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paredness of the system. Besides, if we perform the analysis of S_i values, varying independent dual variables (probability of the system stay in operation modes) then the conclusion can be drawn regarding the strategy of further development or prospects of new systems design.

IV. Conclusions

Combination of the tools of Markov processes and similarity theories enables to obtain criterial model of Markov process, this model can be used for the formation of integral index of automatic control system functioning quality. Such index allows to evaluate in relative units the efficiently of system operation and determine the degree of preparedness to perform its functions.

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CRITERIAL MODELING OF MARKOV PROCESSES IN THE PROBLEMS OF AUTOMATIC CONTROL SYSTEMS FUNCTIONING QUALITY EVALUATION

Abstract. *Similarity model of Markov process has been built, corresponding criterial model has been formed. It is shown that the combination of Markov processes and similarity theories as well as criterial modeling enables to obtain integrated index of automatic control system functioning quality. Such index allows to evaluate in relative units the efficiency of its operation and determine the degree of preparedness for execution of their functions.*

Key wards: *Markov processes, similarity modeling, automatic control system, functioning quality, generalized criterion.*

I. Introduction

Modern automatic control systems (ACS) are multilevel hierarchical systems, upper level of which is the centre of control in the form of local computational network with distributed architecture. On one hand, such architecture improves the reliability and performance of such systems, on the other hand, considerably complicates the possibility of their analysis and analytical study at the stage of design, operation and improvement. In this connection the problem of the selection and determination of the criterion, applying which the current state of ACS could be evaluated, strategy of its improvement could be developed, comparison of various structural realizations could be performed , arises . Especially it concerns the problem of evaluation of ACS efficiency as a result of SMART Grid technologies usage, their impact on technical economic indices of the object of control.

Particularly, taking into account flexible feedbacks, using communication networks for adaptation of separate units to non-stable operation conditions, change of which is of probabilistic character [1].

Taking into consideration characteristic features of ACS, mathematical methods of Markov processes are used for their simulation and study [2, 3] . Markov models are used for evaluation of the performance and reliability of software and hardware facilities of ACS. They include the ability to cover various dependences, the possibility to calculate the parameters of stable, transition and cumulative transition states,

Criterial model can also be used for determination of relative value of ACS functioning quality criterion. For this purpose the method of geometric programming can be used [8].

Criterion of ACS functioning quality.

In geometric programming the duality theory is used for the search of optimal solutions, according to this theory direct problem of function minimization is replaced by the problem of maximization of corresponding dual function [8]. In our case dual function, that corresponds to the model of Markov process has the form:

$$d_*(p) = \prod_{i=1}^{m} \frac{p_i^{P_i}}{p_{0i}^{P_{0i}}} \tag{10}$$

where p_{0i}, p_i – are dual variables, which by their values are probabilities of system stay in initial state and after updating, accordingly. Using the obtained criterial model of functioning quality, it is possible to evaluate the real state of the system relatively its initial state.

Fig. 2 shows the constructed dependences $d_*(p)$. As a rule, it is possible to construct m (amount of states) of such dependences, but it is sufficient to construct one dependence relatively the probability of the state, the system is at the given moment.

Using the obtained results the criterion of operation quality can be determined. Integral criterion of system functioning quality is defined as the area S_i, limited by the curve $d_*(p)$ and straight line $d_* = \delta d_*$:

$$S_i = \int_{p'_{*j}}^{p''_{*j}} \left(d_{*i}(p_{*j})\right)dp - \int_{p'_{*j}}^{p''_{*j}} (\delta d_*)dp$$

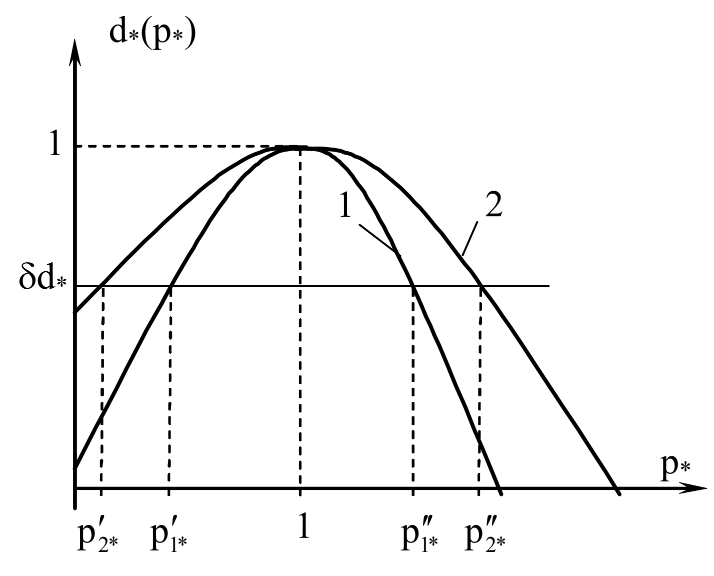

Fig. 2 Function of control system quality in two states

Value of δd_* corresponds to the limit of ACS functioning quality over which the system cannot perform its functions or performs these functions with the accuracy that does not correspond to the requirements regarding the determination of the output effect.

The greater quality criterion value is, the higher is functional pre-

In the given equations the values of similarity criteria π_i are normalized. They show the relative part of each term (5). It is the same as the probability of finding in each i^{th} state of the system is taken into account in f(v), that characterizes it on the whole. Similarly to (7), probabilities p_i of investigated system states are normalized in (1).

Such analogy allows to apply the principles of criterial method for transition to the analysis of system function quality not by equations (2), but according to criterial dependences, the state of optimal control system is taken as the basis [6]. For this purpose it is necessary to construct criterial model of Markov process.

Criterial model of Markov process

In practice, when the problem of ACS updating is of multivariant character, mainly optimization character, the obtained equation (7) that determines similarity of Markov processes is not acceptable for such problems. Evaluation of variants in multivariant problems is carried out relatively the basic variant, that is characterized by the values of parameters f_b , x_{1b}, x_{2b}, …, x_{nb}.The latter, as (5) are restricted by the condition

$$f_b(\nu) = \sum_{i=1}^{m} a_i \prod_{j=1}^{n} x_{jb}^{\nu_{ji}} .$$

Similarity criteria for the given case have the form

$$\pi_{ib} = \frac{a_i \prod_{j=1}^{n} x_j^{\nu_{ji}}}{f_b(\nu)} = \text{idem}\,,\ i = \overline{1,m}\,. \tag{8}$$

If the left and right parts of (5) are divided term-wise by f_b and substituted in the right part the value f_b from (8)

$$f_b(\nu) = \frac{a_i \prod_{j=1}^{n} x_{jb}^{\nu_{ji}}}{\pi_{ib}},$$

then we obtain the following criterial relation

$$f_* = \pi_{1b} \prod_{j=1}^{n} x_{j*}^{\nu_{j1}} + \pi_{2b} \prod_{j=1}^{n} x_{j*}^{\nu_{j2}} + \ldots + \pi_{mb} \prod_{j=1}^{n} x_{j*}^{\nu_{jm}}\,, \tag{9}$$

where $f_* = f(\nu) / f_b(\nu)$, $x_{j*} = x_j / x_{jb}$.

Criterial model (9) allows to determine relative changes of f while deviation of x_j; from basic value including deviation from the optimal value. That is, by means of (9) the sensitivity of optimal solutions, regarding changes in ACS can be evaluated.

Such evaluation is relative but its advantage is that its values are obtained without determination of optimal values of system parameters.

the possibility of expansion to Markov models of reward, useful for the analysis of ACS performance [4].

II. Problem set-up

Owing to distributed structure ACS are able to perform their functions even in case of failure of certain elements, but with reduced efficiency, that is, they can be in several operation states. This feature of ACS requires the determination of such its index (characteristic) that would allow to evaluate the efficiency of its operation and determine the degree of preparedness to perform its functions, that influences directly on the level of optimal state of the object under control. The index of ACS adaptability to the solution of the specific problems is the quality of its operation [5]. The important fact is that this index must characterize ACS at a certain interval of time ,when the performance of ACS changes , but does not exceed the admissible limits. That is, this index must be integral and characterize all operation states of ACS on the whole.

Similar feature for all operation states of ACS, is that their determining parameters and their relation provide functioning of the object under control in the area of optimality. Accordingly, for the analysis of ACS functioning in the system of optimal control it is necessary to replace it by the similar model, properties of which are proportional to corresponding properties of the original. Thus, **the aim of the paper** is similar modeling of Markov process, that enables to suggest the quality index of ACS functioning.

III. Results

Similar modeling of Markov process. As it is known, Markov process is described by the system of differential equations [3]

$$\left.\begin{aligned} \frac{dp}{dt} &= \nu \cdot \mathbf{p}; \\ \sum_{i=1}^{m} p_i &= 1, \end{aligned}\right\}, \tag{1}$$

where **p** – is the vector of investigated system states probabilities; **v** – is the matrix of probability densities of transitions from one state to another; m – is the amount of possible states of investigated system (Fig. 1).

Analyzing the system of equations (1); it is difficult to obtain the criteria, which would meet the suggested requirements. However, it can be used for construction of similar model, that would allow to investigate the properties, parameters and values of variables of which are proportional to corresponding properties, parameters and values of variables of the original [6].

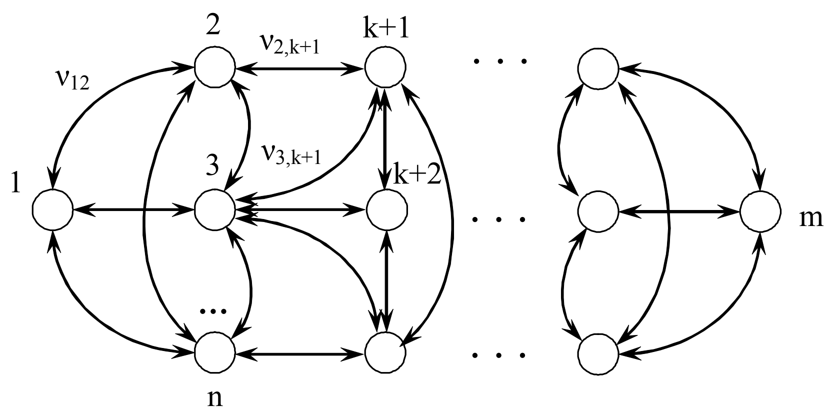

Fig. 1 Graph of ACS states change

For steady states of the system $\frac{dp_i}{dt} = 0$ and the system of equations (1) will be rewritten:

$$\left.\begin{aligned} &\sum_{i=1}^{m} \nu_{ji} p_i = 0, \quad j = \overline{2,n} \\ &\sum_{i=1}^{m} p_i = 1, \end{aligned}\right\}, \tag{2}$$

where ν_{ji} – are constant values (elements of matrix $\boldsymbol{\nu}$), that are algebraic sums of the values of transitions intensities from the i^{th} to j^{th} states; n – is the amount of directions of system states change that leave operation state 1 (Fig 1.)

In more general form (2) is written as

$$\nu \cdot \mathbf{p} = \mathbf{b}, \tag{3}$$

where $\mathbf{b_t}$=[0 0…0 1] and matrix $\boldsymbol{\nu}$ from the expression (1) is completed with the row, that is composed of m units.

To determine the similarity we will construct the polynomial in matrix $\boldsymbol{\nu}$. As any analytical function f(x) can be presented by a convergent series (polynomial), from variable x [7]:

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots = \sum_{s=0}^{\infty} a_s x^s$$

Function from matrix f(x) can be presented in the form of a polynomial in matrix, which formally is obtained by the replacement of scalar variable x by matrix X:

$$f(X) = a_0 + a_1 X + a_2 X^2 + \ldots = \sum_{s=0}^{\infty} a_s X^s$$

If we make use of interpolation polynomial, then transition matrix $\boldsymbol{\nu}$ of the equations system (3) can be reduced to matrix polynomial [7]. For this purpose we use exponential function $f(z) = e^{zt}$. If minimal polynomial (in the given case it is characteristic polynomial $\Delta(Z)$), is composed only of linear multiplies $(z - z_k)$, then it is sufficient to define function $f(z)$ in characteristic points $z_1, z_2, \ldots, z_m$. In this case the system of equations for the coefficients of interpolation polynomial will have the form:

$$f(z_k) = a_0 + a_1 z_k + \ldots + a_{m-1} z_k^{m-1}, \tag{4}$$

or in matrix form

$$\begin{bmatrix} f(z_1) \\ f(z_2) \\ \ldots \\ f(z_m) \end{bmatrix} = \begin{bmatrix} 1 \; z_1 \; z_1^2 \; \ldots \; z_1^{m-1} \\ 1 \; z_2 \; z_2^2 \; \ldots \; z_2^{m-1} \\ \ldots \ldots \ldots \ldots \ldots \\ 1 \; z_m \; z_m^2 \; \ldots \; z_m^{m-1} \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_{m-1} \end{bmatrix}.$$

Having solved this system relatively $a_0, a_1, \ldots, a_{m-1}$ we will obtain polynomial

$$f(\nu) = \sum_{i=1}^{m} a_i \prod_{j=1}^{n} x_j^{\nu_{ji}}. \tag{5}$$

Having made such transformation, all the properties and corollaries of the theorems of similarity theory can be used [6]. To obtain the system of similarity criteria, we will divide the left and right part (5) into *f(ν)*:

$$1 = \frac{a_1 \prod_{j=1}^{n} x_j^{\nu_{j1}}}{f(\nu)} + \frac{a_2 \prod_{j=1}^{n} x_j^{\nu_{j2}}}{f(\nu)} + \ldots + \frac{a_m \prod_{j=1}^{n} x_j^{\nu_{jm}}}{f(\nu)}, \tag{6}$$

where $\pi_i = \frac{a_i \prod_{j=1}^{n} x_j^{\nu_{ji}}}{f(\nu)} = idem$, $i = \overline{1,m}$ – are similarity criteria, determined by the method of integrated analogues.

Taking into account the introduced designations of similarity criteria, (6) will be rewritten as:

$$\pi_1 + \pi_2 + \ldots + \pi_m = 1 \tag{7}$$