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Mathematical model of the visible range optical radiation passing through a water-milk solution

A mathematical model and a mathematical modeling of the passing visible range optical radiation through a water-milk solution are carried out. The mathematical model was created on the basis of theoretical and experimental studies of the spectral characteristics of water-milk solution passing. As a result of the modeling, the dependence of the output voltage of the linear photoreceiver, which based on a pair of photodiode-operating amplifier from the relative mass fraction of milk in the water-milk solution and the wavelength of the optical radiation in the visible range is obtained. The optimal wavelength of infrared radiation was determined by determining the maximum of the ratio of the spectral characteristics of water and milk. The question of passing the visible range optical radiation through the water-milk solution has not been studied at the present time, which determines the relevance of these studies.

Keywords: water-milk solution, visible range, optical radiation passing, detection of water in milk.

Stall milking systems occupy almost half of the world market of milking equipment [1, 2]. They are used for tie-stall housing system of cows and provide machine milking, milk transportation to the milk receiving container, milk transfer from the milk receiving container into a cooling or mixing plant. Typically, the salary of staff is calculated according to the amount of milk they have received, because this indicator characterizes the quality of their work [1-3]. According to the technological regulations of the milking process, each staff has a water tank, which is necessary for the care of animals. In some cases, in order to increase the rates of milk, personnel with the help of a milking machine sucks this water into the milk pipeline. In addition, dishonest suppliers often dilute milk with water and hand it over to reception points. It is impossible to visually identify these cases, therefore, the human eye can not distinguish water-milk solution with a small amount of water from pure milk [4]. Existing means of measuring milk parameters in most cases do not allow to carry out operative control of water availability in milk. By their help, in most cases, it is impossible to identify the staff who falsifies [1-3]. Thus, the development of methods for the operative measurement of the relative mass fraction of milk in the water-milk solution and the creation of their mathematical models is an important and urgent task.

At present, the detection of water content in milk in the laboratory is often carried out by measuring the temperature of its freezing [4]. Such a technique is difficult to apply at the receiving points and can not be used on stall milking systems during the process of milking [5, 6]. In work [3] the counter of milk portions for stall milking systems with the electric conductivity sensor is considered, the principle of which is as follows. The milk portion passes through a sensor, which is a tube with built-in electrodes. The sensor generates a voltage signal that is directly proportional to the electrical conductivity value. In the presence of water in milk, the electrical conductivity of milk decreases [7, 8], but due to the presence of foam and the absence of temperature control, it is impossible to detect a slight dilution of milk when using such sensor. In works [9, 10] an optical method for measuring the relative mass fraction of milk in a water-milk solution is considered, which is based on measuring the degree of reduction of the intensity of lower range infrared radiation when it passes through the solution. In [4], the optimal wavelength of infrared radiation was determined by determining the maximum of the ratio of the spectral characteristics of water and milk. The question of passing the visible range optical radiation through the water-milk solution has not been studied at the present time, which determines the relevance of these studies.

Aim of the research – to create a mathematical model for the passing of the visible range optical radiation through a water-milk solution.

To develop the means for measuring the mass fraction of milk in the water-milk solution in the milk production process, it is necessary to create a mathematical model for the passing of the visible range optical radiation through a water-milk solution based on its experimental passing characteristics. With the help of the obtained mathematical model it is possible to determine the optimal wavelength of the visible range optical radiation for measuring small values of the relative mass fraction of water in the water-milk solution. The development of new means of operative measurement of the water amount in milk will allow to identify the facts of falsification of milk with water.

Let's consider the theoretical principles of the optical method of measuring the relative mass fraction of milk in the water-milk solution using optical radiation of the visible range. The weakening of monochromatic radiation by substance is determined by Bouguer-Lambert-Ber law

$$I(\lambda) = I_0 \cdot 10^{-k(\lambda)cd}, \tag{1}$$

where I_0 — the intensity of the radiation, that falls on the solution; $I(\lambda)$ — the intensity of the radiation that passed through the solution; d — thickness of the solution layer; c — concentration of the substance; λ — wavelength of optical radiation; $k(\lambda)$ — extinction coefficient, which is a function of the wavelength of optical radiation.

The optical density of solution determined by the expression

$$D(\lambda) = \lg\frac{I_0}{I(\lambda)} = k(\lambda)cd. \tag{2}$$

Passing through a solution of the n components, radiation by each of them is absorbed in different ways. The resulting absorption is derived by additive superposition of individual components. Accordingly, the optical density of n — component mix is determined by the expression [11]

$$D(\lambda) = \sum_{i=1}^{n} k_i(\lambda) c_i d, \tag{3}$$

where $k_i(\lambda)$ — extinction coefficient of i — component of the solution; c_i — concentration of i — component of the solution.

For the water-milk solution $n = 2$, accordingly from (3), its optical density is determined by the expression [11]

$$D_{VM}(\lambda) = d\left(k_M(\lambda)c_M + k_V(\lambda)c_V\right), \tag{4}$$

where $k_M(\lambda)$ — milk extinction coefficient; c_M — concentration of the milk in water-milk solution; $k_V(\lambda)$ — water extinction coefficient; c_V — concentration of the water in water-milk solution.

The volume of water-milk solution, that absorbs radiation, is given by

$$V_K = \frac{m_M}{\rho_M} + \frac{m_V}{\rho_V}, \tag{5}$$

where m_M — mass of the milk in water-milk solution; ρ_M — density of milk; m_V — mass of the water in water-milk solution; ρ_V — density of water.

Concentration of the water in water-milk solution defined as [9, 10]

$$c_V = \frac{m_V}{V_K} = \frac{m_V}{\frac{m_M}{\rho_M} + \frac{m_V}{\rho_V}} = m_V \frac{\rho_M \rho_V}{m_M \rho_V + m_V \rho_M}. \tag{6}$$

Concentration of the milk in water-milk solution defined as

$$c_M = \frac{m_M}{V_K} = \frac{m_M}{\frac{m_M}{\rho_M} + \frac{m_V}{\rho_V}} = m_M \frac{\rho_M \rho_V}{m_M \rho_V + m_V \rho_M}. \tag{7}$$

The relative mass fraction of milk in water-milk solution is defined as

$$\eta = \frac{m_M}{m_M + m_V}. \tag{8}$$

From the expression (8), after transformations we get [9, 10]

$$m_V = m_M\left(\frac{1}{\eta} - 1\right), \tag{9}$$

Substituting (9) to (6) and (7), after transformations we obtain expressions that link the concentration of milk and water in the milk-water solution with relative mass fraction of milk.

$$c_V = \frac{\rho_M \rho_V \left(\frac{1}{\eta} - 1\right)}{\rho_V + \rho_M \left(\frac{1}{\eta} - 1\right)}, \tag{10}$$

$$c_M = \frac{\rho_M \rho_V}{\rho_V + \rho_M \left(\frac{1}{\eta} - 1\right)}. \tag{11}$$

Substituting (10) and (11) to (4), and after transformations we obtain the dependence of the optical density of the water-milk solution from the relative mass fraction of milk

$$D_{VM}(\lambda) = \frac{d\rho_M \rho_V \left(k_M(\lambda) + k_V(\lambda)\left(\frac{1}{\eta} - 1\right)\right)}{\rho_V + \rho_M \left(\frac{1}{\eta} - 1\right)}. \tag{12}$$

In view of (2), after transformations we obtain

$$\frac{I(\lambda)}{I_0} = 10^{-\frac{d\rho_M \rho_V \left(k_M(\lambda) + k_V(\lambda)\left(\frac{1}{\eta} - 1\right)\right)}{\rho_V + \rho_M \left(\frac{1}{\eta} - 1\right)}}. \tag{13}$$

Expression (13) links the intensity of optical radiation passing through the solution with relative mass fraction of milk in solution [9, 10].

At Figure 1 the experimental spectral characteristics of the water-milk solution passing (which obtained by using the spectrophotometer SF-4, $d = 10$ mm), which represent the dependence of the parameter $I(\lambda)/I_0$ at different values of η at the wavelength of the optical radiation in the visible range, are shown.

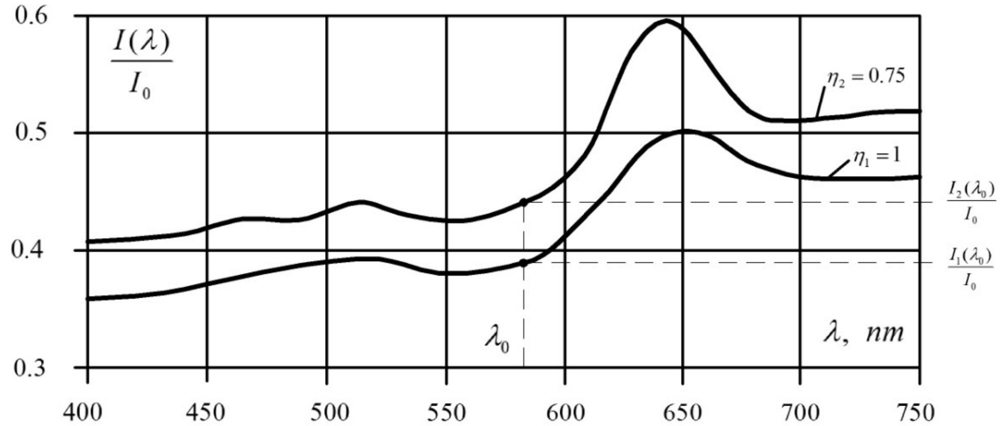

Figure 1. Experimental spectral characteristics of the water-milk solution passing at the visible optical range at the different values of η

As follows from Figure 1, the values of the spectral characteristics vary considerably depending on the wavelength. At a wavelength of about 640 nm, there is a certain resonance due to the diffraction and interference of optical waves in the intervals between fat beads and strongly affects the values of the extinction coefficients.

Consider a system of two equations that determine the parameter $I(\lambda)/I_0$ at a certain wavelength λ_0 and two values of the relative mass fraction of milk in the water-milk solution $\eta_1 = 1$ and $\eta_2 = 0.75$ (see Fig. 1)

$$\begin{cases} \dfrac{I_1(\lambda_0)}{I_0} = 10^{-\frac{d\rho_M\rho_V\left(k_M(\lambda_0)+k_V(\lambda_0)\left(\frac{1}{\eta_1}-1\right)\right)}{\rho_V+\rho_M\left(\frac{1}{\eta_1}-1\right)}}; \\ \dfrac{I_2(\lambda_0)}{I_0} = 10^{-\frac{d\rho_M\rho_V\left(k_M(\lambda_0)+k_V(\lambda_0)\left(\frac{1}{\eta_2}-1\right)\right)}{\rho_V+\rho_M\left(\frac{1}{\eta_2}-1\right)}}. \end{cases} \tag{14}$$

Solve this system of equations relatively to unknown values of extinction coefficients of water and milk, which are functions of the wavelength of optical radiation. After the transformations of the first expression of the system (14) we obtain an expression that correlates the extinction coefficient of milk with the extinction coefficient of water at the wavelength λ_0.

$$k_M(\lambda_0) = -\frac{\left(\rho_V+\rho_M\left(\frac{1}{\eta_1}-1\right)\right)\lg\frac{I_1(\lambda_0)}{I_0}}{d\rho_M\rho_V} - k_V(\lambda_0)\left(\frac{1}{\eta_1}-1\right). \tag{15}$$

An analogous expression is obtained after transformations of the second expression of the system of equations (14)

$$k_M(\lambda_0) = -\frac{\left(\rho_V+\rho_M\left(\frac{1}{\eta_2}-1\right)\right)\lg\frac{I_2(\lambda_0)}{I_0}}{d\rho_M\rho_V} - k_V(\lambda_0)\left(\frac{1}{\eta_2}-1\right). \tag{16}$$

Equating the right sides of expressions (15) and (16) and solving the obtained equation, we obtain the expression for the coefficient of water extinction

$$k_V(\lambda_0) = \frac{\left(\rho_V+\rho_M\left(\frac{1}{\eta_1}-1\right)\right)\lg\frac{I_1(\lambda_0)}{I_0} - \left(\rho_V+\rho_M\left(\frac{1}{\eta_2}-1\right)\right)\lg\frac{I_2(\lambda_0)}{I_0}}{d\rho_M\rho_V\left(\frac{1}{\eta_2}-\frac{1}{\eta_1}\right)}, \tag{17}$$

Substituting (17) to (15) we obtain the expression for the extinction coefficient of milk

$$k_M(\lambda_0) = -\frac{\left(\rho_V+\rho_M\left(\frac{1}{\eta_1}-1\right)\right)\lg\frac{I_1(\lambda_0)}{I_0}}{d\rho_M\rho_V} - \frac{\left(\rho_V+\rho_M\left(\frac{1}{\eta_1}-1\right)\right)\lg\frac{I_1(\lambda_0)}{I_0} - \left(\rho_V+\rho_M\left(\frac{1}{\eta_2}-1\right)\right)\lg\frac{I_2(\lambda_0)}{I_0}}{d\rho_M\rho_V\left(\frac{1}{\eta_2}-\frac{1}{\eta_1}\right)}\left(\frac{1}{\eta_1}-1\right). \tag{18}$$

Using the expressions (18), (17) and the experimental dependences in Fig. 1 it is possible to determine the extinction coefficients of water and milk in the water-milk solution for any value of the wavelength of the optical radiation in the visible range.

To measure the intensity of optical radiation, linear photoreceiver based on a pair of photodiode-operational amplifier are widely used. In accordance with [9, 12], the dependence of the linear photoreceiver output voltage from the relative mass fraction of milk in the water-milk solution is determined by the expression

$$U_F(\eta) = I_0\cdot S_{I0}(\lambda)\cdot R\cdot S\cdot 10^{-\frac{d\rho_M\rho_V\left(k_M(\lambda)+k_V(\lambda)\left(\frac{1}{\eta}-1\right)\right)}{\rho_V+\rho_M\left(\frac{1}{\eta}-1\right)}}, \tag{19}$$

where $S_{I0}(\lambda)$ — integral current sensitivity of the photodiode; S — area of the photosensitive layer of the photodiode, which is illuminated; R — resistance in the feedback loop of the operational amplifier.

As follows from expression (19), the output voltage of the photoreceiver based on the pair of photodiode - operating amplifier is directly proportional to the intensity of the optical radiation of the visible range, which passed through the water-milk solution. After measuring the intensity of light radiation that passed through the water-milk solution, it is possible to determine the relative mass fraction of milk in the water-solution in terms of expression [10]

$$\eta = \frac{\rho_M \lg \frac{U_F(\eta)}{I_0 \cdot S_{I0}(\lambda) \cdot R \cdot S} + d\rho_M \rho_V k_V(\lambda)}{(\rho_M - \rho_V)\lg \frac{U_F(\eta)}{I_0 \cdot S_{I0}(\lambda) \cdot R \cdot S} + d\rho_M \rho_V \left(k_V(\lambda) - k_M(\lambda)\right)}. \quad (20)$$

Let's conduct a mathematical modeling of the visible range optical radiation passing through a water-milk solution. To do this, let's interpolate the experimental spectral passing characteristics, which are shown in Figure 1, by usual cubic splines [13]

$$\frac{I_1(\lambda)}{I_0} = \begin{cases} a_{1M} + b_{1M}(\lambda - \lambda_1) + c_{1M}(\lambda - \lambda_1)^2 + d_{1M}(\lambda - \lambda_1)^3, \ \lambda \in [\lambda_1, \lambda_2]; \\ a_{2M} + b_{2M}(\lambda - \lambda_2) + c_{2M}(\lambda - \lambda_2)^2 + d_{2M}(\lambda - \lambda_2)^3, \ \lambda \in [\lambda_2, \lambda_3]; \\ \dots \\ a_{N-1\ M} + b_{N-1\ M}(\lambda - \lambda_{N-1}) + c_{N-1\ M}(\lambda - \lambda_{N-1})^2 + d_{N-1\ M}(\lambda - \lambda_{N-1})^3, \ \lambda \in [\lambda_{N-1}, \lambda_N], \end{cases} \quad (21)$$

$$\frac{I_2(\lambda)}{I_0} = \begin{cases} a_{1V} + b_{1V}(\lambda - \lambda_1) + c_{1V}(\lambda - \lambda_1)^2 + d_{1V}(\lambda - \lambda_1)^3, \ \lambda \in [\lambda_1, \lambda_2]; \\ a_{2V} + b_{2V}(\lambda - \lambda_2) + c_{2V}(\lambda - \lambda_2)^2 + d_{2V}(\lambda - \lambda_2)^3, \ \lambda \in [\lambda_2, \lambda_3]; \\ \dots \\ a_{N-1\ V} + b_{N-1\ V}(\lambda - \lambda_{N-1}) + c_{N-1\ V}(\lambda - \lambda_{N-1})^2 + d_{N-1\ V}(\lambda - \lambda_{N-1})^3, \ \lambda \in [\lambda_{N-1}, \lambda_N], \end{cases} \quad (22)$$

where a_{iM}, b_{iM}, c_{iM}, d_{iM} — coefficients of interpolating cubic splines of the experimental spectral passing characteristic $I_1(\lambda)/I_0$; a_{iV}, b_{iV}, c_{iV}, d_{iV} — coefficients of interpolating cubic splines of the experimental spectral passing characteristic $I_2(\lambda)/I_0$; $\lambda_1 ... \lambda_N$ — abscisses of the interpolation function nodes.

The set of expressions (17)–(22) is a mathematical model for the visible range optical radiation passing through a water-milk solution. For mathematical modeling, a package of mathematical applications Maple 9 [14] was used. The result of the mathematical modeling of the linear photoreceiver output voltage dependence from the relative mass fraction of milk in the water-milk solution and the wavelength of the optical radiation in the visible range is shown in Figure 2 and Figure 3.

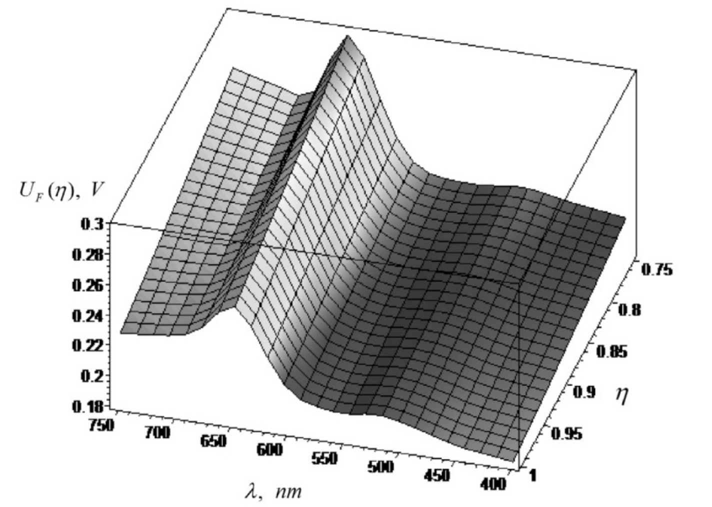

Figure 2. Dependence of the linear photoreceiver output voltage from the relative mass fraction of milk in the water-milk solution and the wavelength of the optical radiation in the visible range

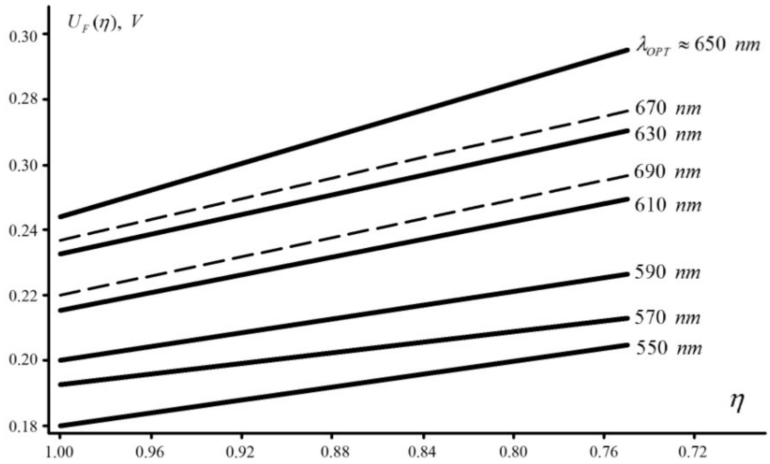

Figure 3. The family of dependencies of the linear photoreceiver output voltage from the relative mass fraction of milk in the water-milk solution at different values of the wavelengths at the visible range optical radiation

The modeling was carried out in the visible optical radiation range for wavelengths from 400 nm to 750 nm and the range of relative mass fraction of milk in the water-milk solution from 0.75 to 1 and $d = 10$ mm. During the modeling, it was assumed that the integral sensitivity of the photodiode is constant throughout the wavelength range. The choice of the relative mass fraction of milk range in the water-milk solution is due to the fact that, according to the authors' observations, with $\eta < 0.75$ the human eye begins to distinguish between water and milk solution from pure milk. As it follows from Figure 3, there is an optimal wavelength ($\lambda_{OPT} \approx 650$ *nm* if $d = 10$ mm), in which the sensitivity of the measuring channel of the relative mass fraction of milk in the water-milk solution will be maximal.

A mathematical model is proposed and a mathematical modeling of the passing of visible range optical radiation through a water-milk solution is carried out. The mathematical model was created on the basis of theoretical and experimental studies of the spectral characteristics of water-milk solution passing. As a result of the modeling, the dependence of the output voltage of the linear photoreceiver which based on a pair of photodiode - operating amplifier from the relative mass fraction of milk in the water-milk solution and the wavelength of the optical radiation in the visible range is obtained. Depending on the thickness of the water-milk solution layer, there is an optimal wavelength of visible range optical radiation, at which the sensitivity of the measuring channel of the relative mass fraction of milk in the water-milk solution will be maximal. Promising is the use of the proposed mathematical model for the creation of means for measuring the relative mass fraction of milk in the water and milk solution to detect the facts of milk falsification.

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Су-сүтті ерітінді арқылы көрінетін ауқымдағы оптикалық сәулелену өтуінің математикалық моделі

Су-сүтті ерітінді арқылы көрінетін диапазонның оптикалық сәулеленуін математикалық модельдеу ұсынылды. Математикалық модель су-сүтті ерітіндісі өтуінің спектралдық сипаттамаларды теориялық және тәжірибелік зерттеулер негізінде құрылды. Модельдеу нәтижесінде су-сүтті ерітіндідегі сүттің салыстырмалы массалық үлесі және көрінетін диапазондағы оптикалық сәулеленудің толқын ұзындығы бойынша фотодиод-операциондық күшейткішке негізделген сызықтық фотодетектордың шығу кернеуінің тәуелділігі орын алды. Инфрақызыл сәулеленудің оңтайлы толқын ұзындығы судың және сүттің спектралды сипаттамаларының максималды арақатынасын анықтау арқылы сипатталды. Су-сүтті ерітінді арқылы көрінетін диапазонның оптикалық сәулелену мәселесі қазіргі уақытта жан-жақты зерттелмеген, сол себепті осы сияқты зерттеудің өзектілігін айқындайды.

Кілт сөздер: су-сүтті ерітінді, көрінетін диапазон, оптикалық сәулелену, сүттегі су мөлшерін анықтау.

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Математическая модель прохождения оптического излучения видимого диапазона через водно-молочный раствор

Предлагаются разработанная математическая модель и результаты математического моделирования прохождения оптического излучения видимого диапазона через водно-молочный раствор. Математическая модель была создана на основе теоретических и экспериментальных исследований спектральных характеристик пропускания водно-молочного раствора. В результате математического моделирования получена зависимость выходного напряжения линейного фотоприемника на основе пары фотодиод-операционный усилитель от относительной массовой доли молока в водно-молочном растворе и длины волны оптического излучения видимого диапазона. Оптимальная длина волны инфракрасного излучения определялась путем определения максимума отношения спектральных характеристик воды и молока. Вопрос о прохождении оптического излучения видимого диапазона через водно-молочный раствор в настоящее время не изучен, что и определяет актуальность этих исследований.

Ключевые слова: водно-молочный раствор, видимый диапазон, прохождение оптического излучения, определение наличия воды в молоке.

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