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TENSOR MODELING OF A QUANTUM SYSTEM

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Abstract

The issues of geometric tensor modeling of a quantum system considered. The quantum system understood as an ensemble of distinguished entities which are bound by the complex wave function of a local-point. The set of local points forms the definition area for the wave function; this area is organized in a closed compact and everywhere continuous topological space embracing the time/space independent coordinates. An interpretation of a tensor model of quantum system presented in terms of information network.

Анотація

Розглянуто питання геометричного тензорного моделювання квантової системи. Квантова система розуміється як ансамбль виділених сутностей, пов'язаних комплексною хвильовою функцією від локальної точки. Набір локальних точок утворює область визначення для хвильової функції; ця область організована у замкнутий компактний і скрізь безперервний топологічний простір, що охоплює незалежні координати часу/простору. Наведена інтерпретація тензорної моделі квантової системи в термінах інформаційної мережі.

Introduction

Classical system theories such as "General systems theory" ([1], 1968) and "Open systems theory" ([2], 2008) are being challenged recently by quantum system theory, primarily because of that classic system approach assumes systems to be a whole in a single scalability ([3], 2014). The quantum system theory takes its beginning from 1900–1925; it was rather a set of heuristic amendments and additions to classical mechanics ([4], 1967), which is now understood as semi-classical approximation [5] to modern quantum mechanics ([6], 2014). The basic concepts and methods of open quantum systems presented in [7, 2003]. The quantum system methodology is closely related to measurement problem; this view promises emergence of truly nanotechnologies that rely on the counter-intuitive properties of individual objects and can often outperform conventional decisions ([8], 2010). Nowadays, the quantum theory goes beyond physics and is increasingly used in various engineering tasks ([9], 2012). The modern directions of tiny-system modeling are quantum linear systems theory ([10], 2016) and linear dynamical quantum systems intended for communications and control ([11], 2017). However, the methodology of quantum systems analysis is not yet enough transparent and popular among the specialists of network engineering, particular with respect to apparatus of tensor algebra. *This work aims to formulate the core principles of quantum tensor analysis in terms of network applications*.

1. The "quantum system" characteristics

Consider the principal distinctive features of a quantum system (QS) in comparison with traditional conservative-continuum systems (CS); among them we will focus the three following characteristics: openness, indeterminacy, scalability.

The *QS-openness* means that classic hypothesis of the so called "conservative object" (which is closed and isolated from external disturbances) does not correspond to modern concepts of subtle interactions at the weak energies level. Moreover, the object interaction with the outer environment is the most interesting aspect of system analysis, where the system is constituted by at least two counteracting entities: the object of study (O) and the subject of research (S). The subject could be an individual person as it is (or, maybe provided by special instruments). So, we say that QS="O/S".

From this premise, the only way to comprehensively investigate a smart object is an accurate interaction between the object O and subject S. This interaction affects the object, and therefore, one can't observe the object as it is (without our impact). Only object disturbed states can be observed, and these states triggered by the concrete subject. Thus, the QS triggered behavior is non-fully objective (instead, it depends on a subject to some extent). The QS-openness causes the so called O/S-uncertainty dualism as a part of general QS-uncertainty. The system openness is not an exclusive privilege of quantum physics and elementary particles models. In modern IT-world one cannot imagine any cognitive activity beyond the tight communication with information environment. Towards system openness paradigm, we note that classic graph theory does not accentuate the open graph as network model. Again, many math textbooks and manuals for university students still based on the closed one-layer set theory of Cantor (suffering known logical contradictions).

The QS- indeterminacy means that contrary to the classical idea of complete matter observability, stochastic approximation is presumed for object particles behavior description. From this premise, one cannot exactly observe and control the QS-entity everywhere and any moment, or expect to find an absolute optimal decision. Everything is stochastically uncertain and approximate; everywhere the noise of indeterminacy is present. The stochastic character of QS is caused by the huge number of various factors of influence on the object of observation, which lead to exponent complexity growth of the QS-model. In general case, one may hope to estimate the probability of distinct particles appearance while regular experiments, along with the probability distribution of particle`s coupling. The probability distribution is formalized by the so called wave function $\psi(x)$ which is normalized as $\|\psi(x)\| = 1$. Here $x \in X$ is the set of points as definition area for wave function; this area is organized in a closed compact and everywhere continuous topological space embracing the time/space independent coordinates.

The square of wave function determines the probability distribution for particles coupling: $\rho(k,n)_x = [\psi(x)]^2 = \psi(x) \cdot \psi^*(x)$. The $\rho(k,n)_x$ is an Hermitian matrix, which diagonal elements $\rho(n,n)_x$ are real numbers; these numbers determine the proper degree of freedom for each particle. In case of symmetric coupling the $\rho(k,n)_x$ is a real symmetric matrix. The wave function admits geometric presentation in the form tensor operator. A case study of wave function could be normalized data flow distribution among the nodes of an open telecommunication network.

The *QS-scalability* means that modeling a complex object performed as multilayered dually opened formal system, which elements are scaled unlimitedly from above and below, in order to keep an overall system complexity at any layer. Thus, the whole complex object or system of low case level evolves to an utterly small primitive while going up to higher level of formalism. The total number of basic primitives at any hierarchy layer remains approximately even, though the primitives decisively change their essence and behavior. The greater system primitives are (towards observing subject), the more predictable and deterministic behavior shows the system. Instead, the less particles of matter investigated, the more stochastic and uncertain model of QS results.

2. Geometric tensor model of a quantum system

Visual thinking with geometric images is a powerful tool of the person's intellectual activity, including the understanding of complex objects and systems. A convenient form of representing geometric images is the classical tensor analysis, where the set of interacting objects is mapped onto the set of vectors in a local Euclidian space (real or complex). The simple case of vector presentation is a real positive symmetric matrix R which is interpreted as scalar product of a vector set (Riemann metric tensor): $R(k,n) = \{\vec{V}_k \times \vec{V}_n\}$. In this case, a real matrix A exists that $A \cdot A^T = R(k,n)$, or $A = \sqrt{R(k,n)}$.

Matrix A presents the normal coordinates of vectors V_k in a real Euclidian space with the orthonormal basis $\vec{E} = \{\vec{e}_k\}$. Under certain conditions, the wave function square $H = [\psi(x)]^2$ has a form of Hermitian matrix H; however, it is not obligatory positive (i.e. some eigen values λ_k of H could be negative or zero). Because of that, a special technique needed to map the wave function $\psi(x)$ onto the linear system of vectors. If no zero eigen values (but diverse signs of eigen values shown) then a particular case of complex Euclidian space can be used for vector presentation (e.g. complex bundle of Euclidean space). If some zero eigen values appeared, then reduced form of linear system applied (less number of vectors presenting the wave function).

A lot of network tasks can be formulated in terms of related wave functions and their tensor presentation. Among those study cases are: the matrix of network nodes connectivity; the matrix of the shortest paths between the network nodes; the matrix of data flows between the open network poles etc.

Conclusion

The quantum system theory emerged on demand of new experimental facts in chemistry and physics of elementary particles. However, today this theory has gone far beyond this framework and finds its application in many fields of science and technology, including in the field of telecommunications, computer engineering and information systems.

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