СЕКЦІЯ 2.
Методи та засоби захисту інформації

Numeral Systems with Irrational Bases for Mission-Critical Applications. The Basic Concepts and Scientific Results

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Системи числення з ірраціональними основами для критично важливих застосувань. Головні концепції та наукові результати

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***Abstract*— This speech is of an overview nature. Its main goal is to substantiate the basic concepts and scientific results of a new direction in the theory of coding-the numeral system with irrational bases, and their application in mission-critical systems. This scientific direction began to develop in the Taganrog Radiotechnical Institute after the defense of the doctoral dissertation of the author of this speech (1972), and then successfully continued to develop in the Vinnytsya Polytechnic Institute at the Department of Computer Technology. Nowadays this scientific direction is developing in Canada (computer firm FibTech (Fibonacci Technology)) and at Sumy University.**

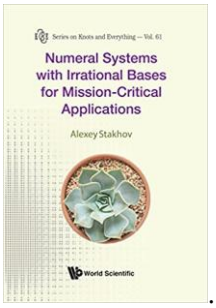
***Анотація*—Ця доповідь має оглядовий характер. Її основна мета – обгрунтувати головні концепції та наукові результати нового напрямку в теорії кодування – системи числення з ірраціональними основами, та їх застосування в критично-важливих системах. Цей науковий напрямок почав розвиаатися в Таганрогському радіотехнічному інституті після захисту докторської дисертації автора цієї доповіді (1972), потім успішно продовжив розвиватися в Вінницькому політехнічному інституті на кафедрі обчислювальної техніки. Зараз цей науковий напрямок розвивається в Канаді (компьютерна фірма FibTech (Fibonacci Technology) та в Сумському університеті.**

Keywords*—*mission-critical systems; Bergman's system; Fibonacci p-codes; codes of the golden p-proportions, ternary mirror-symmetrical arithmetc.

Ключові слова*—*критично-важливі системи; система Бергмана; р-коди Фібоначчі; коди золотої р-пропорції, трійкова зеркально-симетрична арифметика

INTRODUCTION

In 2017 the International Publishing House "World Scientific" has published new book of the author "Numeral Systems with Irrational Bases for Mission-Critical Applications".



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Advertising information on the book is posted at the site of "World Scientific"
(http://www.worldscientific.com/worldscibooks/10.1142/10671) and the site of Amazon.com (https://www.amazon.com/Numeral-Irrational-Mission-Critical-Applications-Everything/dp/981322861X)

Abstract of the book

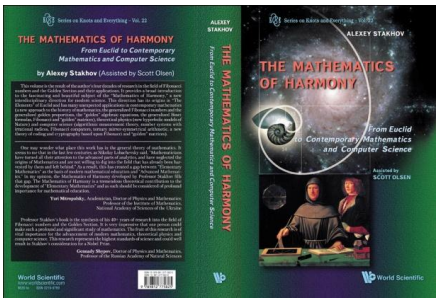
This volume is the result of the author's many-years of research in this field. These results were presented in the author's two books, Introduction to the Algorithmic Measurement Theory(Moscow, Soviet Radio, 1977), and Codes of the Golden Proportion (Moscow, Radio and Communications, 1984), which had not been translated into English and are therefore not known to English-speaking audience. This volume

sets forth new informational and arithmetical fundamentals of computer and measurement systems based on Fibonacci p-codes and codes of the golden p-proportions, and also on Bergman's system and "golden" ternary mirror-symmetrical arithmetic. The book presents some new historical hypotheses concerning the origin of the Egyptian calendar and the Babylonian numeral system with base 60 (dodecahedral hypothesis), as well as about the origin of the Mayan's calendar and their numeral system with base 20 (icosahedral hypothesis). The book is intended for the college and university level. The book will also be of interest to all researchers, who use the golden ratio and Fibonacci numbers in their subject areas, and to all readers who are interested to the history of mathematics. Readership: Researchers in mathematics and computer science.

The main goal of this speech is to substantiate the basic concepts and scientific results of a new direction in the coding theory - the numeral system with irrational bases, and their application in mission-critical systems.

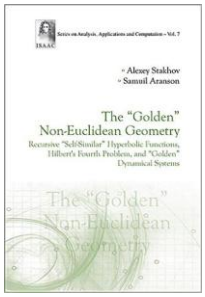
In addition to this book, the Publishing House "World Scientific" published two fundamental author's books:

1. Alexey Stakhov. Assisted by Scott Olsen. "The Mathematics of Harmony. From Euclid to Contemporary Mathematics and Computer Science", World Scientific, 2009
http://www.worldscientific.com/worldscibooks/10.1142/6635



2. Alexey Stakhov, Samuil Aranson. Assisted by Scott Olsen. The "Golden" Non-Euclidean Geometry: Hilbert's Fourth Problem, "Golden" Dynamical Systems, and the Fine-Structure Constant, World Scientific, 2016.

http://www.worldscientific.com/worldscibooks/10.1142/9603



All these three books are author's contribution to the development not only of world science, but also of Ukrainian science, in particular, of Ukrainian mathematics, computer science and digital metrology.

The author dedicates the last book "Numeral Systems with Irrational Bases for Mission-Critical Applications" to the 45th anniversary of the Computer Technology Department, Vinnytsia National Technical University.

BASIC CONCEPTS AND THE MAIN SCIENTIFIC RESULTS:

1. Mission-critical applications. At the present time the computer science and digital metrology are passing to new stage of their development, to the stage of designing computing and measuring systems for **mission-critical applications.** In the Wikipedia article "Mission critical" we read:

*"**Mission critical** refers to any factor of a system (components, equipment, Personnel, process, procedure, software, etc.) that is essential to business operation or to an organization. Failure or disruption of **mission critical** factors will result in serious impact on business operations or upon an organization, and even can cause social turmoil and catastrophes. Therefore, it is extremely critical to the organization's "mission" (to avoid Mission Critical Failures).*

***Mission critical system** is a system whose failure may result in the failure of some goal-directed activity. Mission essential equipment and mission critical application are also known as mission-critical system. Examples of mission critical systems are: an online banking system, railway / aircraft operating and control systems, electric power systems, and many other computer systems that will adversely affect business and society seriously if downed. A good example of a mission critical system is a navigational system for a spacecraft."*

This puts forward new requirements for ensuring informational reliability of such systems. The most important requirement is to prevent the occurrence of "false signals" at the output of the mission-critical systems that can lead to technological disasters.

2. "**Philosophy" of error detection for the error-correcting codes (ECC)**. Modern methods of providing informational reliability of mission-critical systems (in particular, the use of error-correcting codes) do not always provide the required informational reliability of the mission-critical systems. In particular, the theory of ECC mainly is focused on the detection and correction of the errors of low multiplicity (for example, single-bit and double-bit errors) as the most probable. *With regard to the errors of high multiplicity, the theory of ECC simply ignores them because of their low probability; this follows from the model of "symmetrical channel".* Such "philosophy" of error detection is absolutely unacceptable for the case of the mission-critical systems, because these undetectable errors can be the source of "false signals" at the output of mission-

critical systems what can lead to enormous social and technological disasters.

3. Paradox of Hamming code. The main paradox of Hamming code and its analogs (for example, Hsiao code) consists of the fact that the Hamming and Hsiao codes perceive many-bit errors of the odd multiplicity (3,5,7,9,…) as single-bit errors, and for these cases they begin "false correction" by adding new errors to the erroneous code word. That is, for this case the Hamming and Hsiao codes are turned out into **anti-ECC**, because they are "ruining" the Hamming and Hsiao code words. This "paradoxical" property of the Hamming and Hsiao codes is well known to experts in the field of ECC, but many consumers do not always know about this. For such cases, the main arguments for customers consist in the fact that the errors of large multiplicity are unlikely, but *such arguments are unacceptable for mission-critical applications*.

4. Row hammer effect is a new phenomenon in the field of electronic memory. The main reason of this phenomenon is microminiaturization of electronic memory, which leads to mutual electrical interaction between nearby memory rows. This interaction is altering the contents of nearby memory rows that were not addressed in the original memory access. No effective methods of fighting against *row hammer effect* have been proposed until now. Possibly, the only reasonable proposal is to introduce restrictions on microminiaturization of electronic memory. But then the question arises how we have to design nano-electronic memory?

5. "Trojan horse" of the binary system. The prominent American scientist, physicist and mathematician John von Neumann (1903–1957), together with his colleagues from the Prinstone Institute Goldstein and Berks after careful analysis of the strengths and weaknesses of the first electronic computer ENIAC gave *strong preference to the binary system as a universal way of coding of data in electronic computers*. However, this proposal contains in itself a great danger for the case of mission-critical systems. The classical binary code has zero code redundancy what excludes a possibility detecting any errors in computer structures. This danger was called "Trojan horse" of binary system by the Russian academician Yaroslav Khetagurov. Because of the "Trojan Horse" phenomenon, humanity becomes a hostage to the binary system for the case of mission-critical applications. *From here, it follows the conclusion that the binary system is unacceptable for designing computational and measuring systems for mission-critical applications.*

5. Bergman's system, introduced in 1957 by the American 12-year-old wunderkind George Bergman, is an unprecedented case in the history of mathematics. The mathematical discovery of the young American mathematician returns mathematics to the Babylonian positional numeral system, that is, to the initial period in the development of mathematics, when the numeral systems and rules of performing basic arithmetic operations stood at the center of mathematics. But the most important is the fact that the famous irrational number $\Phi = \frac{1+\sqrt{5}}{2}$ (the *golden ratio*) is the base of *Bergman's system* what puts forward the irrational numbers on the first position among the numbers. *It can be argued that the Bergman's system is the greatest modern mathematical discovery in the field of numeral systems, which changes our ideas about numeral systems and alters both the number theory and computer science.*

6.The "golden" number theory and new properties of natural numbers is the first important consequence, following from Bergman's system. For many mathematicians in the field of number theory, it is a great surprise that new properties of natural numbers (*Z*-property, *D*-property, *F*-code, *L*-code) were discovered in the 21st century, that is, 2.5 millennia after the writing of Euclid's *Elements*, in which systematic studying the properties of natural numbers started. *Bergman's system* is the source for the *"golden" number theory* what once again emphasizes a fundamental nature of the mathematical discovery of George Bergman.

7. Ternary mirror-symmetrical numeral system and new ternary mirror-symmetrical arithmetic are the main applied scientific results, following from *Bergman's system*. These results alter our ideas about ternary numeral system. The *property of mirror symmetry* is the main checking property, which allows detecting errors in all arithmetical operations.

8. Fibonacci *p*-codes and Fibonacci arithmetic based on the basic micro-operations. The new computer arithmetic consists in the sequential execution of the so-called "basic micro-operations." The errors are detected by built-in error-detection device simultaneously with the execution of the micro-operations in the moment of errors occurrence what ensures the high information reliability of the arithmetic device for mission-critical applications.

9. Codes of the golden *p*-proportions, "golden" resistive divisors and self-correcting ADC and DAC. The codes of the golden *p*-proportions with the base Φ_p (the positive root of the algebraic equation $x^{p+1} - x^p - 1 = 0, p = 0.1, 2, 3, \ldots$) are a wide generalization of the *binary system* (p=0) and *Bergman's system* (p=1). The "golden" resistive divisors, based on the golden *p*-proportions Φ_p, have unique electrical properties, which allow to design self-correcting analog-to-digital and digital-to-analog converters. Metrological parameters of such ADCs and DACs remain unchanged in the process of temperature changing and elements aging, what is important for mission-critical applications.

10.The final concept. The above theory of numeral systems with irrational bases are a new direction in the field of coding theory, intended for increasing informational reliability and noise-immunity of specialized computing and measuring systems. This direction does not set itself the task of replacing the classical binary system in those cases where the use of the binary system does not threaten an appearance of

technological disasters and where informational reliability and noise immunity can be ensured by traditional methods. The main task of this direction is preventing or significantly reducing the probability of "false signals" at the output of information systems that can lead to social or technological disasters. This scientific direction is at the initial stage of its development and can lead to new technical solutions in the field of computer science and digital metrology.

11. **The main conclusion** of author's book **"Numeral Systems with Irrational Bases for Mission-Critical Applications**" is the fact that the mission-critical applications are that major area of computer science and digital metrology, where numeral systems with irrational bases (Fibonacci codes and codes of the golden proportion) get their natural applications and can realize all their basic advantages.

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