УДК 538.975

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WAVE EQUATION FOR A QUANTUM PARTICLE CONFINED BY MÖBIUS STRIPE

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Abstract
We have considered an example of one side surface, namely the Möbius strip, in order to establish in explicit form the Hamiltonian of confined electrons. Applying well known general approach for pointing out purely geometric effects we have found the kinetic energy and geometric potential operators in terms of curvilinear coordinates.
Key words: one-side surface, geometric potential, Gauss curvature, main curvature, confinement, metric tensor, Beltrami-Laplace operator.

Анотація
Нами розглянуто приклад односторонньої поверхні, а саме смугу Мебіуса, з метою встановлення у явній формі оператора Гамільтон електронів, утримуваних поверхнею.Застосуванням добре відомого загального методу виділення чисто геометричних ефектів одержано оператори кінетичної енергії і геометричного потенціалу, виражені в термінах криволінійних координат.
Ключові слова: одностороння поверхня, геометричний потенціал, гаусівська кривизна, середня кривизна, утримування, метричний тензор, оператор Белтрамі-Лапласа.

This communication is dedicated to study some peculiarities of quantum mechanics describing a movement of particle along the Möbius strip. We consider the strip with one only twisting given by following two-parametric equations [1]:

$$x(u,v)=a\left(1+\frac{hv}{2a}Cos\frac{u}{2}\right)Cosu \qquad y(u,v)=a\left(1+\frac{hv}{2a}Cos\frac{u}{2}\right)Sinu$$

$$z(u,v)=\frac{hv}{2}Sin\frac{u}{2} \qquad (1)$$

where u and v correspondently run values $u\in[0\ ,\ 2\pi]$, $v=[-1\ ,\ 1]$. As far as another parameters, that is, a and h ,they note radius of the main circle and width of strip.

Möbius strip gained significant interest because of its intriguing and unexpected topologic properties, following from parameterization (1). The first of them reduces to the fact that the stripe has only one side. When one starts at some point of surface and paints a continuous line this line runs over the entire stripe without intersecting stripe’s edge. The second surprising property corresponds to edge of strip/ Going along the edge one can just to verify that the Möbius strip has only one edge/ These properties as well as another, no mentioned here, fascinating features motivate a lot of scientific groups from various branches of mathematics and physics to investigate one side surfaces beginning from its topology and ending its applications as templates for syntheses nanostructures in physics, medicine and biology.

It is quiet obvious that such attractive object like the Möbius surface couldn’t stay away from Quantum Mechanics area of interest. Some quantum mechanics properties such as a movement of a spinning particle [2] or energy levels and its manifestations in magnetic field [3] for a particle forced to move along the Möbius strip have been investigated. However, certain fundamental features, namely, role of twisting and formation of geometric potential still stayed away from more or less complete investigation. That is why we considered here a construction of quantum particle Hamiltonian beginning from basic principles. We assume that a wave equation corresponds to a particle moving in the uniform 3D space has its usual form and consist of kinetic energy and confining potential energy operators. Then describing the Möbius strip by Eqs (1) we carry out the thin-layer procedure proposed by authors [4] and come up to the Hamilton's operator given as follows:

$$\hat{H} = -\frac{\hbar^2}{2m} g^{-1/2} \partial_i g^{1/2} g^{ij} \partial_j - \frac{\hbar^2}{2m}(M^2 - K) \qquad (2)$$

The first term of the Eq (1) corresponds to the operator of 2D kinetic energy while the last one is determined as geometric potential and it describes all the effects related to geometric factors. The geometric potential is expressed over two invariants of surface, namely its mean curvature M and Gauss's curvature K. As far as kinetic energy it is modified due to passing to curvilinear coordinates by means of the contra variant g_{ij} and covariant g^{ij} metric tensor. Applying well-known methods [1] we have found metric tensor corresponds to the parameterization (1) which assumes the following form

$$\hat{g} = Diag\left\{ a^2\left(1 + \frac{hv}{2a} Cos\frac{u}{2}\right)^2 + \left(\frac{hv}{4}\right)^2 \;\; ; \;\; \left(\frac{h}{2}\right)^2 \right\} \qquad (3)$$

Taking in account this result we came up to explicit expression

$$\hat{H}_c = -\frac{\hbar^2}{2m}\left\{ \frac{4}{h^2}\frac{\partial^2}{\partial v^2} + \left[\left(\frac{hv}{4}\right)^2 + a^2\left(1+\frac{hv}{2a}Cos\frac{u}{2}\right)^2\right]^{-1} \left[\frac{\partial^2}{\partial u^2} + \frac{1}{a^2} \frac{\left(1+\frac{hv}{2a}Cos\frac{u}{2}\right)^2 \frac{hv}{2a} Sin\frac{u}{2}}{\left(\frac{hv}{4}\right)^2 + a^2\left(1+\frac{hv}{2a}Cos\frac{u}{2}\right)^2} \frac{\partial}{\partial u} \right] \right\} \qquad (4)$$

for kinetic energy operator.

The Gauss's and mean curvatures demand for its calculations knowledge of the first and the second fundamental forms of surface. Our result for Gauss's curvature is given by formula

$$K = -\frac{1}{4}\left[\left(\frac{hv}{4a}\right)^2 + \left(1 + \frac{hv}{2a}Cos\frac{u}{2}\right)^2\right]^{-2} \qquad (5)$$

and it coincides with well-known one (see, for example reference [1]). We have found independent result for the mean curvature too but due to the lack of place it will not write in this text.

Paying attention to expressions (4) , (5) and above mentioned the mean curvature we obtained complete form of Hamilton's operator which allowed to write the Schrödinger equation for a wave function of quantum particle localized in the Möbius strip.

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