quad (1)$$

The method consists in finding such $\Delta y$, which gives:

$$MSE = \frac{\sqrt{\sum_{i=1}^{n} \left[ (y_{A_i} + \Delta y) - y_i \right]^2}}{n} \to \min . \quad (2)$$

To do this, the $MSE$ to $\Delta y$ dependency graph is built (Fig. 3). Iteratively increasing $\Delta y$ from 0 to 1 algorithm looks for minimum $MSE$. It allows to achieve better image quality, especially with high values of preassigned maximum allowable deviation $\varepsilon$ (tolerance).

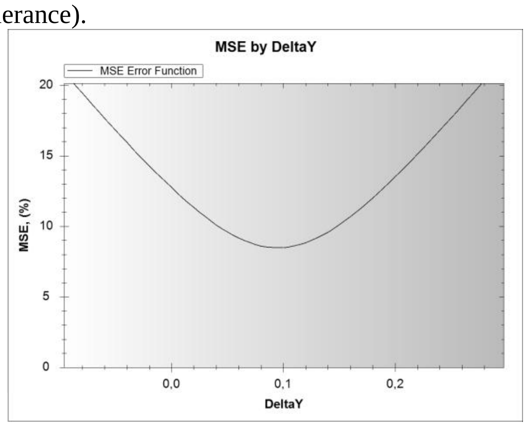


Fig.3. Mean square error as function of offset along $Y$ axis.

The results of using the algorithm for brightness function approximation for the test image for the maximum allowable deviation (tolerance) are presented in Fig. 4.

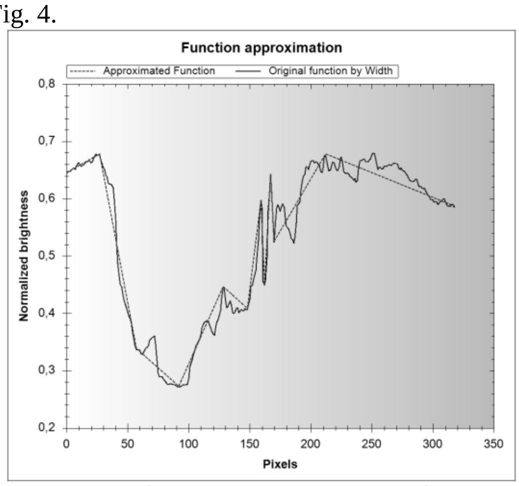


Fig.4. Brightness function approximation by modified Ramer–Douglas–Peucker algorithm (t=0,075)

## COMPRESSION RESULTS

Comparison of the Ramer–Douglas–Peucker algorithm results for different values of the maximum allowable deviation (tolerance) is presented in Table 1.

Table 1. Image compression by piecewise linear approximation

| | tolerance | | |
|---|---|---|---|
| | 0,128 | 0,1 | 0,08 |
| "Lena" (256x256) |  |  |  |
| (MSE), % | 4, 43 | 3,32 | 2,65 |
| Compression ratio | 7,28 | 5,39 | 4,25 |
| Time s | 0,66 | 0,705 | 0,735 |

So, the table 1 demonstrates decreasing compression ratio and MSE according to the decreasing tolerance value. The compression ratio is calculated by a reference on original image size that is a number of an image pixels. We can not compare these results with jpeg algorithm, because the last one is very complicated and consists of at least three powerful components; discrete cosine transformation, special type of matrix scanning and the Huffman algorithm for file compression. The piecewise linear approximation has the same principles as the DCT algorithm. The last one neglects high frequencies by ignoring the DCT matrix coefficients. Our approximation does it when some impulse curves are changed by lines. We can see an example on fig. 5.

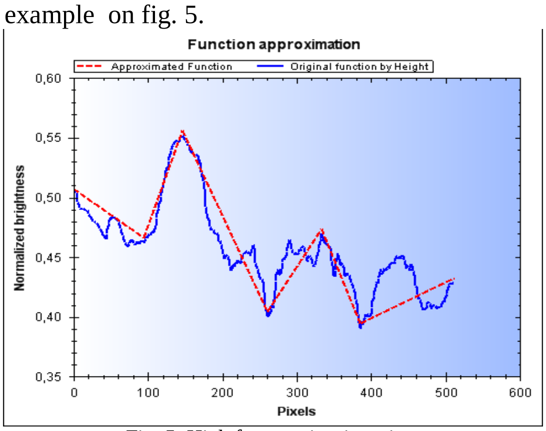


Fig. 5. High frequencies ignoring by piecewise linear approximation

The proposed algorithm was analyzed by compression of the "1a" face image. For one tolerance value t=0,054 three compression experiments were held: one, two and three columns approximation by piecewise linear functions. As input image the original "1a" face and its pixel matrix were taken. In three cases the compression ratio was 6,08, 12, 6 та 17,1. The compressed images were characterized by corresponding MSE from original image : 1,83, 2,14, 2,33. Then the compressed images were saved by jpeg-format. As we can see from Table 2 sizes of files are close to the original image and between them. Also we can see that sizes of the coding arrays for two last compressed images (by 2 and 3 columns) are smaller than size of the jpeg file of the original image. So, here we see possibility to improve a compression ratio by compressing a coding array itself.

Table2.Image compression by different steps of piecewise linear approximation

| | Number of columns (step) | | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| "1a" (260 x 360) |  |  |  |  |
| (MSE), % | | 1,83 | 2,14 | 2,33 |
| Compression ratio | | 6,08 | 12,6 | 17,1 |
| Time, s | | 0,39 | 0,24 | 0,18 |
| Coding array (b) | | 15383 | 7422 | 5472 |
| jpeg file | 9291 | 9147 | 8608 | 8856 |

## CONCLUSION

Thus, two methods of images encoding – based on original and modified Ramer-Douglas-Peucker algorithms intended to reduce mean square error of compressed image – allow to achieve image compression indicators comparable with other approaches.

## REFERENCES


[1] *A. Skodras, C. Christopoulos, T. Ebrahimi*, "The JPEG 2000 Still Image Compression Standard", IEEE Signal processing magazine, 23 p., 2001.

[2] *M. R. Nelson*, "LZW data compression", Dr. Dobb's Journal, pp.28-36, 1989.

[3] *U. Ramer*, "An iterative procedure for the polygonal approximation of plane curves", Computer Graphics and Image Processing, 1(3): pp.244–256, 1972.

[4] *D. Douglas, T. Peucker*, "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature", The Canadian Cartographer, 10(2): pp.112–122, 1973.

[5] *S. Kundu, V.A. Ubhaya*, "Fitting a Least Squares Piecewise Linear Continuous Curve in Two Dimensions", Computers and Mathematics with Applications, 41(7/8): pp.1033-1041, 2001.

[6] *D.G. Wilson*, "Piecewise linear approximations of fewest line segments", Proe. AFIPS 1972 SJCC, vol. 40, AFIPS Press, Montvale, N.J., pp. 187-198, 1972.

[7] *N. Golovchenko*, "Least-squares Fit of a Continuous Piecewise Linear Function", 7p., 2004.

[8] *P. S. Heckbert, M. Garland*, "Survey of Polygonal Surface Simplification Algorithms", Tech. rep., Carnegie Mellon University, 31p., 1997.

[9] *D. H. Ballard, C. M. Brown*, "Computer Vision", Prentice Hall, Englewood Cliffs, NJ, 523p., 1982.