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OPTIMIZATION OF FUZZY CLASSIFICATION KNOWLEDGE BASES USING IMPROVING TRANSFORMATIONS

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Запропоновано метод оптимізації класифікаційних нечітких баз знань з використанням поліпшувальних підстановок у вигляді розв'язків нечітких реляційних рівнянь. Поліпшувальні підстановки дозволяють формалізувати процес генерування та відбору варіантів нечіткої бази знань за критеріями «точність – складність», що спрощує процес налаштування

Ключові слова: оптимізація нечітких баз знань, min-max кластеризація, розв'язання нечітких реляційних рівнянь

Предложен метод оптимизации классификационных нечетких баз знаний с использованием улучшающих подстановок в виде решений нечетких реляционных уравнений. Улучшающие подстановки позволяют формализовать процесс генерирования и отбора вариантов нечеткой базы знаний по критериям «точность – сложность», что упрощает процесс настройки

Ключевые слова: оптимизация нечетких баз знаний, min-max кластеризация, решение нечетких реляционных уравнений

1. Introduction

Fuzzy classification knowledge base design is carried out according to the criteria of accuracy, complexity, and interpretability. The design criteria are provided by gradual transformations of the initial model. In the theory of defect-free design of human-machine systems [1, 2], formalization of such transformations is achieved by the use of improving transformations.

Then improving transformations correspond to the addition (removal) of output classes, input terms, and rules. Improving transformations allow formalization of the process of generating the fuzzy knowledge base variants with the simultaneous establishment of control variables that affect the accuracy and complexity of the model. At the same time, the system of fuzzy logic equations follows from the description of the tuning process in the language of V. M. Glushkov algorithmic algebras [3].

In practice, fuzzy classification knowledge bases are tuned by generating the candidate rules with a further selection of rules. Improving transformations for a given granularity [4], when at the stage of generating the candidate rules it is possible to obtain the required number of input terms are called formalized. In this case, only the weights of rules are sufficient for selection. However, such an approach provides lower inference accuracy. In general, the knowledge base optimization problem is a problem of granular *min-max* clustering [5], where the relation between design criteria is achieved by the level of granularity of interval rules (hyperboxes).

2. Literature review and problem statement

Candidate rules for the *min-max* clustering problem [5] are generated by means of multi-objective genetic algorithms [6, 7] or *min-max* neural networks [8, 9]. Dimensions of hyperboxes are selected until the required inference accuracy is achieved [6–9]. The preliminary method of partition is determined by the boundaries of triangular membership functions of candidate terms. However, the initial model is redundant. The methods of candidate rule generation do not guarantee the optimum number of rules and optimum granularity of input variables.

The number of terms and rules is determined during the multi-objective evolutionary selection [10–12]. The purpose of selection is to obtain a simplified and interpretable knowledge base. Selection is implemented by choosing the best configuration of terms and rules of zero option. The fitness function is constructed on the basis of granularity measures, which allow estimating the redundancy among fuzzy rules [13]. Linguistic interpretation of the solutions obtained requires the adjustment of preference relations between the candidate terms [14, 15]. Criteria for the selection of terms are the level of detail and the maximum size of the hyperbox [16]. The selection process becomes more complicated with increasing number of criteria, in particular, when taking into account the rule length [12]. In addition to weights of rules, such selection requires signs of the terms or their absence in each rule.

The considered approach differs in computational complexity due to the increasing number of rules. Both at the stage of candidate rule generation and at the selection stage, the control variables are the membership function parameters in each rule. The proposed method consists in replacing the tuning algorithm fragments with other fragments that provide accuracy at lower cost. Improving transformations of fuzzy classification knowledge bases are based on solving fuzzy relational equations [17, 18]. As improving transformations, the following is proposed: transition to a composite model for selecting the number of output classes and rules [19–21]; transition to a relational model for selecting the number of input terms [22, 23].

Composite transformation replaces the fragment of candidate rule generation. The problem of *min-max* clustering is solved by generating composite rules in the form of interval solutions of the trend system of equations [19–21]. The number of rules in a class is determined by the number of solutions, and the granularity is determined by intervals of values of input variables in rules. The set of minimum solutions provides the minimum rule length [17, 18]. Simplification of the candidate rule generation process is achieved through the sequential generation of solutions of the trend equation system.

Relational transformation replaces the selection fragment. Since the number of rules is already known, selection of terms lies in the maximum approximation to the partition by interval solutions of the trend equation system. Linguistic interpretation of the resulting solutions is performed by the relational partition of the space of input variables [22, 23]. The level of detail and the density of coverage are determined by the “input terms – output classes” relational matrix, and the dimensions of hyperboxes are adjusted using triangular membership functions. Simplification of the selection process is achieved through a concise presentation of rules in the form of a relational matrix.

3. The aim and objectives of the study

The aim of the work is to develop a method for optimization of fuzzy classification knowledge bases using improving transformations. The method should provide the construction of accurate, compact and interpretable knowledge bases. At the same time, consistent use of composite and relational models shall provide design process simplification.

To achieve this aim, the following objectives were accomplished:

– to develop logic-algorithmic models of improving transformations;

– to develop a genetic algorithm for the fuzzy knowledge base optimization.

4. Models and method of fuzzy classification knowledge base optimization

4. 1. Logic-algorithmic models of improving transformations

Algorithms for tuning the fuzzy classification knowledge base for the object $y=f(x_1,\ldots,x_n)$ were described in the language of V. M. Glushkov algorithmic algebras [3].

The linear operator structure corresponds to the algorithm of tuning by means of an expert (without tuning to the experimental data):

– for the zero option

$$A_0 = L_1^0 L_2^0, \tag{1}$$

– for the composite model

$$A_1 = L_1^R L_2^R L_1^c L_2^c, \tag{2}$$

– for the relational model

$$A_2 = L_1^r L_2^r. \tag{3}$$

Here L_1 and L_2 are the operators of tuning the structure and parameters of expert fuzzy rules: L_1^0 and L_2^0 – for the zero option, L_1^R, L_2^R and L_1^c, L_2^c for the trend and composite models, L_1^r and L_2^r for the relational model; A_0, A_1, A_2 are equivalent operators of the linear structure for the zero option, composite and relational models, respectively.

To increase the inference accuracy, the operator of improving by tuning to experimental data is used.

An alternative operator structure corresponds to the partial case of the "accuracy control – tuning" algorithm for the specified output classes:

– for the zero option

$$B_0 = A_0 \underset{\alpha_0}{(} Y \vee D_1^0), \tag{4}$$

– for the composite model

$$B_1 = A_1 \underset{\alpha_c(\omega_R)}{(} Y \vee D_1^R D_1^c), \tag{5}$$

– for the relational model

$$B_2 = A_2 \underset{\omega_r}{(} Y \vee D_1^r). \tag{6}$$

Here D_1 is the tuning operator in case of the specified output classes: D_1^0 – for the zero option, D_1^R and D_1^c – for the trend and composite models, D_1^r – for the relational model; α and ω are the conditions that are checked during the control, α, ω=1(0) – if the rules or relations satisfy (do not satisfy) the inference accuracy requirements: α_0 – for the zero option, ω_R and α_c – for the trend and composite models, ω_r – for the relational model; Y is the identity operator, corresponding to the fixation of the tuning results; B_0, B_1, B_2 are equivalent operators of the alternative structure for the zero option, composite and relational models, respectively.

The iterative operator structure corresponds to the general case of the "control-tuning" algorithm for unknown output classes:

– for the zero option

$$C_0 = A_0 \underset{\alpha_0}{\{} D_2^0\}, \tag{7}$$

– for the composite model

$$C_1 = A_1 \underset{\alpha_c(\omega_R)}{\{} D_2^R D_2^c\}, \tag{8}$$

– for the relational model

$$C_2 = A_2 \underset{\omega_r}{\{} D_2^r\}. \tag{9}$$

Here D_2 is the tuning operator in case of unknown output classes: D_2^0 – for the zero option, D_2^R and D_2^c – for the trend and composite models, D_2^r – for the relational model; C_0, C_1, C_2 are equivalent operators of the iterative structure for the zero option, composite and relational models, respectively.

It is assumed that the tuning operator D_2 is executed until the conditions α and ω become true. The truth of the conditions α and ω is determined by the correctness of knowledge base construction. The condition – indicator of correctness of knowledge base construction is described by a series-parallel logical structure:

– for the zero option [18]

$$\alpha_0 = \underset{p=\overline{1,z_j^0}}{\vee}\left[\underset{i=\overline{1,n}}{\wedge}\alpha_i^{jp}(x_i = a_i^{jp})\right] \to y = d_j^0, \quad j = \overline{1,m_0}, \tag{10}$$

– for the composite model [19, 20]

$$\omega_R = \underset{i=\overline{1,n}}{\wedge}\left\{\underset{l=\overline{1,k_i}}{\vee}\omega_{il}^J(x_i = c_{il})\right\} \to y = E_J, \quad J = \overline{1,M},$$

$$\alpha_c(\omega_R) = \underset{p=\overline{1,z_j}}{\vee}\left[\underset{i=\overline{1,n}}{\wedge}\left\{\underset{l=\overline{1,k_i}}{\vee}\alpha_{il}^{jp}(x_i = a_{il}^{jp})\right\}\right] \to y = d_j,$$
$$j = \overline{1,m}, \tag{11}$$

– for the relational model [23]

$$\omega_r = \underset{i=\overline{1,n}}{\wedge}\left\{\underset{l=\overline{1,k_i}}{\vee}\left[\underset{s=\overline{1,g_{il}}}{\vee}\omega_{ils}^j(x_i = a_{ils})\right]\right\} \to y = d_j, \quad j = \overline{1,m}. \tag{12}$$

Here d_j^0, E_J and d_j are output classes of the zero option, trend and composite models, respectively; m_0, M and m are the number of output classes; α_i and ω_i are the micro conditions of correctness of terms of the variable x_i in rules and relations: α_i^{jp} – for the term a_i^{jp} in the rule jp of the class d_j^0 of the zero option; ω_{il}^J – for the initial term c_{il} in the relation $c_{il}\times E_J$ of the trend model; α_{il}^{jp} – for the linguistic modification a_{il}^{jp} of the term c_{il} in the rule jp of the class d_j of the composite model; ω_{ils}^j – for the term a_{ils} in the relation $a_{ils}\times d_j$ of the relational model; z_j^0 and z_j are the number of rules in the class of the zero option and composite model; k_i and g_{il} are the number of primary terms and linguistic modifiers to evaluate the variable x_i.

The parameters of improving operators are the number of input terms, output classes, rules, and parameters of membership functions in logical conditions (10)–(12).

The stages of generation and selection of the zero option rules are described by the same operator and logical structures. Then, as a result of improving transformations, we obtain variants of the fuzzy knowledge base:

– without tuning to experimental data

$$A_0 = A_1 A_2,$$

– with tuning to data for the specified output classes

$$B_0 = B_1 B_2,$$

– with tuning to data for unknown output classes

$$C_0 = C_1 C_2.$$

In this case, the condition of correctness of knowledge base construction takes the form:

$$\alpha_0 = \omega_R \alpha_c \omega_r.$$

The rules of improving transformations (2)–(12) allow representing the generation of the knowledge base variants as inference in a formal grammar [1]:

$$G = \langle V_t, V_n, S_0, P\rangle,$$

where V_t is the set of operator and logical functional units (terminals), $\{L_1^0, L_2^0, L_1^R, L_2^R, L_1^c, L_2^c, L_1^r, L_2^r\}$ and $\{\alpha_i^{jp}\}$; V_n is the set of operator and logical functional structures (nonterminals), $\{A_0, B_0, C_0\}$ and $\{\alpha_0\}$; S_0 is the zero option of a fuzzy knowledge base; P is the set of improving transformations: $\{D^0_{1,2}, D^R_{1,2}, D^c_{1,2}, D^r_{1,2}\}$ and $\{\omega^J_{il}, \alpha^{jp}_{il}, \omega^j_{ils}\}$ for operators and logical conditions; $\{A_1, A_2, B_1, B_2, C_1, C_2\}$ and $\{\omega_R, \alpha_c, \omega_r\}$ for operator and logical structures.

Fuzzy logic equations follow directly from the algorithmic descriptions (10)–(12).

4. 2. Problems of fuzzy classification knowledge base optimization

We denote the set of input terms of the trend, composite and relational models as:

$$\{c_{11},...,c_{1k_1},...,c_{n1},...,c_{nk_n}\}=\{T_1,...,T_N\},$$

$$\{a_1,...,a_q\}$$

and

$$\{a_{11},...,a_{1g_1},...,a_{N1},...,a_{Ng_N}\}=\{t_1,...,t_K\},$$

where N, q and K are the number of input terms, $N=k_1+...+k_n$, $K=g_1+...+g_N$.

Let:

$$\underline{\mathbf{Y}}_E=(\underline{y}^{E_1},...,\underline{y}^{E_M}),\ \ \overline{\mathbf{Y}}_E=(\overline{y}^{E_1},...,\overline{y}^{E_M})$$

and

$$\underline{\mathbf{Y}}_d=(\underline{y}^{d_1},...,\underline{y}^{d_m}),\ \ \overline{\mathbf{Y}}_d=(\overline{y}^{d_1},...,\overline{y}^{d_m})$$

are the vectors of the lower (upper) boundaries of the output classes E_J and d_j;

$$\underline{\mathbf{B}}_T=(\underline{\beta}^{T_1},...,\underline{\beta}^{T_N}),\ \overline{\mathbf{B}}_T=(\overline{\beta}^{T_1},...,\overline{\beta}^{T_N}),$$

$$\underline{\mathbf{B}}_a=(\underline{\beta}^{a_1},...,\underline{\beta}^{a_q}),\ \overline{\mathbf{B}}_a=(\overline{\beta}^{a_1},...,\overline{\beta}^{a_q})$$

and

$$\underline{\mathbf{B}}_t=(\underline{\beta}^{t_1},...,\underline{\beta}^{t_K}),\ \overline{\mathbf{B}}_t=(\overline{\beta}^{t_1},...,\overline{\beta}^{t_K})$$

are the vectors of the lower (upper) boundaries of triangular membership functions of fuzzy input terms of the trend, composite and relational models;

$\mathbf{R}\subseteq c_{il}\times E_J=[r^J_{il}]$ and $\mathbf{P}\subseteq a_{ils}\times d_j=[p^j_{ils}]$ – "input terms – output classes" fuzzy relational matrices before and after linguistic modification.

Then the solution vectors are:

$$\mathbf{\Psi}_1=(M,N,m,q,\underline{\mathbf{Y}}_E,\overline{\mathbf{Y}}_E,\underline{\mathbf{Y}}_d,\overline{\mathbf{Y}}_d,\mathbf{R},\underline{\mathbf{B}}_T,\overline{\mathbf{B}}_T,\underline{\mathbf{B}}_a,\overline{\mathbf{B}}_a)$$

and

$$\mathbf{\Psi}_2=(K,m,\underline{\mathbf{Y}}_d,\overline{\mathbf{Y}}_d,\mathbf{P},\underline{\mathbf{B}}_t,\overline{\mathbf{B}}_t)$$

for the composite and relational transformations, respectively.

We will estimate the fuzzy model quality based on the mean square error $e(\mathbf{\Psi})$, and complexity – based on the number of rules $Z(\mathbf{\Psi})$ [20, 23].

For each improving transformation, the knowledge base optimization problem is formulated in a direct and dual statement.

Direct statement. To find the vector $\mathbf{\Psi}$, for which $Z(\mathbf{\Psi})\rightarrow$ $\rightarrow$min and $e(\mathbf{\Psi})\leq\overline{e}$, where $\overline{e}$ is the maximum permissible inference error.

Dual statement. To find the vector $\mathbf{\Psi}$, for which $e(\mathbf{\Psi})\rightarrow$ $\rightarrow$min and $Z(\mathbf{\Psi})\leq\overline{Z}$, where $\overline{Z}$ is the maximum permissible number of rules.

4. 3. Genetic algorithm

For the composite transformation, the chromosome contains elements of the vector $\mathbf{\Psi}_1$, and for the relational – elements of the vector $\mathbf{\Psi}_2$. The number of classes, rules, and terms is determined using arrangement vectors [23]. The elements of arrangement vectors are 1(0), which means the addition (removal) of classes, rules, and terms.

For the composite transformation, the initial population is generated for all output classes, and for the relational – for all input terms. This ensures the maximum approximation to the partition by interval solutions of the trend equation system. Then the elements of the partition matrix determine the degree of coverage of projections of hyperboxes $[\underline{\beta}^{jp}_i, \overline{\beta}^{jp}_i]$ by the interval $[\underline{\beta}^{t_k}, \overline{\beta}^{t_k}]$.

Let us define the fitness function for the solution $\mathbf{\Psi}$. For the direct problem, we choose the function $1/Z(\mathbf{\Psi})$, inverse to the target one. For the dual problem, we choose the function $1-\delta(\mathbf{\Psi})$, where $\delta(\mathbf{\Psi})=e(\mathbf{\Psi})\big/(\overline{y}-\underline{y})$ is the normalized mean square error.

The accounting of restrictions of constrained optimization problems is carried out by fining the chromosomes, and the fitness function combines the target and penalty functions. According to [2], when leaving the admissible area, the chromosome is subject to fining, reducing the value of the target function. The amount of a penalty depends on the distance to a zone of permissible solutions and reflects the quality of the current population.

Thus, the fitness function for the solution $\mathbf{\Psi}$ has the form [2]:

– for the direct problem

$$F'(\mathbf{\Psi})=\begin{cases}\dfrac{1}{Z(\mathbf{\Psi})}, \text{if } \mathbf{\Psi} - \text{permissible solution},\\ \dfrac{1-\Omega'(\mathbf{\Psi})}{Z(\mathbf{\Psi})}, \text{otherwise;}\end{cases}$$

– for the dual problem

$$F''(\mathbf{\Psi})=\begin{cases}(1-\delta(\mathbf{\Psi})), \text{if } \mathbf{\Psi} - \text{permissible solution},\\ (1-\delta(\mathbf{\Psi}))(1-\Omega''(\mathbf{\Psi})), \text{otherwise,}\end{cases}$$

where $\Omega'(\mathbf{\Psi})\in[0,1]$ and $\Omega''(\mathbf{\Psi})\in[0,1]$ are penalty functions that take into account the violation of constraints of direct and dual optimization problems by the chromosome $\mathbf{\Psi}$.

The penalty function is defined as the relation [2]:

$$\Omega'(\mathbf{\Psi})=\frac{\Delta e(\mathbf{\Psi})}{\Delta e_{\max}} \text{ or } \Omega''(\mathbf{\Psi})=\frac{\Delta Z(\mathbf{\Psi})}{\Delta Z_{\max}}, \tag{13}$$

where $\Delta e(\mathbf{\Psi})=e(\mathbf{\Psi})-\overline{e}$ or $\Delta Z(\mathbf{\Psi})=Z(\mathbf{\Psi})-\overline{Z}$ is the deviation from the permissible value for the solution $\mathbf{\Psi}$; $\Delta e_{\max}$ or $\Delta Z_{\max}$ is the maximum deviation in the current population.

Penalties (13) vary for the same violations of constraints in different populations of chromosomes. An adaptation of penalties (13) to the population quality allows distancing high-quality solutions from low-quality ones during the selection [2]. Following [2], it is expedient to use tournament selection, which does not require tuning the penalty coefficients to take into account constraints.

5. Example: time series forecasting

The problem of forecasting the number of comments after the publication of a post in a social network is considered [24]. Experimental data were obtained from [25].

Input parameters are: x_1–x_3 – the number of comments for the last, the penultimate and the first day of observation, respectively, $x_{1\text{-}3}\in[0,\ 250]$; x_4 – observation window, $x4\in[1, 72]$ hours; x_5 – forecasting time-frame (H), $x_5\in[1, 24]$ hours. The output parameter is: y – the number of comments in the next H hours, $y\in[0, 250]$.

The indicator of the model accuracy is: P_{top10} – the probability of a correct forecast of top 10 posts, which will receive the largest number of comments. The task was to find an option of the knowledge base, which provides: $\underline{Z}\to$min and $\overline{P}_{top10}\geq 0.7$ in a direct statement; $P_{top10}\to$max and $\overline{Z}\leq 30$ in a dual statement.

As a result of the relational transformation, the number of primary input and output terms for the direct and dual problems was M=3, $k_{1\text{-}5}$=3.

The system of fuzzy relational equations for rule generation has the form:

$$\begin{aligned}
\mu^{E_1} &= [(\mu^{c_{11}}\wedge 0.90)\vee(\mu^{c_{12}}\wedge 0.41)\vee(\mu^{c_{13}}\wedge 0.22)]\wedge\\
&\wedge[(\mu^{c_{21}}\wedge 0.99)\vee(\mu^{c_{22}}\wedge 0.89)\vee(\mu^{c_{23}}\wedge 0.62)]\wedge\\
&\wedge[(\mu^{c_{31}}\wedge 0.87)\vee(\mu^{c_{32}}\wedge 0.64)\vee(\mu^{c_{33}}\wedge 0.26)]\wedge\\
&\wedge[(\mu^{c_{41}}\wedge 0.45)\vee(\mu^{c_{42}}\wedge 0.59)\vee(\mu^{c_{43}}\wedge 0.93)]\wedge\\
&\wedge[(\mu^{c_{51}}\wedge 0.95)\vee(\mu^{c_{52}}\wedge 0.46)\vee(\mu^{c_{53}}\wedge 0.30)];\\
\mu^{E_2} &= [(\mu^{c_{11}}\wedge 0.35)\vee(\mu^{c_{12}}\wedge 0.83)\vee(\mu^{c_{13}}\wedge 0.56)]\wedge\\
&\wedge[(\mu^{c_{21}}\wedge 0.17)\vee(\mu^{c_{22}}\wedge 0.61)\vee(\mu^{c_{23}}\wedge 0.84)]\wedge\\
&\wedge[(\mu^{c_{31}}\wedge 0.23)\vee(\mu^{c_{32}}\wedge 0.89)\vee(\mu^{c_{33}}\wedge 0.47)]\wedge\\
&\wedge[(\mu^{c_{41}}\wedge 0.68)\vee(\mu^{c_{42}}\wedge 0.82)\vee(\mu^{c_{43}}\wedge 0.32)]\wedge\\
&\wedge[(\mu^{c_{51}}\wedge 0.62)\vee(\mu^{c_{52}}\wedge 0.90)\vee(\mu^{c_{53}}\wedge 0.68)];\\
\mu^{E_3} &= [(\mu^{c_{11}}\wedge 0.11)\vee(\mu^{c_{12}}\wedge 0.50)\vee(\mu^{c_{13}}\wedge 0.74)]\wedge\\
&\wedge[(\mu^{c_{21}}\wedge 0.08)\vee(\mu^{c_{22}}\wedge 0.75)\vee(\mu^{c_{23}}\wedge 0.38)]\wedge\\
&\wedge[(\mu^{c_{31}}\wedge 0.12)\vee(\mu^{c_{32}}\wedge 0.50)\vee(\mu^{c_{33}}\wedge 0.75)]\wedge\\
&\wedge[(\mu^{c_{41}}\wedge 0.92)\vee(\mu^{c_{42}}\wedge 0.46)\vee(\mu^{c_{43}}\wedge 0.12)]\wedge\\
&\wedge[(\mu^{c_{51}}\wedge 0.20)\vee(\mu^{c_{52}}\wedge 0.77)\vee(\mu^{c_{53}}\wedge 0.89)].
\end{aligned}\tag{14}$$

Using the genetic algorithm, a set of solutions for β-parameters of candidate rules was obtained (Table 1).

Table 1

A set of values of β-parameters in solutions of the trend system of equations

No.	IF					THEN
	x_1	x_2	x_3	x_4	x_5	y
11	[0, 25]			[30, 40]	[0, 4]	d_1, [0, 50]
12	[0, 25]	[25, 112]		[36, 50]	[0, 4]	
13	[0, 25]	[0, 125]	[0, 25]	[55, 72]	[0, 4]	
21	52			[30, 45]	[0, 5]	d_2, [35, 100]
22	52	[52, 105]		[36, 55]	[0, 5]	
23	[0, 52]	[79, 105]	[100, 140]	[61, 72]	[0, 8]	
24	[0, 52]	52	[86, 100]	[61, 72]	[0, 8]	
31	[105, 140]			[20, 29]	[0, 16]	d_3^*, [75, 110]
32	[73, 105]	[89, 175]		[29, 43]	[6, 24]	
33	[105, 163]	200		[29, 43]	[6, 24]	
34	[73, 105]	89	[105, 163]	[46, 72]	[6, 24]	
35	[105, 140]	[55, 89]	[160, 197]	[46, 72]	[6, 24]	
41	[125, 180]			[10, 30]	[4, 18]	d_4, [90, 150]
42	[87, 125]	[168, 250]		[39, 50]	[7, 24]	
43	[125, 180]	[120, 140]		[39, 50]	[7, 24]	
44	[125, 180]	[168, 210]	[90, 125]	[61, 72]	[7, 24]	
45	[87, 125]	[120, 168]	[160, 197]	[61, 72]	[7, 24]	
51	[160, 197]			[12, 30]	[7, 18]	d_5, [130, 190]
52	[112, 160]	[174, 210]		[32, 50]	[12, 24]	
53	[160, 197]	210	[112, 160]	[46, 72]	[12, 24]	
54	[160, 185]	[140, 174]	[160, 197]	[46, 72]	[12, 24]	
61	203			[0, 29]	[11, 16]	d_6^*, [175, 220]
62	203	165		[29, 43]	[20, 24]	
63	[184, 203]	[165, 203]	[202, 250]	[40, 60]	[20, 24]	
64	[184, 203]	203	[171, 202]	[40, 60]	[20, 24]	
71	[217, 250]			[0, 27]	[12, 18]	d_7, [190, 250]
72	[217, 250]	[118, 153]		[20, 42]	[22, 24]	
73	[217, 250]	[160, 207]		[31, 50]	[22, 24]	
74	[168, 250]	[118, 153]	[200, 250]	[40, 46]	[22, 24]	
75	[168, 250]	[160, 207]	[167, 196]	[40, 46]	[22, 24]	

*Note: * – output classes (rules) for the dual problem*

The set of interval rules (Table 1) corresponds to the set of solutions of the system of equations (14), where the minimum solutions determine the rule length:

$$S(d_1)=\{0.86\cup[0.86,1]-\mu^{c_{11}},\mu^{c_{31}},\mu^{c_{43}},\mu^{c_{51}};$$
$$[0.86,1]\cup[0,0.86]-\mu^{c_{21}},\mu^{c_{22}};$$
$$0.23-\mu^{c_{23}};[0,0.23]-\mu^{c_{32}},\mu^{c_{33}},\mu^{c_{52}},\mu^{c_{53}};$$
$$[0,0.11]-\mu^{c_{12}},\mu^{c_{13}};[0,1]-\mu^{c_{41}},\mu^{c_{42}}\};$$

$$S(d_2)=\{0.61\cup[0.61,1]-\mu^{c_{11}},\mu^{c_{43}},\mu^{c_{51}};$$
$$[0.61,1]\cup[0,0.61]-\mu^{c_{21}},\mu^{c_{31}};$$
$$[0.61,1]\cup[0.35,0.61]\cup[0,0.61]-\mu^{c_{22}},\mu^{c_{32}};$$
$$[0.35,1]-\mu^{c_{42}};[0.35,0.50]\cup[0,0.50]-\mu^{c_{33}};$$
$$0.35\cup[0,0.35]-\mu^{c_{23}};$$
$$[0,0.20]-\mu^{c_{52}},\mu^{c_{53}};[0,0.11]-\mu^{c_{12}},\mu^{c_{13}};[0,1]-\mu^{c_{41}}\};$$

$$S(d_3)=\{0.57\cup[0.57,1]-\mu^{c_{23}},\mu^{c_{32}};$$
$$[0,0.57]\cup[0.57,1]-\mu^{c_{13}},\mu^{c_{41}},\mu^{c_{42}},\mu^{c_{52}},\mu^{c_{53}};$$
$$[0,0.57]\cup[0.41,0.57]\cup[0.57,1]-\mu^{c_{12}};$$
$$[0,0.50]-\mu^{c_{33}};[0,0.46]-\mu^{c_{51}};0.41\cup[0,0.41]-\mu^{c_{11}};$$
$$[0,0.38]-\mu^{c_{22}};[0,1]-\mu^{c_{21}},\mu^{c_{31}},\mu^{c_{43}}\};$$

$$S(d_4)=\{0.83\cup[0.83,1]-\mu^{c_{12}},\mu^{c_{23}},\mu^{c_{32}},\mu^{c_{42}},\mu^{c_{52}};$$
$$[0,0.64]-\mu^{c_{31}};[0,0.59]-\mu^{c_{43}};$$
$$0.50-\mu^{c_{22}},\mu^{c_{41}};[0,0.50]-\mu^{c_{13}},\mu^{c_{33}};$$
$$[0,0.46]-\mu^{c_{51}};[0,0.41]-\mu^{c_{11}};[0,1]-\mu^{c_{21}},\mu^{c_{53}}\};$$

$$S(d_5)=\{0.68\cup[0.68,1]-\mu^{c_{12}},\mu^{c_{23}},\mu^{c_{32}};$$
$$[0.68,1]\cup[0,0.68]-\mu^{c_{41}},\mu^{c_{42}},\mu^{c_{52}},\mu^{c_{53}};$$
$$[0,0.50]-\mu^{c_{13}},\mu^{c_{33}};0.46\cup[0,0.46]-\mu^{c_{22}};$$
$$[0,0.41]-\mu^{c_{11}},\mu^{c_{51}};$$
$$[0,1]-\mu^{c_{21}},\mu^{c_{31}},\mu^{c_{43}}\};$$

$$S(d_6)=\{0.54\cup[0.54,1]-\mu^{c_{13}},\mu^{c_{22}},\mu^{c_{33}},\mu^{c_{41}};$$
$$[0,0.54]\cup[0.54,1]-\mu^{c_{52}},\mu^{c_{53}};[0,0.54]-\mu^{c_{23}},\mu^{c_{42}};$$
$$[0,0.47]-\mu^{c_{32}},\mu^{c_{51}};[0,0.22]-\mu^{c_{11}},\mu^{c_{12}};$$
$$[0,1]-\mu^{c_{21}},\mu^{c_{31}},\mu^{c_{43}}\};$$

$$S(d_7)=\{0.74\cup[0.74,1]-\mu^{c_{13}},\mu^{c_{22}},\mu^{c_{33}},\mu^{c_{41}};$$
$$[0.74,1]\cup[0,0.74]-\mu^{c_{52}},\mu^{c_{53}};$$
$$[0,0.68]-\mu^{c_{42}};[0,0.61]-\mu^{c_{23}};[0,0.47]-\mu^{c_{32}},\mu^{c_{51}};$$
$$[0,0.22]-\mu^{c_{11}},\mu^{c_{12}};[0,1]-\mu^{c_{21}},\mu^{c_{31}},\mu^{c_{43}}\}.$$

As a result of the composite transformation, the number of output classes and rules was m=5, Z=21 for the direct problem; m=7, Z=30 for the dual problem.

As a result of the relational transformation, the number of output classes and input terms was m=5, q_{1-3}=5, q_4=6, q_5=5 for the direct problem; m=7, $q_{1\text{-}3}$=7, q_4=6, q_5=5 for the dual problem.

The fuzzy partition matrix for the direct and dual problems is presented in Table 2. Composite terms were obtained through linguistic modification: *little (L) – very little (vL); average (A) – lower (higher) than average (lA, hA); big (B) – very big (vB).*

The relation matrix (Table 2) is a concise form of presentation of the rules, the length of which can be variable. This means that for the incomplete composite "inputs – output" rules (Table 1), the relational "input – output" rules are absent in Table 2. Therefore, for each element of the partition matrix, the number of the composite rule in the output class is specified. This allows reproducing the measurability of hyperboxes over the partition matrix.

Table 2

Fuzzy "input terms – output classes" partition matrix

IF		THEN y						
		vL	*L*	*lA**	*A*	*hA*	*B**	*vB*
x_1	*vL*	0.96^{1-3}	0.92^{1-4}	0	0	0	0	0
	L	0	$0.98^{1,2}$	$0.71^{2,4}$	$0.34^{2,5}$	0	0	0
	*lA**	0	0	0.62^{1-5}	$0.91^{2,5}$	0	0	0
	A	0	0	$0.76^{1,3,5}$	0.64^{1-5}	0.83^{2}	0	0
	hA	0	0	0.60^{3}	$0.85^{1,3,4}$	0.79^{1-4}	$0.51^{1,2}$	$0.62^{4,5}$
	*B**	0	0	0	0	$0.68^{1,3}$	0.84^{1-4}	$0.91^{4,5}$
	vB	0	0	0	0	0	0	0.95^{1-5}
x_2	*vL*	$0.68^{2,3}$	0	0	0	0	0	0
	L	$0.94^{2,3}$	0.83^{2-4}	0.85^{5}	0	0	0	0
	*lA**	$0.85^{2,3}$	$0.86^{2,3}$	$0.69^{2,4,5}$	0	0	0	0
	A	0.54^{3}	$0.38^{2,3}$	0.94^{2}	$0.87^{3,5}$	0.64^{4}	0	$0.56^{2,4}$
	hA	0	0	0.67^{2}	$0.69^{2,4,5}$	$0.61^{2,4}$	$0.73^{2,3}$	0.64^{2-5}
	*B**	0	0	0.89^{3}	$0.90^{2,4}$	$0.76^{2,3}$	$0.79^{3,4}$	$0.87^{3,5}$
	vB	0	0	0	$0.81^{2,4}$	$0.50^{2,3}$	0	$0.39^{3,5}$
x_3	*vL*	0.57^{3}	0	0	0	0	0	0
	L	0	0.50^{4}	0	0	0	0	0
	*lA**	0	$0.62^{3,4}$	0.65^{4}	0.82^{4}	0	0	0
	A	0	0.91^{3}	0.86^{4}	0.91^{4}	0.75^{3}	0	0
	hA	0	0	$0.61^{4,5}$	0.86^{5}	$0.72^{3,4}$	0.69^{4}	0.54^{5}
	*B**	0	0	0.83^{5}	0.88^{5}	0.80^{4}	$0.84^{3,4}$	$0.90^{4,5}$
	vB	0	0	0	0	0.43^{4}	0.75^{3}	0.93^{4}
x_4	*L*	0	0	0	0.50^{1}	0.59^{1}	0.90^{1}	0.97^{1}
	lA	0	0	0.84^{1-3}	0.81^{1}	$0.75^{1,2}$	$0.91^{1,2}$	$0.96^{1,2}$
	A	0.68^{1}	0.74^{1}	$0.89^{2,3}$	$0.63^{2,3}$	0.90^{2}	0.82^{2}	$0.78^{2,3}$
	hA	0.65^{2}	0.86^{2}	0.80^{2-5}	$0.72^{2,3}$	0.74^{2-4}	0.68^{2-4}	0.70^{3-5}
	B	0.98^{3}	$0.80^{3,4}$	$0.97^{4,5}$	$0.80^{4,5}$	$0.93^{3,4}$	$0.74^{3,4}$	0
	vB	1.00^{3}	$0.95^{3,4}$	$0.95^{4,5}$	$0.94^{4,5}$	$0.98^{3,4}$	0	0
x_5	*L*	0.90^{1-3}	0.92^{1-4}	0.46^{1-5}	0.48^{1}	0	0	0
	lA	0	$0.84^{3,4}$	0.83^{1-5}	0.69^{1-5}	0.75^{1}	0	0
	A	0	0	0.87^{1-5}	0.72^{1-5}	0.77^{1-4}	0.80^{1}	0.60^{1}
	hA	0	0	0.60^{1-5}	0.65^{1-5}	0.63^{1-4}	0.59^{1-4}	0.65^{1}
	B	0	0	0.92^{2-5}	0.94^{2-5}	0.88^{2-4}	0.92^{2-4}	0.91^{2-4}

*Note: * – output classes and input terms for the dual problem*

Parameters of membership functions of the trend and composite models are shown in Table. 3. The membership functions of fuzzy terms are presented in Fig. 1.

Linguistic interpretation of β-parameters for the direct and dual problems is presented in Table 4.

Table 3

Parameters of membership functions of the trend and composite models

Input	Parameter	Trend model			Composite model						
		L	*A*	*B*	*vL*	*L*	*lA*	*A*	*hA*	*B*	*vB*
$x_{1\text{-}3}$	$\underline{\beta}$	0	55	160	0	40	80	92	137	177	186
	$\overline{\overline{\beta}}$	114	197	250	49	98	115	152	195	206	250
x_4	$\underline{\beta}$	1	20	46	–	1	16	28	37	52	59
	$\overline{\overline{\beta}}$	30	50	72	–	22	34	44	56	65	72
x_5	$\underline{\beta}$	1	7	16	–	1	4	9	14	20	–
	$\overline{\overline{\beta}}$	10	18	23	–	6	12	17	22	24	–

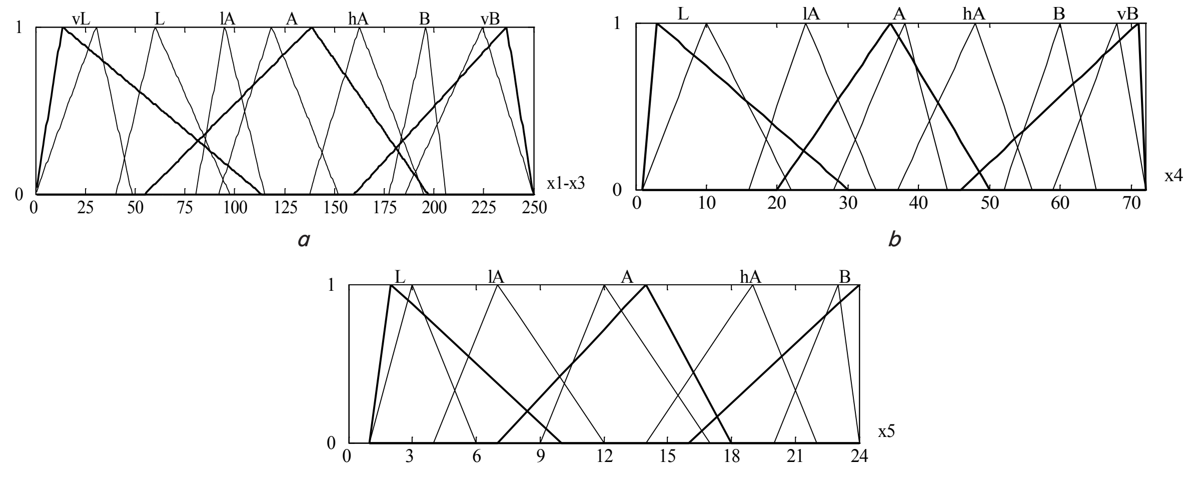

Fig. 1. Membership functions of fuzzy terms of variables: *a* – $x_{1\text{-}3}$; *b* – x_4; *c* – x_5

Table 4

Linguistic interpretation of solutions of the direct/dual problems

No.	IF					THEN
	x_1	x_2	x_3	x_4	x_5	y
11	*vL*			*A*	*L*	*vL*
12	*vL*	*vL-L/vL-lA*		*hA*	*L*	
13	*vL*	*vL-A*	*vL*	*B-vB*	*L*	
21	*L*			*A*	*L*	*L*
22	*L*	*L-A/L-lA*		*hA*	*L*	
23	*vL-L*	*L-A/L-lA*	*A/lA-A*	*B-vB*	*L-lA*	
24	*vL-L*	*L*	*L/L-lA*	*B-vB*	*L-lA*	
31	*lA-A*			*lA*	*L-hA*	*lA**
32	*L-lA*	*lA-hA*		*lA-hA*	*L-B*	
33	*lA-hA*	*B*		*lA-hA*	*L-B*	
34	*L-lA*	*lA*	*lA-hA*	*hA-vB*	*L-B*	
35	*lA-A*	*L-lA*	*hA-B*	*hA-vB*	*L-B*	
41	*A-hA*			*L-lA*	*L-hA*	*A*
42	*L-A/lA-A*	*hA-vB*		*A-hA*	*lA-B*	
43	*A-hA*	A		*A-hA*	*lA-B*	
44	*A-hA*	*hA-vB/hA-B*	*A/lA-A*	*B-vB*	*lA-B*	
45	*L-A/lA-A*	*A-hA*	*hA/hA-B*	*B-vB*	*lA-B*	
51	hA/hA-B			*L-lA*	*lA-hA*	*hA*
52	*A-hA*	*hA-vB/hA-B*		*lA-hA*	*A-B*	
53	*hA/hA-B*	*vB/B*	*A-hA*	*hA-vB*	*A-B*	
54	*hA*	*A-hA*	*hA-vB/hA-B*	*hA-vB*	*A-B*	
61	*B*			*L-lA*	*A-hA*	*B**
62	*B*	*hA*		*lA-hA*	*hA-B*	
63	*hA-B*	*hA-B*	*B-vB*	*hA-B*	*hA-B*	
64	*hA-B*	*B*	*hA-B*	*hA-B*	*hA-B*	
71	*vB*			*L-lA*	*A-hA*	*vB*
72	vB	A-hA		*lA-A*	*B*	
73	*vB*	*hA-vB/hA-B*		*A-hA*	*B*	
74	*hA-vB/B-vB*	*A-hA*	*vB/B-vB*	*hA*	*B*	
75	*hA-vB/B-vB*	*hA-vB/hA-B*	*hA/hA-B*	*hA*	*B*	

*Notes: * – output classes (rules) for the dual problem*

The linguistic model (Table 4) ensures the forecasting correctness at the level of P_{top10}=0.71 for the direct problem; P_{top10}=0.83 for the dual problem.

6. Discussion of the results of effectiveness estimation of improving transformations

The papers [19–23] proposed methods for simplifying the process of tuning a fuzzy knowledge base for the specified (unknown) output classes and input terms.

The given method uses these results as improving transformations that replace fragments of the granular *min-max* clustering algorithm. The principal difference of this method is the establishment of control variables in improving transformations that allow formalizing the fuzzy knowledge base generation. The effectiveness estimates of improving transformations are given below.

Candidate rule generation requires solving the optimization problem with $2nZ_0+2m_0$ variables for boundaries of interval rules and boundaries of output classes [4, 5, 7–11]. Application of the composite transformation allows reducing the number of tuning parameters by solving Z optimization problems for $2m$ boundaries of output classes [19–21]. $2N$ variables for boundaries of -parameters of the rules are subject to tuning. At the same time, the rule length is determined by minimum solutions. Rule generator tuning is the optimization problem with $2N+2M+NM$ variables for the relational matrix and boundaries of triangular membership functions.

Selection, that is, finding the best configuration of terms and rules of zero option, requires solving the optimization problem with $2K_0+2m_0+nZ_0$ variables. The boundaries of triangular membership functions, a term sign with the possibility of tuning the rule length, as well as the degree of relevance of rules are subject to tuning [4, 5, 7–11, 15, 16]. In the case of Z solutions of the trend system of equations, selection is reduced to maintaining the level of detail and density of coverage. Application of the relational transformation reduces the number of tuning parameters to $2K+2m+Km$ for the partition matrices and boundaries of triangular membership functions [23].

For time series forecasting problems, the model tuning time from the moment the new experimental data appear shall not exceed the forecasting time-frame. The minimum value of this parameter for the problem of forecasting the number of comments in a social network is 1 hour. The time of tuning according to the method [4, 5, 7–11, 15, 16] is 72 min, which exceeds the forecasting time-frame. The tuning time for this method is 21 min (Intel Core 2 Duo P7350 2.0 GHz processor).

A restriction of the proposed method is the use of improving transformations for fuzzy knowledge bases of classification type.

7. Conclusions

1. Logic-algorithmic models of improving transformations for the fuzzy classification knowledge base are proposed. Such transformations are transition to a composite or relational fuzzy model. The composite model represents interval solutions of the trend system of fuzzy logic equations. The number of rules in a class is determined by the number of solutions, and the granularity is determined by intervals of values of input variables in rules. The set of minimum solutions provides the minimum rule length. The relational model represents a linguistic interpretation of the resulting solutions. The level of detail and the density of coverage are determined by the "input terms – output classes" relational matrix, and the dimensions of hyperboxes are tuned using triangular membership functions. Composite transformation provides the choice of the number of output classes and rules, the relational – the choice of the number of input terms.

2. The method of fuzzy classification knowledge base optimization using improving transformations is proposed. Improving transformations allow formalizing the generation of fuzzy knowledge base variants with the simultaneous establishment of control variables that affect the accuracy and complexity of the model. This solves the problem of redundancy of terms and rules in *min-max* clustering problems. The choice of control variables (the number of classes, terms, and rules) is carried out using the genetic algorithm. At the same time, consistent use of composite and relational improving transformations provides tuning process simplification.

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