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RESEARCH OF THE «DETERMINED CHAOS» PHENOMENON IN THE RL-DIODE ELECTRIC CIRCUIT OF SINUSOIDAL CURRENT

Annotation. *Reasons and conditions of chaotic oscillations in the nonlinear RL-diode electric circuit of sinusoidal current are analyzed. A diode is presented by the circuit of substitution that generally includes a nonlinear resistor and two nonlinear capacities – barrier and diffusive. To calculate the transition process a differential equation, which due to the non-linearity of a number of parameters is modified by the method of polygonal approximation, is proposed. The dependence of attractor form on a linear inductor and frequency is explored.*

When creating parametric resistive transducers for metrology (in particular, resolution ability) it is quite often needed to convert very small changes of the output resistance, for example, strain measurements.

This in turn leads to increased random noise on a useful signal that increases the random error of measurement. That is why the increased sensitivity of resistive transducers while ensuring a low level of random noise is an aim.

One way to accomplish this task is the use of RL-diode generators of chaotic oscillations [1-7]. However, in scientific literature the problem of mathematical modeling of physical processes in the RL-diode circuit and the causes of deterministic chaos in it are not considered in detail.

Analysis of the causes and conditions of chaotic oscillations in RL-diode circuits is the subject of the research described in this paper. For this purpose the diode substitution scheme for a diode is considered [8].

Substitution scheme for a diode

The substitution scheme for a diode in the mode of small signal (more generally) is presented on fig.1.

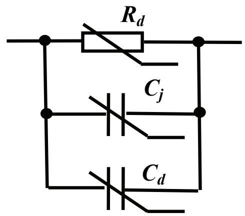

Fig. 1 – Substitution scheme for a diode in the mode of small signal

According to this scheme a diode is a parallel connection of nonlinear resistor R_d and two nonlinear capacities – barrier C_j and diffusive C_d.

A barrier capacity is determined by the formula:

$$C_j = \frac{C_{j0}}{\left(1 - \frac{U}{U_D}\right)^n}, \qquad (1)$$

where C_{j0} – a barrier capacity at a zero voltage of diode; U – voltage of diode; U_D – diffusive voltage of diode; n – technology ratio in the range $(\frac{1}{3}...\frac{2}{3})$.

A diffusive capacity is determined by:

$$C_d = \frac{\tau_B I_S}{mU_T} e^{\frac{U}{mU_T}}, \qquad (2)$$

where I_S – thermal current of diode; τ_B – life-time of non-core charge carrier; U –diode voltage; U_T – thermal voltage of diode; m – coefficient of emission.

It should be noted that in the mode of direct voltage at $U \geq U_D$ it is possible to ignore a barrier capacity. In the mode of reverse voltage it is possible to ignore a diffusive capacity.

Analysis of the mode of operations of RL-diode electric circuit of sinusoidal current

The processes in the RL-diode circuit (Fig. 2) with the input sinusoidal voltage are considered. In the most general case, the substitution scheme for this circuit is shown in Figure 3. This scheme contains three nonlinear elements: resistor R_d, C_j and diffusive capacity C_d. Thus, the resistance of the resistor depends on the diode current; the capacities depend on the diode voltage.

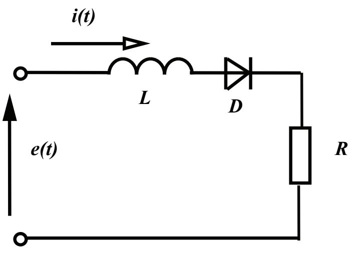

Fig. 2 – RL-diode electric circuit of sinusoidal current

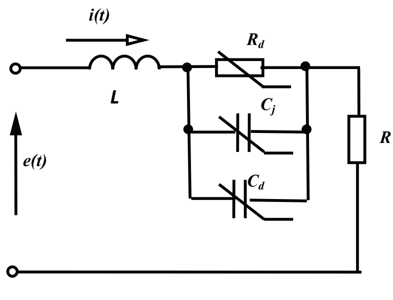

Fig. 3 – Substitution scheme for the RL-diode electric circuit of sinusoidal current

The system of equations for the scheme according to Kirchhoff's laws is shown in Figure 3. It is assumed that the current i_1 passes through the resistor R_d, the current i_2 through the capacity C_j and the current i_3 through the capacity C_d. As the voltage is the same it is denoted by U_C.

$$
\begin{aligned}
& i = i_1 + i_2 + i_3 \\
& L\frac{di}{dt} + Ri + R_d(i_1)i_1 = e \\
& R_d(i_1)i_1 = U_C \\
& i_2 = C_j(U_C)\frac{dU_C}{dt} \\
& i_3 = C_d(U_C)\frac{dU_C}{dt}
\end{aligned} \tag{3}
$$

It follows

$$
i_1 = \frac{U_C}{R_d(i_1)},
$$

$$
i = \frac{U_C}{R_d(i_1)} + C_j(U_C)\frac{dU_C}{dt} + C_d(U_C)\frac{dU_C}{dt},
$$

and, finally,

$$
\begin{aligned}
& L\frac{d\left(\frac{U_C}{R_d(i_1)} + C_j(U_C)\frac{dU_C}{dt} + C_d(U_C)\frac{dU_C}{dt}\right)}{dt} + \\
& + R\left(\frac{U_C}{R_d(i_1)} + C_j(U_C)\frac{dU_C}{dt} + C_d(U_C)\frac{dU_C}{dt}\right) + U_C = \\
& = \frac{L}{R_d(i_1)}\frac{dU_C}{dt} + LC_j(U_C)\frac{d^2U_C}{dt^2} + LC_d(U_C)\frac{d^2U_C}{dt^2} + \\
& + \frac{R}{R_d(i_1)}U_C + RC_j(U_C)\frac{dU_C}{dt} + RC_d(U_C)\frac{dU_C}{dt} + U_C = e.
\end{aligned}
$$

The final expression for the differential equations of the second order that subjects the behavior of the electrical circuit is proposed.

$$
\begin{aligned}
& \left(LC_j(U_C) + LC_d(U_C)\right)\frac{d^2U_C}{dt^2} + \\
& + \left(\frac{L}{R_d(i_1)} + RC_j(U_C) + RC_d(U_C)\right)\frac{dU_C}{dt} + \\
& + \left(\frac{R}{R_d(i_1)} + 1\right)U_C = e.
\end{aligned}
$$

This differential equation is performed by an operator in a general form. The operator equation of the 2-th Kirchhoff's law for this circuit is:

$$
I(s)\left(R + sL + Z_d(s)\right) = E(s),
$$

where $Z_d(s)$ – full operator resistance of the substitution scheme for a diode that is defined as

$$Z_d(s)=\frac{R_d(i_1)\frac{1}{sC_j(U_C)}\frac{1}{sC_d(U_C)}}{R_d(i_1)\frac{1}{sC_j(U_C)}+R_d(i_1)\frac{1}{sC_d(U_C)}+\frac{1}{sC_j(U_C)}\frac{1}{sC_d(U_C)}}.$$

Then, operator image of the current of the circuit is

$$I(s)=\frac{E(s)}{(R+sL+Z_d(s))}.$$

Due to the substantial nonlinearity $Z_d(s)$ it is impossible to get the original circuit current *i(t)* in a general view.

Therefore, let's solve a problem in numerical form for one of the types of diodes and the specific values *e(t)*, *R*, *L*.

Let: $e(t)=3\sin(\omega t)$ [B], $f=10$ [kGc], $L=50$ [mGn], $R=2$ [kOm] and the type of diode – *1N457*.

Note, the parameters of the diode are taken from a database of the MicroCap software. The parameters required to define the formulas (1, 2) C_j and C_d partially are taken from a database of the MicroCap software and partially are taken from in the common form for a wide class of the diode models. Taking into account that the diode parameters are significantly different for forward and reverse voltage, the circuit has a modified mode analysis by polygonal approximation using transient characteristics and Duhamel integral. All calculations are performed in MathCAD.

Due to the labour-consuming mode the calculations are performed for four oscillation periods. Based on the results of the calculations the dependence *U2=f(U1)* was built, where *U2=IR*. The graph of this function is shown in Figure 4. The graph demonstrates that at the above-mentioned parameters the circuit mode has chaotic oscillations. Certainly, the form of these oscillations of the actual diode may differ significantly from the calculation form because, firstly, the parameters required to define the formulas (1, 2) C_j and C_d are taken from in the common form for a wide class of the diode models and, secondly, any numerical calculations in nonlinear circuits always give tolerance.

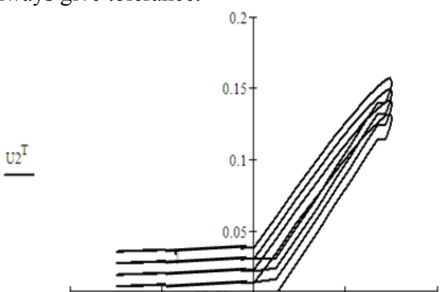

Fig. 4 – Graph of dependence *U2=f(U1)*: at $f=10$ [kGc], $L=50$ [mGn], $R=2$ [kOm]

The calculations of the circuit mode are also performed at other values of inductance and frequency in order to evaluate the influence of these parameters on chaotic oscillations.

Conclusion

The current in the circuit doesn't die out to zero and a new period begins with a non-zero initial conditions at the end of the period in the RL-diode circuits of sinusoidal current if the period of oscillations is commensurate with the time constant of the transition process. Thus, due to essential non-linearity of parameters of the substitution scheme for a diode there is practically the ongoing transitional process in the circuit that takes the form of chaotic oscillations. While increasing the inductance of the circuit the scale of the attractor increases. In the case of increasing the frequency of the input voltage there is a tendency of some displacement of the attractor towards negative voltages.

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