PATTERN recognition and INFORMATION processing

PRIP '99

volume 2

18 – 20 May 1999 Minsk, Republic of Belarus

Proceedings of Fifth International Conference

IAPR

БААРИ

Belarussian & Polish Centres IEE in Development

Minsk – Szczecin
Belarus – Poland

Fifth International Conference on Pattern Recognition and Information Processing

Sponsors

Ministry of Education of Belarus

Basic Research Foundation of Belarus

International Association for Pattern Recognition

Organized by

BELARUSIAN STATE UNIVERSITY
OF INFORMATICS AND RADIOELECTRONICS

DEPARTMENT OF COMPUTER SYSTEMS
AND LABORATORY "PATTERN RECOGNITION AND IMAGE ANALYSIS"

TECHNICAL UNIVERSITY OF SZCZECIN

INSTITUTE OF COMPUTER SCIENCE & INFORMATION SYSTEMS (POLAND)

BELARUSIAN ASSOCIATION FOR IMAGE ANALYSIS AND RECOGNITION

In cooperation with

Belarusian State University,
Faculty of Applyed Mathematics and Computer Sciences

Belarusian Academy of Sciences,
Institute of Engineering Cybernetics

Belarusian State Economic University,
Laboratory of New Information Technologies in Economic

Belarusian & Polish IEE Centers in Development

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В сборнике (Том 2) собраны материалы докладов 5-й международной конференции по распознаванию образов и обработке информации PRIP-99, проходившей 18 - 20 мая 1999 в г. Минске, Республика Беларусь. Тираж 70 экз. Объем 21 усл. печ. л.

CONTENTS

CHAPTER 1
PATTERN RECOGNITION

Chapter 4
Signal Processing

Chapter 5
Systems and Parallel Architectures for Signal and Image Processing

FAST ORTHOGONAL MULTIVARIATE TRANSFORMS

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Abstract

Modifications of Nussbaumer-Quandalle Fourier Transform fast algorithm for code lendths $2^n \times 2^n$ are considered. The possibility of calculating of all coefficients for two steps offered. At first step the even first index coefficients has calculated in analogy Nussbaumer-Quandalle algorithm; and at the second step the odd first index coefficients has calculated. The presented calculating order assured a parallel processing and structure algorithm improvement without essential or with conversed computional complexity changes.

Keywords: Fourier transform, concurency, parallel processing

1. Introduction

Although the discrete orthogonal transforms (including discrete Fourier transform) knowns long ago, efficient aplianse in digital signal processing they received after theirs fast calculating algorithms has been discovered.The impulse to unitary transforms rast algorithms designing is known to be Cooley, Tukey work [1], which was published in 1965. In those described discreet Fourier transform (DFT) calculating method known as Cooley-Tukey fast Fourier transform.

In further, many of fast Fourier transform algorithms have been designed [2]. Theirs common particularity is that n-pointed DFT calculating is converted to DFT calculating with less than n code lenght. The fast Fourier transform (FFT) algorithm made in algorithms basis not only in Fourier transform, but also in other ortohgonal transforms .

The multivariate Fourier transform calculating problem appear, on the first hand, in digital signal processing, which at substance is multivariate, but on the other hand, as monovariate Fourier transform calculating means. The simple multivariate Fqurier transform calculating algorithm is based on sequential monovariate transform performanse at each directs. Therefore, all fast monovariate transforms algorithms may be converted on multivariate case. Proper multivariate algorithms has arose later [2]. Most broad applianse they discovered in the latter time in junction of booster developing digital processing and transmitting system. Although of that multivariate FFT has allowed to dramaticaly decrease of operations number, speed of a computer increasing are still remain essential for solution of image processing and transmitting in real time. One of the way to afford this aim is parallel processing appliance. The new electronic technology has allowed to realize of multiprocessing system with extend of throughput coefficient to be equal N by N processors. However, concurrency at multivariate FFT using is required a new approach, being designed with account to concurrency degree maximalization, to FFT algorithm design.

The aim of this paper is to examine the Nussbaumer-Quandalle algorithm modifications of multivariate fast Fourier transform appliance for the parallel system.

2. The principal concurrency conceptions

Concurrency at the basis is suppoused an idea to make use of any processors for one task resolving. Availability of any processors is not meant that task may be parallel performed. In task estimatial of concurrency of-fit may be following tests:

- <u>Concurrency power</u>. Number of operations, which may be parallel executed.
- <u>Mean concurrency power</u>. Full operations number and stage number ratio.

$$s = \frac{B}{c}$$

Certainly, to apply this tests in algorithms, it is neccessary that «B» were equal, or approximately equal.

3. Nussbaumer-Quandalle algorithm with account concurrency requires synthesis

Nussbaumer-Quandalle algorithm has based on that combinations number is limited for all exp(-iφ) multipliers. Suppose that code lendth is $n \times n$, $n = 2^m$.

Odd coefficients Fourier transform computing at the Nussbaumer Quandalle algorithm based on following theorem:

Theorem 1. *Let k' – fixed integer number which mutualy common with n. Let Q(x) – circular polynomial root of w^k element. Then $V_{k'k''}$ components calculating may be realised by following thtee steps:*

(1) calculate polynomial transform

$$V'_k(x)=\sum_{i''=0}^{n-1} v_{i''}(x)\, x^{i''k} \pmod{Q(x)}, \qquad k=0, \ldots, n-1;$$

where $v_{i''}(x)=\sum_{i'=0}^{n-1} v_{i',i''} x^{i'}$, $i''=0, \ldots, n-1$; $Q(x)=x^{n/2}+1$;

(2) calculate n trimed Fourier transforms

$$V''_{k',k}=\sum_{i'=1}^{n-1} w^{i'k'}\, V'_{i',k}, \qquad k'\text{: GCD (k}$$

(3) make of transposition

$$V_{k',k''}=V''_{k',((k''/k'))}, \qquad k'\text{: GCD}(k',n)=1,\ k''=0, \ldots, n-1.$$

An arrangement Fourier transform calculation according to Nussbaumer-Quandalle algorithm at $n=2^m$ is followed:

1) $V_{k',k''}$ components with odd k' and $k''=0, \ldots, n-1$ is calculated according to theorem 1.

2) $V_{k',k''}$ components with even k' and odd k'' is calculated according to theorem 1 by exchange k' and k'' by place.

3) The rest of $V_{k',k''}$ components calculated as code length 2^{m-1} FFT.

Stage number have to calculating Nussbaumer-Quandalle algorithm will

$$c=2+2\log_2 n+a_{FT}+c_{FT},\ n>4,$$

(2) where a_{FT} – stage number of trimed Fourier transforms; c_{FT} –stage number of bivariate code length n/2 Fourier transform.

Total number of arithmetic operations

$$B=(2+2\log_2 n)\frac{n^2}{2}+\frac{n^2}{4}nA_{FT}+C_{FT},\ n>4$$

Then mean concurrency power will be

$$s'=\frac{(2+2\log_2 n)\cdot\frac{n^2}{2}+\frac{n^2}{4}+n\cdot A_{FT}+C_{FT}}{2+2\log_2 n+a_{FT}+c_{FT}},$$

where A_{FT} – arithmetic operations number for trimed Fourier transforms of length n; C_{FT} – arithmetic operations number for bivariate code length n/2 Fourier transform.

Deficiency of FFT- Nussbaumer-Quandalle algorithm for code length 2^m calculating is that mean concurrency power is low. More effective algorithm is even output components calculating, which is based on polynomial transform symbols transpositions for k and k' .

Let to introduce temperary variables:

$$a_{i,j}=v_{i,j}-v_{i+n/2,j},$$

$a'_{i,j} = v_{i,j} + v_{i+n/2,j}$, i=0, ... , n/2-1, j=0, ... , n-1,

where $v_{i,j}$ — correspond input components.The further arrangement output even rows components explained the following theorem:

Theorem 2. *Let k' – fixed even number. Let V'_k – n polynomial code length $(2^m)^2$ transforms (see theorem 1):*

$$V'_k(x) = \sum_{i''=0}^{n-1} a_{i''}(x) x^{i''k} \pmod{Q(x)},$$

k=0, ... , n-1,

which $v_{i'}(x) = \sum_{i'=0}^{n-1} v_{i',j} \cdot x^{i'}$ $i''=0, \ldots, n-1; Q(x)=x^{n/2}+1$.

Then $V_{k',k''}$ component calculating may be realised by following thtee steps:

(1) replace $a_{i',i''}$ by $a'_{i',i''}$;

(2) Make transposition for $a_{i',i''}$, multiplied by $(-1)^{integer(2(-i'+a)/n)}$, ($i' \neq 0, k'=0, \ldots, n-2$), to multiplier x^b where

$$b=(-i'+a+k'i')(\bmod n/2)$$

(3) make V_l components of $n^2/2$ trimed code length nFourier transforms calculation:

$$V''_{k',k} = \sum_{i'=1}^{n-2} w^{i'} V'_{i'k}, \quad k': odd, \qquad k=0, \ldots, n-2.$$

Optimisation by additions in the those polynomial transforms may be organized by Cooley-Tukey algorithm by base-radix 2.

Applianse of this theotem in the case where $n=2^m$ is allowing to calculate of all Fourier transform coefficients for two steps:

1) calculate odd rows coefficients under teorem 1:

2) calculate even rows coefficients under teorem 2.

The offered algorithm assimptotically not to relinquish Nussbaumer-Quandalle algorithm by computional complexity. But, the main advantage offered way is higher mean concurrency power.

Total number of arithmetic operations and total number of stages is such:

$$B=n^2(1+\log_2 n)+nA_{FT}+A'_{FT}, \qquad c=3+\log_2 n + a_{FT},$$

where A_{FT} – arithmetic operations number of trimed code length n Fourier transform number code length n: a_{FT} – stage number for trimed code length n Fourier transform calculation; A'_{FT} – for $n^2/2$ trimed Fourier transform componentls V_l arithmetic operation number The mean concurrency power is equal:

$$s = \frac{n^2 \cdot (1+\log_2 n) + n \cdot A_{FT} + A'_{FT}}{3 + \log_2 n + a_{FT}}$$

In such a manner, the offered way had assured mean concurrency power increasing at n>4.

4. Nussbaumer Quandalle algorithm applianse in other transforms

In common case Nussbaumer Quandalle algorithm modifications is possible to use in transforms, which contains isomorph polynomial transform calculating:

$$V_{k',k''} = \sum_{i'=0}^{n-1} x^{i'k'} \sum_{i''=0}^{n-1} x^{i''k''} v_i(x) (\mathrm{mod}\ p(x)) \quad , \quad k',k''=0, \ldots, n-1,$$

where p(x) – m power circular polynomial, $v_{i',j''}(x)$, i',i''=0, . . . n-1 – polynomial above F field of not more than m power. For instanse, in bivariate discreet Laplas transform

$$V_{k'k''} = \sum_{i'=0}^{n-1} w^{i'k'+c_1} \sum_{i''=0}^{n-1} w^{i''k''+c_2} v_{i',j''}, \ k',k''=0, \ldots, n-1, \ c_1, c_2 \text{— integer},$$

where Fourier transform is particular case, construction has differed by transposition polynomial transform coefficients. However, Nussbaumer-Quandalle algorithm applianse immediate to transforms with circular polynomials power not large than 4 will not may advantage in comparison with Cooley-Tukey algorithm. In certain cases possibly to use relationship with DFT spectr. For instance, relationship between energy spectr Walsh transform ordered under Adamar, and DFT [3]:

$$P_h(0)=V_0^2; \qquad P_h(r)= \sum_{m=2^{r-1}}^{2^r-1} |V_{<m>}|^2, \qquad r=1, \ldots, \log_2 n$$

where $P_h(r)$ – spectr points 2^r— periodical Walsh transform sequences, <m> – got through binary converse m number [3].

5. Conclusions

1. Made analysis of main multivariate FFT-algorithms show that Nussbaumer-Quandalle algorithm on most gauge correspond to concurrency conceptions.

2. Offered new manner to DFT-calculating which is based on Nussbaumer—Quandalle FFT-algorithm and ensured of mean concurrency power increasing approximately in two ones.

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