Inverse Problem Solving based on IF-THEN Rules and Genetic Algorithms

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Abstract

This paper proposes an approach for inverse problem solving based on the description of the interconnection between unobser-ved and observed parameters of an object with the help of fuzzy IF-THEN rules. The essence of the approach proposed consists in formulating and solving the optim-ization problems, which, on the one hand, find the roots of fuzzy logical equations, corresponding to IF-THEN rules, and on the other hand, tune the fuzzy model on the readily available experimental data. The genetic algorithms are proposed for the optimization problems solving.

***Index Terms*— inverse problem, fuzzy IF-THEN rules, fuzzy logical equations solving**

1. Introduction

The wide class of the problems, arising in engineering, medicine, economics and other domains, belongs to the class of the so called inverse problems. The essence of the inverse problem consists in the following. The dependency *Y=f(X)* is known, which connects the vector *X* of the unobserved parameters with the vector *Y* of the observed parameters. It is necessary to ascertain the unknown values of the vector *X* through the known values of the vector *Y*. The typical representative of the inverse problem is the problem of medical and technical diagnosis, which amounts to the restoration and the identification of the unknown causes of the disease or the failure through the observed effects, i.e. the symptoms or the external signs of the failure.

The classical theory of the inverse problems [11] envisages the possibility of description of the dependency *Y=f(X)* with the help of differential or other equations. In the cases, when it is impossible to obtain such equations, the dependency between unobserved and observed parameters can be modelled using the means of fuzzy sets theory [12, 13]: fuzzy relations and fuzzy IF-THEN rules. The analytical [1, 2, 6] and numerical [8 – 10] methods of solving the inverse problems of diagnosis on the basis of fuzzy relations and Zadeh's compositional rule of inference are the most developed ones.

In this paper we propose an approach for solving inverse problem based on description of the dependency *Y=f(X)* with the help of fuzzy IF-THEN rules. These rules enable to consider complex combinations in cause-effect connections simpler and more naturally, which are difficult to model with fuzzy relations. For example, the expert interconnection of the unobserved and the observed parameters (causes and effects) in the fuel pipe diagnosis problem can look as follows:

> IF feed pressure is *high* and leakage is *low* and pipe resistance is *low,* THEN delivery head is *high* and productivity is *high*.

This example has three input (unobserved) parameters and two output (observed) parameters. Each parameter is evaluated by the fuzzy term. The problem consists not only in solving system of fuzzy logical equations, which correspond to IF-THEN rules, but also in selection of such forms of the fuzzy terms membership functions and such weights of the fuzzy IF – THEN rules, which provide maximal proximity between model and real results of diagnosis.

The essence of the proposed approach consists in formulating and solving the optimization

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problems, which, on the one hand, find the roots of fuzzy logical equations, corresponding to IF-THEN rules, and, on the other hand, tune the fuzzy model on the readily available experimental data. The genetic algorithms are proposed for the formulated optimization problems solving

2. Fuzzy Model of the Object

Inputs–outputs connection can be represented with use of expert matrix of knowledge (see Table 1) [12]. The fuzzy knowledge base below corresponds to this matrix:

Table 1: Fuzzy knowledge base

Rule number	Inputs				Outputs				Rule weight
	x_1	x_2	...	x_n	y_1	y_2	...	y_m	
1	a_{11}	a_{21}	...	a_{n1}	b_{11}	b_{21}	...	b_{m1}	w_1
2	a_{12}	a_{22}	...	a_{n2}	b_{12}	b_{22}	...	b_{m2}	w_2
...	...	...	...	...	...	...	...	...	...
K	a_{1K}	a_{2K}	...	a_{nK}	b_{1K}	b_{2K}	...	b_{mK}	w_K

IF $x_1 = a_{11}$ and $x_2 = a_{21}$ … and $x_n = a_{n1}$
THEN $y_1 = b_{11}$ and $y_2 = b_{21}$ … and $y_m = b_{m1}$
with weight w_1,
IF $x_1 = a_{12}$ and $x_2 = a_{22}$ … and $x_n = a_{n2}$
THEN $y_1 = b_{12}$ and $y_2 = b_{22}$ … and $y_m = b_{m2}$
with weight w_2,
…
IF $x_1 = a_{1K}$ and $x_2 = a_{2K}$ … and $x_n = a_{nK}$
THEN $y_1 = b_{1K}$ and $y_2 = b_{2K}$ … and $y_m = b_{mK}$
with weight w_K, (1)

where a_{il} is a fuzzy term for variable x_i evaluation in the rule with number l; b_{jl} is a fuzzy term for variable y_j evaluation in the rule with number l; w_l is a rule weight, i.e. a number in the range [0, 1], characterizing the subjective measure of confidence of an expert relative to the statement with number l; K is the number of fuzzy rules.

The problem of inverse logical inference is set in the following way: it is necessary to restore and identify the values of the input parameters $(x_1^*, x_2^*, ..., x_n^*)$ through the values of the observed output parameters $(y_1^*, y_2^*, ..., y_m^*)$. The restoration of the inputs amounts to the solution of the system of fuzzy logical equations, which is derived from relation (1):

$$\mu^{a_{11}}(x_1) \wedge \mu^{a_{21}}(x_2) ... \wedge \mu^{a_{n1}}(x_n) = w_1 \cdot (\mu^{b_{11}}(y_1) \wedge \mu^{b_{21}}(y_2) ... \wedge \mu^{b_{m1}}(y_m))$$
$$\mu^{a_{12}}(x_1) \wedge \mu^{a_{22}}(x_2) ... \wedge \mu^{a_{n2}}(x_n) = w_2 \cdot (\mu^{b_{12}}(y_1) \wedge \mu^{b_{22}}(y_2) ... \wedge \mu^{b_{m2}}(y_m))$$
$$\dots \quad \dots \quad \dots \quad \dots$$
$$\mu^{a_{1K}}(x_1) \wedge \mu^{a_{2K}}(x_2) ... \wedge \mu^{a_{nK}}(x_n) = w_K \cdot (\mu^{b_{1K}}(y_1) \wedge \mu^{b_{2K}}(y_2) ... \wedge \mu^{b_{mK}}(y_m)). \quad (2)$$

Here $\mu^{a_{il}}(x_i)$ is a membership function of a variable x_i to the fuzzy term a_{il}; $\mu^{b_{jl}}(y_j)$ is a membership function of a variable y_j to the fuzzy term b_{jl}.

Taking into account the fact that operation $\wedge$ is replaced by *min* in fuzzy set theory [13], system (2) is rewritten in the form

$$\min_{i=1,n}\left[\mu^{a_{i1}}(x_i)\right] = w_1 \cdot \min_{j=1,m}\left[\mu^{b_{j1}}(y_j)\right]$$
$$\min_{i=1,n}\left[\mu^{a_{i2}}(x_i)\right] = w_2 \cdot \min_{j=1,m}\left[\mu^{b_{j2}}(y_j)\right]$$
$$\dots$$
$$\min_{i=1,n}\left[\mu^{a_{iK}}(x_i)\right] = w_K \cdot \min_{j=1,m}\left[\mu^{b_{jK}}(y_j)\right]$$

or

$$\min_{i=1,n}\left[\mu^{a_{il}}(x_i)\right] = \mu^{B_l}(w_l, Y), \quad l = \overline{1,K}, \quad (3)$$

where $\mu^{B_l}(w_l, Y)$ is the measure of the effects combination significance in the rule number l.

The use of fuzzy logical equations provides for the presence of the fuzzy terms membership functions included in the knowledge base. We use a bell-shaped membership function model of variable u to arbitrary term T in the form [7]:

$$\mu^T(u) = \frac{1}{1 + \left(\frac{u - \beta}{\sigma}\right)^2}, \quad (4)$$

where β is a coordinate of function maximum, $\mu^T(\beta) = 1$; σ is a parameter of concentration.

Correlations (3) and (4) define the generalized fuzzy model of an object as follows:

$$F_Y(X,\mathrm{B}_C,\Omega_C) = \mu^B(Y,W,\mathrm{B}_E,\Omega_E), \qquad (5)$$

where $X=(x_1,x_2,...,x_n)$ is the vector of input variables; $Y=(y_1,y_2,...,y_m)$ is the vector of output variables; $\mu^B=(\mu^{B_1},\mu^{B_2},...,\mu^{B_K})$ is the vector of effects combinations significances measures in the IF–THEN rules; $W=(w_1,w_2,...,w_K)$ is the vector of fuzzy rules weights; $\mathrm{B}_C=(\beta^{C_1},\beta^{C_2},...,\beta^{C_N})$ and $\Omega_C=(\sigma^{C_1},\sigma^{C_2},...,\sigma^{C_N})$ are the vectors of β- and σ- parameters for input variables membership functions to the fuzzy terms C_1, C_2, … , C_N; $\mathrm{B}_E=(\beta^{E_1},\beta^{E_2},...,\beta^{E_M})$ and $\Omega_E=(\sigma^{E_1},\sigma^{E_2},...,\sigma^{E_M})$ are the vectors of β- and σ- parameters for output variables membership functions to the fuzzy terms E_1, E_2, … , E_M; N is the total number of fuzzy terms for input variables; M is the total number of fuzzy terms for output variables; F_Y is the operator of inputs – outputs connection, corresponding to formulae (3), (4).

3. Solving Fuzzy Logical Equations

Following the approach, proposed in [8 – 10], the problem of solving system of fuzzy logical equations (3) is formulated as follows. Vector $\mu^C=(\mu^{C_1},\mu^{C_2},...,\mu^{C_N})$ of the membership degrees of the inputs to fuzzy terms C_1, C_2, … , C_N, should be found which satisfies the constraints $\mu^{C_k}\in[0,1]$, and also provides the least distance between model and observed measures of effects combinations significances, that is between the left and the right parts of each system equation (3):

$$F=\sum_{l=1}^{K}\left[\min_{\mathrm{i}=\overline{1,\mathrm{n}}}\left[\mu^{a_{il}}(x_i)\right]-\mu^{B_l}(Y)\right]^2=\min_{\mu^C}. \qquad (6)$$

In accordance with [1, 2, 6], in the general case system (3) has a solution set $S(\mu^B)$, which is completely characterized by the unique minimal solution $\underline{\mu}^C$ and the set of maximal solutions $S^*(\mu^B)=\left\{\overline{\mu}_t^C,\, t=\overline{1,T}\right\}$:

$$S(\mu^B) = \bigcup_{\overline{\mu}_t^C\in S^*}\left[\underline{\mu}^C,\overline{\mu}_t^C\right]. \qquad (7)$$

Here $\underline{\mu}^C=(\underline{\mu}^{C_1},\underline{\mu}^{C_2},...,\underline{\mu}^{C_N})$ and $\overline{\mu}_t^C=(\overline{\mu}_t^{C_1},\overline{\mu}_t^{C_2},...,\overline{\mu}_t^{C_N})$ are the vectors of the lower and upper bounds of the membership degrees of the inputs to the terms C_k, where the union is taken over all $\overline{\mu}_t^C\in S^*(\mu^B)$.

Formation of solution set (7) begins with the search for the null solution of optimization problem (6). As the null solution of optimization problem (6) we designate $\mu_0^C=(\mu_0^{C_1},\mu_0^{C_2},...,\mu_0^{C_N})$, where $\mu_0^{C_k}\geq\underline{\mu}^{C_k}$, $\mathrm{k}=\overline{1,\mathrm{N}}$. The modified vector of the effects combinations significances measures $\mu_0^B=(\mu_0^{B_1},\mu_0^{B_2},...,\mu_0^{B_K})$, which corresponds to the obtained null solution μ_0^C, provides the analytical solvability of the system of fuzzy logical equations (3). Formation of the solution set $S(\mu_0^B)$ for the modified vector μ_0^B is accomplished by exact analytical methods [1, 2, 6] supported by the free software [6].

The real-coded genetic algorithm is used for the null solution finding [3 – 5]. We define the chromosome as the vector of real parameters μ^{C_k}, $\mathrm{k}=\overline{1,\mathrm{N}}$. The multi-crossover operation [3] provides a more accurate adjusting direction for evolving offsprings that allows to systematically reduce the size of the search region. The non-uniform mutation whose action depends on the age of the population provides generation of the non-dominated solutions. We used the roulette wheel selection procedure giving priority to the best solutions. We choose criterion (6) as the fitness function. While performing the genetic algorithm the size of the population stays constant. That is why after crossover and mutation operations it is necessary to remove the chromosomes having the worst values of the fitness function from the obtained population.

4. Fuzzy Model Tuning

It is assumed that the training data which is given in the form of L pairs of experimental data is known: $\left\langle \hat{X}_p, \hat{Y}_p \right\rangle$, $p = \overline{1, L}$, where $\hat{X}_p = \left(\hat{x}_1^p, \hat{x}_2^p, ..., \hat{x}_n^p\right)$ and $\hat{Y}_p = \left(\hat{y}_1^p, \hat{y}_2^p, ..., \hat{y}_m^p\right)$ are the vectors of the values of the input and output variables in the experiment number p.

The essence of tuning of the fuzzy model (5) consists of finding such vector of fuzzy rules weights W and such vectors of membership functions parameters B_C, Ω_C, B_E, Ω_E, which provide the least distance between model and experimental vectors of the effects combinations significances measures:

$$\sum_{\mathrm{p}=1}^{\mathrm{L}} \left[F_Y(\hat{\mathrm{X}}_{\mathrm{p}}, \mathrm{B}_{\mathrm{C}}, \Omega_{\mathrm{C}}) - \hat{\mu}^B(\hat{Y}_p, W, \mathrm{B}_{\mathrm{E}}, \Omega_{\mathrm{E}})\right]^2 = \min \tag{8}$$

The chromosome needed in the multi-crossover real-coded genetic algorithm for solving this optimization problem is defined as the vector of real parameters W, B_C, Ω_C, B_E, Ω_E. Fitness function is built on the basis of criterion (8).

5. Computer Experiment

The aim of the experiment consists of checking the performance of the above proposed models and algorithms with the help of the target "inputs – outputs" model. The target model were some analytical functions $y_1 = f_1(x_1, x_2)$ and $y_2 = f_2(x_1, x_2)$. These functions were approximated by the combined fuzzy knowledge base, and served simultaneously as training and testing data generator. The input values (x_1, x_2), restored for each output combination (y_1, y_2), were compared with the target level lines.

The target model is given by the formulae: $y_1 = f_1(x_1, x_2) = \frac{1}{10}(2z_1 - 0{,}9)\,(7z_1 - 1)\,(17z_2 - 19)\,(15z_2 - 2)$, $y_2 = f_2(x_1, x_2) = -y_1 + 3.4$, where

$$z_1 = \frac{(x_1 - 2{,}9)^2 + (x_2 - 2{,}9)^2}{39}, \quad z_2 = \frac{(x_1 - 3{,}1)^2 + (x_2 - 3{,}1)^2}{41}.$$

The target model is represented in Figure 1.

The fuzzy IF-THEN rules correspond to this model (see Table 2).

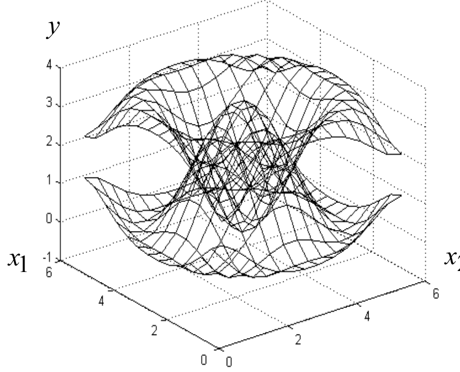


Figure 1: «Inputs – outputs» model-generator

Table 2: Fuzzy IF-THEN rules for target model

Rule	Inputs		Outputs	
	x_1	x_2	y_1	y_2
1	*L*	*L*	*lA*	*hA*
2	*A*	*L*	*hL*	*lH*
3	*H*	*L*	*lA*	*hA*
4	*L*	*A*	*hL*	*lH*
5	*A*	*A*	*H*	*L*
6	*H*	*A*	*hL*	*lH*
7	*L*	*H*	*lA*	*hA*
8	*A*	*H*	*hL*	*lH*
9	*H*	*H*	*lA*	*hA*

In Table 2 the total number of the terms for the input and output variables consists of: C_1 *Low (L)*, C_2 *Average (A)*, C_3 *High (H)* for x_1, C_4 *(L)*, C_5 *(A)*, C_6 *(H)* for x_2; E_1=*higher than Low* (*hL*), E_2= *lower than Average (lA)*, E_3= *High (H)* for y_1; E_4=*Low (L)*, E_5=*higher than Average (hA)*, E_6=*lower than High (lH)* for y_2.

Fuzzy logical equations take the following form:

$$\begin{aligned}
&\mu^{C_1} \wedge \mu^{C_4} = w_1 \cdot (\mu^{E_2} \wedge \mu^{E_5})\\
&\mu^{C_2} \wedge \mu^{C_4} = w_2 \cdot (\mu^{E_1} \wedge \mu^{E_6})\\
&\mu^{C_3} \wedge \mu^{C_4} = w_3 \cdot (\mu^{E_2} \wedge \mu^{E_5})\\
&\mu^{C_1} \wedge \mu^{C_5} = w_4 \cdot (\mu^{E_1} \wedge \mu^{E_6})\\
&\mu^{C_2} \wedge \mu^{C_5} = w_5 \cdot (\mu^{E_3} \wedge \mu^{E_4})\\
&\mu^{C_3} \wedge \mu^{C_5} = w_6 \cdot (\mu^{E_1} \wedge \mu^{E_6})\\
&\mu^{C_1} \wedge \mu^{C_6} = w_7 \cdot (\mu^{E_2} \wedge \mu^{E_5})\\
&\mu^{C_2} \wedge \mu^{C_6} = w_8 \cdot (\mu^{E_1} \wedge \mu^{E_6})\\
&\mu^{C_3} \wedge \mu^{C_6} = w_9 \cdot (\mu^{E_2} \wedge \mu^{E_5}).
\end{aligned} \tag{9}$$

The parameters of the fuzzy model before (after) tuning are given in Tables 3, 4. The results of solving the inverse problem after tuning are shown in Figure 2. The same figure depicts the membership functions of the fuzzy terms for the input and output variables.

Table 3: Rules weights

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
1 (0.93)	1 (0.97)	1 (0.92)	1 (0.95)	1 (1.00)	1 (0.97)	1 (0.92)	1 (0.96)	1 (0.93)

Table 4: Membership functions parameters for the input and output variables fuzzy terms

	Fuzzy terms					
	C_1	C_2	C_3	C_4	C_5	C_6
β -	0 (0.03)	3.0 (3.04)	6.0 (5.97)	0 (0.02)	3.0 (3.05)	6.0 (5.96)
σ -	0.5 (0.41)	1.0 (0.82)	0.5 (0.39)	0.5 (0.43)	1.0 (0.9)	0.5 (0.4)

	Fuzzy terms					
	E_1	E_2	E_3	E_4	E_5	E_6
β -	0.5 (0.52)	1.0 (0.91)	3.4 (3.35)	0.1 (0.10)	2.5 (2.57)	3.0 (3.03)
σ -	0.3 (0.28)	0.3 (0.16)	2.0 (1.95)	2.0 (1.93)	0.3 (0.14)	0.3 (0.26)

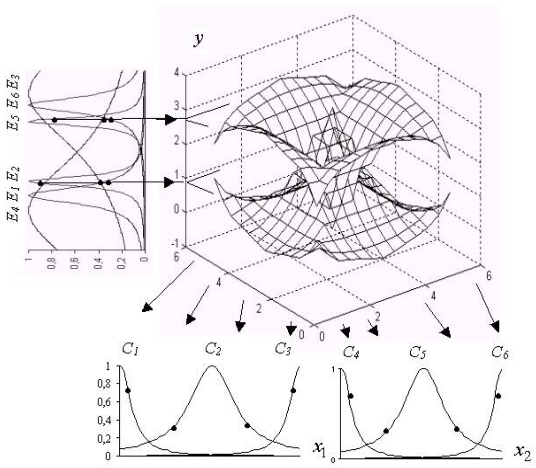


Figure 2: Solution to the inverse problem

Let the specific values of the output variables consists of y_1^*=0.95 and y_2^*=2.65. The degrees of membership of the outputs to the fuzzy terms $E_1 \div E_6$ for these values can be defined with the help of the membership functions in Figure 2

$\mu^{E_1}(y_1^*)$=0.30; $\mu^{E_2}(y_1^*)$=0.94; $\mu^{E_3}(y_1^*)$=0.40;

$\mu^{E_4}(y_2^*)$=0.36; $\mu^{E_5}(y_2^*)$=0.75; $\mu^{E_6}(y_2^*)$=0.32

Taking into account the rules weights (Table 3), the vector of the effects combinations significances measures takes the following form:

$$\mu^B(Y^*) = (\mu^{B_1} = 0.70; \mu^{B_2} = 0.29; \mu^{B_3} = 0.70; \mu^{B_4} = 0.29; \mu^{B_5} = 0.36; \mu^{B_6} = 0.29; \mu^{B_7} = 0.70; \mu^{B_8} = 0.29; \mu^{B_9} = 0.70).$$

The null solution was obtained with the help of the genetic algorithm

$$\mu_0^C = (\mu_0^{C_1} = 0.9, \mu_0^{C_2} = 0.3, \mu_0^{C_3} = 0.8, \mu_0^{C_4} = 0.7, \mu_0^{C_5} = 0.3, \mu_0^{C_6} = 0.7),$$

for which the modified vector of effects combinations significances measures corresponds

$$\mu_0^B = (\mu_0^{B_1} = 0.7, \mu_0^{B_2} = 0.3, \mu_0^{B_3} = 0.7, \mu_0^{B_4} = 0.3, \mu_0^{B_5} = 0.3, \mu_0^{B_6} = 0.3, \mu_0^{B_7} = 0.7, \mu_0^{B_8} = 0.3, \mu_0^{B_9} = 0.7).$$

The optimization criterion (6) takes the value of F=0.0040.

This modified vector allows us to use the implemented in MATLAB Fuzzy Relational Calculus Toolbox [6] for finding the solution set $S(\mu_0^B)$. Using the standard solver ***solve_flse*** [6] we obtain the following results. The solution set $S(\mu_0^B)$ for the modified vector μ_0^B is completely determined by the minimal solution

$$\underline{\mu}^C = (\underline{\mu}^{C_1} = 0.7, \underline{\mu}^{C_2} = 0.3, \underline{\mu}^{C_3} = 0.7, \underline{\mu}^{C_4} = 0.7, \underline{\mu}^{C_5} = 0.3, \underline{\mu}^{C_6} = 0.7)$$

and the two maximal solutions $S^* = \{\bar{\mu}_1^C, \bar{\mu}_2^C\}$

$$\bar{\mu}_1^C = (\bar{\mu}_1^{C_1} = 0.7, \bar{\mu}_1^{C_2} = 0.3, \bar{\mu}_1^{C_3} = 0.7, \bar{\mu}_1^{C_4} = 1.0, \bar{\mu}_1^{C_5} = 0.3, \bar{\mu}_1^{C_6} = 1.0);$$

$$\bar{\mu}_2^C = (\bar{\mu}_2^{C_1} = 1.0, \bar{\mu}_2^{C_2} = 0.3, \bar{\mu}_2^{C_3} = 1.0, \bar{\mu}_2^{C_4} = 0.7, \bar{\mu}_2^{C_5} = 0.3, \bar{\mu}_2^{C_6} = 0.7).$$

Thus the solution of the system (9) of fuzzy logical equations can be represented in the form of intervals:

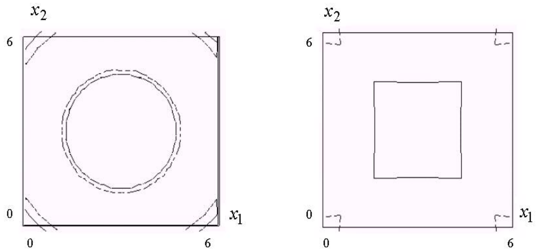
$$S(\mu^E) = \{\mu^{C_1} = 0.7,\ \mu^{C_2} = 0.3,\ \mu^{C_3} = 0.7,\ \mu^{C_4} \in [0.7, 1.0],\ \mu^{C_5} = 0.3,\ \mu^{C_6} \in [0.7, 1.0]\} \cup \{\mu^{C_1} \in [0.7, 1.0],\ \mu^{C_2} = 0.3, \mu^{C_3} \in [0.7, 1.0],\ \mu^{C_4} = 0.7,\ \mu^{C_5} = 0.3,\ \mu^{C_6} = 0.7\}. \quad (10)$$

The intervals of the values of the input variable for each interval in solution (10) can be defined with the help of the membership functions in Figure 2:

x_1^*=0.3 or $x_1^* \in [0, 0.3]$ for C_1;

x_1^*=1.8 or x_1^*=4.3 for C_2;
x_1^*=5.7 or $x_1^* \in [5.7, 6.0]$ for C_3;
$x_2^* \in [0, 0.3]$ or x_2^*=0.3 for C_4;
x_2^*=1.7 or x_2^*=4.4 for C_5;
$x_2^* \in [5.7, 6.0]$ or x_2^*=5.7 for C_6.

The restoration of the input set for y_1^*=0.95 and y_2^*=2.65 is shown in Figure 2. The values of the membership degrees of the inputs to fuzzy terms $C_1 \div C_6$ are marked. The comparison of the target and restored level lines for y_1^*=0.95 and y_2^*=2.65 is shown in Figure 3.



a) b)

Figure 3: Comparison of the target (a) and restored (b) level lines for y_1^* (__) and y_2^* (---)

6. Example of Technical Diagnosis

We shall consider faults causes diagnosis of the hydraulic elevator (for dump truck body, excavator ladle etc.). Input parameters are (variation ranges are indicated in parentheses): x_1 – engine speed (30–50 r/s); x_2 – inlet pressure (0,02–0,15 kg/cm^2); x_3 – clearance of the feed change gear (0.1–0.3 mm); x_4 – oil leakage (0.5–2.0 cm ^{3}/min). Output parameters of the hydro elevator are: y_1 – productivity (17–22 l/min); y_2 – force main pressure (13–24 kg/cm^2); y_3 – consumed power (2.1–3.0 kw); y_4 – suction conduit pressure (0.5–1.0 kg/cm^2).

The IF-THEN rules are used for hydro elevator diagnosis (see Table 5).

Table 5: Fuzzy rules for hydroelevator diagnosis

	Inputs				Outputs			
	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
1	*D*	*D*	*D*	*I*	*D*	*D*	*D*	*I*
2	*D*	*I*	*D*	*D*	*D*	*D*	*I*	*D*
3	*I*	*D*	*D*	*I*	*D*	*I*	*D*	*I*
4	*I*	*D*	*D*	*D*	*I*	*I*	*D*	*D*
5	*I*	*I*	*D*	*I*	*D*	*I*	*I*	*D*
6	*I*	*I*	*I*	*D*	*I*	*D*	*I*	*I*
7	*I*	*I*	*I*	*I*	*D*	*D*	*I*	*I*

In Table 5 the total number of the causes and effects consists of: C_1 *Decrease* (*D*) and C_2 *Increase* (*I*) for x_1, C_3 (*D*) and C_4 (*I*) for x_2, C_5 (*D*) and C_6 (*I*) for x_3, C_7 (*D*) and C_8 (*I*) for x_4; E_1 (*D*) and E_2 (*I*) for y_1, E_3 (*D*) and E_4 (*I*) for y_2, E_5 (*D*) and E_6 (*I*) for y_3, E_7 (*D*) and E_8 (*I*) for y_4.

Fuzzy logical equations take the following form:

$$
\begin{aligned}
&\mu^{C_1} \wedge \mu^{C_3} \wedge \mu^{C_5} \wedge \mu^{C_8} = w_1(\mu^{E_1} \wedge \mu^{E_3} \wedge \mu^{E_5} \wedge \mu^{E_8})\\
&\mu^{C_1} \wedge \mu^{C_4} \wedge \mu^{C_5} \wedge \mu^{C_7} = w_2(\mu^{E_1} \wedge \mu^{E_3} \wedge \mu^{E_6} \wedge \mu^{E_7})\\
&\mu^{C_2} \wedge \mu^{C_3} \wedge \mu^{C_5} \wedge \mu^{C_8} = w_3(\mu^{E_1} \wedge \mu^{E_4} \wedge \mu^{E_5} \wedge \mu^{E_8})\\
&\mu^{C_2} \wedge \mu^{C_3} \wedge \mu^{C_5} \wedge \mu^{C_7} = w_4(\mu^{E_2} \wedge \mu^{E_4} \wedge \mu^{E_5} \wedge \mu^{E_7})\\
&\mu^{C_2} \wedge \mu^{C_4} \wedge \mu^{C_5} \wedge \mu^{C_8} = w_5(\mu^{E_1} \wedge \mu^{E_4} \wedge \mu^{E_6} \wedge \mu^{E_7})\\
&\mu^{C_2} \wedge \mu^{C_4} \wedge \mu^{C_6} \wedge \mu^{C_7} = w_6(\mu^{E_2} \wedge \mu^{E_3} \wedge \mu^{E_6} \wedge \mu^{E_8})\\
&\mu^{C_2} \wedge \mu^{C_4} \wedge \mu^{C_6} \wedge \mu^{C_8} = w_7(\mu^{E_1} \wedge \mu^{E_3} \wedge \mu^{E_6} \wedge \mu^{E_8})
\end{aligned}
\quad (11)
$$

For the fuzzy model tuning we used the results of diagnosis for 340 hydroelevators. The results of the fuzzy model tuning are given in Tables 6, 7 and Figure 4.

Let us represent the vector of the observed parameters for a specific elevator: Y^*=(y_1^*=18 l/min; y_2^*=21.5 kg/cm^2; y_3^*=2.35 kw; y_4^*=0.8 kg/cm^2). The degrees of membership of the outputs to the effects $E_1 \div E_8$ for these values can be defined with the help of the membership functions in Figure 4,b:

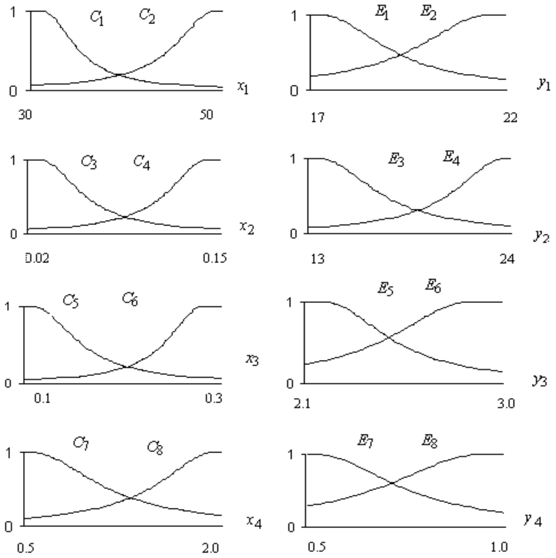
Table 6: Parameters of the membership functions for the causes and effects fuzzy terms

Parameter	Fuzzy terms							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
β-	31.20	49.15	0.03	0.14	0.11	0.28	0.55	1.94
σ-	3.87	4.82	0.03	0.03	0.05	0.04	0.59	0.50

Parameter	Fuzzy terms							
	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
β-	17.27	21.52	13.50	22.65	2.19	2.84	0.52	0.94
σ-	1.88	2.10	3.22	2.84	0.33	0.41	0.24	0.28

Table 7: Rules weights

w_1	w_2	w_3	w_4	w_5	w_6	w_7
0.80	0.65	0.99	0.95	0.98	0.92	0.53



a) b)

Figure 4: Membership functions of the causes (a) and effects (b) fuzzy terms after tuning

$\mu^{E_1}(y_1^*)=0.87;\ \mu^{E_2}(y_1^*)=0.26;\ \mu^{E_3}(y_2^*)=0.14;$
$\mu^{E_4}(y_2^*)=0.86;\ \mu^{E_5}(y_3^*)=0.81;\ \mu^{E_6}(y_3^*)=0.41;$
$\mu^{E_7}(y_4^*)=0.42;\ \mu^{E_8}(y_4^*)=0.80.$

Taking into account the rules weights (Table 7), the vector of the effects combinations significances measures takes the following form:

$$\mu^B(Y^*)=(\mu^{B_1}=0.11;\ \mu^{B_2}=0.09;\ \mu^{B_3}=0.80;\ \mu^{B_4}=0.25;\ \mu^{B_5}=0.40;\ \mu^{B_6}=0.13;\ \mu^{B_7}=0.07).$$

The null solution was obtained with the help of the genetic algorithm

$$\mu_0^C=(\mu_0^{C_1}=0.1,\mu_0^{C_2}=0.9,\mu_0^{C_3}=1.0,\mu_0^{C_4}=0.4,\ \mu_0^{C_5}=0.8,\mu_0^{C_6}=0.1,\mu_0^{C_7}=0.25,\mu_0^{C_8}=0.9),$$

for which the modified vector of effects combinations significances measures corresponds

$$\mu_0^B=(\mu_0^{B_1}=0.1,\mu_0^{B_2}=0.1,\mu_0^{B_3}=0.8,\mu_0^{B_4}=0.25,\ \mu_0^{B_5}=0.4,\mu_0^{B_6}=0.1,\mu_0^{B_7}=0.1).$$

The optimization criterion (6) takes the value of *F*=0.0020.

Using the standard solver ***solve_flse*** [6] we obtain the following results. The solution set $S(\mu_0^B)$ for the modified vector μ_0^B is completely determined by the minimal solution

$$\underline{\mu}^C=(\underline{\mu}^{C_1}=0.1,\underline{\mu}^{C_2}=0.8,\underline{\mu}^{C_3}=0.8,\underline{\mu}^{C_4}=0.4,\ \underline{\mu}^{C_5}=0.8,\underline{\mu}^{C_6}=0.1,\underline{\mu}^{C_7}=0.25,\underline{\mu}^{C_8}=0.8)$$

and the four maximal solutions $S^*=\{\overline{\mu}_1^C,\overline{\mu}_2^C,\overline{\mu}_3^C,\overline{\mu}_4^C\}$

$$\overline{\mu}_1^C=(\overline{\mu}_1^{C_1}=0.1,\overline{\mu}_1^{C_2}=0.8,\overline{\mu}_1^{C_3}=1.0,\overline{\mu}_1^{C_4}=0.4,\ \overline{\mu}_1^{C_5}=1.0,\overline{\mu}_1^{C_6}=0.1,\overline{\mu}_1^{C_7}=0.25,\overline{\mu}_1^{C_8}=1.0)$$

$$\overline{\mu}_2^C=(\overline{\mu}_2^{C_1}=0.1,\overline{\mu}_2^{C_2}=1.0,\overline{\mu}_2^{C_3}=0.8,\overline{\mu}_2^{C_4}=0.4,\ \overline{\mu}_2^{C_5}=1.0,\overline{\mu}_2^{C_6}=0.1,\overline{\mu}_2^{C_7}=0.25,\overline{\mu}_2^{C_8}=1.0)$$

$$\overline{\mu}_3^C=(\overline{\mu}_3^{C_1}=0.1,\overline{\mu}_3^{C_2}=1.0,\overline{\mu}_3^{C_3}=1.0,\overline{\mu}_3^{C_4}=0.4,\ \overline{\mu}_3^{C_5}=0.8,\overline{\mu}_3^{C_6}=0.1,\overline{\mu}_3^{C_7}=0.25,\overline{\mu}_3^{C_8}=1.0)$$

$$\overline{\mu}_4^C=(\overline{\mu}_4^{C_1}=0.1,\overline{\mu}_4^{C_2}=1.0,\overline{\mu}_4^{C_3}=1.0,\overline{\mu}_4^{C_4}=0.4,\ \overline{\mu}_4^{C_5}=1.0,\overline{\mu}_4^{C_6}=0.1,\overline{\mu}_4^{C_7}=0.25,\overline{\mu}_4^{C_8}=0.8).$$

Thus the solution of the system (11) of fuzzy logical equations can be represented in the form of intervals

$$S(\mu^B)=\{\mu^{C_1}=0.1,\mu^{C_2}=0.8,\mu^{C_3}\in[0.8,1],\ \mu^{C_4}=0.4,\ \mu^{C_5}\in[0.8,1],\ \mu^{C_6}=0.1,\ \mu^{C_7}=0.25,\ \mu^{C_8}\in[0.8,1]\}$$
$$\cup\{\mu^{C_1}=0.1,\mu^{C_2}\in[0.8,1],\ \mu^{C_3}=0.8,\ \mu^{C_4}=0.4,\ \mu^{C_5}\in[0.8,1],\ \mu^{C_6}=0.1,\ \mu^{C_7}=0.25,\ \mu^{C_8}\in[0.8,1]\}$$
$$\cup\{\mu^{C_1}=0.1,\mu^{C_2}\in[0.8,1],\mu^{C_3}\in[0.8,1],\mu^{C_4}=0.4,\ \mu^{C_5}=0.8,\ \mu^{C_6}=0.1,\ \mu^{C_7}=0.25,\ \mu^{C_8}\in[0.8,1]\}$$
$$\cup\{\mu^{C_1}=0.1,\mu^{C_2}\in[0.8,1],\mu^{C_3}\in[0.8,1],\ \mu^{C_4}=0.4,\ \mu^{C_5}\in[0.8,1],\ \mu^{C_6}=0.1,\ \mu^{C_7}=0.25,\mu^{C_8}=0.8\}. \quad (12)$$

The resulting solution (12) allows analyst to make the following conclusions. The causes C_2, C_3, C_5 and C_8 are the causes of the observed elevator state, so that $\mu^{C_2}>\mu^{C_1}$, $\mu^{C_3}>\mu^{C_4}$, $\mu^{C_5}>\mu^{C_6}$, $\mu^{C_8}>\mu^{C_7}$.

The intervals of the values of the input variables for these causes can be defined with the help of the membership functions in Figure 4,a:

$x_1^*\in[47, 50]$ r/s for C_2;

$x_2^*\in[0.020, 0.043]$ kg/cm^2 for C_3;

$x_3^*\in[0.100, 0.135]$ mm for C_5;

$x_4^*\in[1.69, 2.00]$ cm^3/min for C_8.

Thus, the causes of the observed elevator state should be located and identified as the increase of the engine speed to 47-50 r/s, the decrease of the inlet pressure from 0.043 to 0.020 kg/cm^2, the decrease of the feed change gear clearance to 100-135 mk, and the increase of the oil leakage to 1.69-2.00 cm^3/min.

To test the fuzzy model we used the results of diagnosis for 210 hydro elevators with different kinds of faults. The tuning algorithm efficiency characteristics for the testing data are given in Table 8. The fault causes diagnosis obtained an accuracy rate of 95% after 10000 iterations of the genetic algorithm (20 min on Celeron 700).

Table 8: Tuning algorithm efficiency characteristics

Cause (diagnose)	Number of cases in the data sample	Probability of the correct diagnose	
		before tuning	after tuning
c_1	56	47 / 56 = 0.84	54 / 56 = 0.96
c_2	154	125 / 154 = 0.81	147 / 154 = 0.95
c_3	100	76 / 100 = 0.76	98 / 100 = 0.98
c_4	110	80 / 110 = 0.72	105 / 110 = 0.95
c_5	167	132 / 167 = 0.79	162 / 167 = 0.97
c_6	43	38 / 43 = 0.88	41 / 43 = 0.95
c_7	92	74 / 92 = 0.80	89 / 92 = 0.97
c_8	118	98 / 118 = 0.83	115 / 118 = 0.97

7. Conclusion

This paper proposes an approach for inverse problem solving based on the description of the interconnection between unobserved and observed parameters of an object with the help of fuzzy IF-THEN rules. The restoration and identification of the inputs through the observed outputs is accomplished by way of solving system of fuzzy logical equations, which correspond to IF-THEN rules, and tuning the fuzzy model on the available experimental data. The approach proposed can find application not only in engineering but also in medicine, economics, military affairs, and other domains, in which the necessity of interpreting the experimental observations arises.

References

[1] B. De Baets. Analytical solution methods for fuzzy relational equations. In: Fundamentals of Fuzzy Sets, The Handbooks of Fuzzy Sets Series, D. Dubois and H. Prade (eds.), Vol.1, Kluwer Academic Publ., 2000, pp. 291–340.

[2] A. Di Nola, S. Sessa, W. Pedrycz, E. Sanchez. Fuzzy Relation Equations and Their Applications to Knowledge Engineering. Dordrecht, Kluwer Acad. Press, 1989, 278 p.

[3] W. Chang. Nonlinear system identification and control using a real-coded genetic algorithm. *Applied Mathematical Modelling*, vol. 31 (3), pp. 541-550, 2007.

[4] O. Cordon, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena. Ten years of genetic fuzzy systems: current framework and new trends. *Fuzzy Sets and Systems*, vol. 141 (1), pp. 5-31, 2004.

[5] O. Cordon, F. Herrera, F. Hoffmann, L. Magdalena. Genetic Fuzzy Systems. Evolutionary tuning and learning of fuzzy knowledge bases. World Scientific, 2001, 492 p.

[6] K. Peeva, Y. Kyosev. Fuzzy Relational Calculus Theory, Applications and Software. World Scient. Publ., 2004, 292 p, http//mathworks.net

[7] A. Rotshtein. Design and Tuning of Fuzzy Rule-Based Systems for Medical Diagnosis. In N.-H. Teodorescu, A. Kandel, L. Gain (ed.): Fuzzy and Neuro-Fuzzy Systems in Medicine. CRC Press, 1998, pp. 243–289.

[8] A. Rotshtein, M. Posner, H. Rakytyanska. Cause and effect analysis by fuzzy relational equations and a genetic algorithm. *Reliability Engineering and System Safety*, vol. 91(9), pp. 1095-1101, 2006.

[9] A. Rotshtein, H. Rakytyanska. Genetic algorithm of fuzzy logic equations solving in expert systems of diagnosis. *Engineering of Intelligent Systems*, L. Monostori, J. Vancza, M. Ali, Ed. Lecture Notes in Computer Science, Springer Verlag, 2001, pp. 139-148.

[10] A. Rotshtein, H. Rakytyanska. Diagnosis Problem Solving using Fuzzy Relations. *IEEE Trans. on Fuzzy Systems*. (in press)

[11] A. Tihonov, V. Arsenin. Methods of uncorrect problems solving. Moscow, Nauka, 1974, 223 p.

[12] R.R. Yager, D.P. Filev. Essentials of Fuzzy Modelling and Control. John Wiley & Sons, 1994, 388 p.

[13] L. Zadeh, The Concept of Linguistic Variable and It’s Application to Approximate Decision Making. Moscow, Mir, 1976, 176 p.