

Szkoła Główna  
Politechniczna z siedzibą  
w Nowym Sączu



**GENERAL AND COMPLEX PROBLEMS  
OF TECHNICAL SCIENCES: EXPERIENCE  
OF EU COUNTRIES AND IMPLEMENTATION  
IN THE PRACTICE OF UKRAINE**

**Collective monograph**

Nowy Sącz, Poland  
2019

*Recommended for publication  
by the Academic Council of Polytechnic University  
with headquarters in Nowy Sącz*

**Responsible for release:** prof. nadzw. dr inż. **Sameer Ayyoub**, rector,  
Polytechnic University with headquarters in Nowy Sącz

**General and complex problems of technical sciences: experience of EU countries and implementation in the practice of Ukraine** : Collective monograph. Riga : Izdevniecība "Baltija Publishing", 2019. 384 p.

# CONTENTS

Каталітичні властивості й втрати платиноїдів у процесі окиснення аміаку <b>Авіна С. І.</b> .....	1
Electronics and high-voltage pulse technique for preservation of foodstuffs, water disinfection and purification, and conversion of gas emissions of various productions <b>Boyko M. I., Makogon A. V.</b> .....	20
The method of translation the additive and multiplicative error of a torque measuring means in the instrumental component of measurement uncertainty <b>Vasilevskiy O. M., Didych V. M.</b> .....	40
Problems of computer modeling of nonlinear filtration processes of underground hydrodynamics <b>Hladka O. M., Karpovych I. M.</b> .....	58
Шкіряні натуральні матеріали поліфункціонального призначення <b>Данилкович А. Г., Ліщук В. І.</b> .....	77
Integrated approach to the modernization of preparation technology of bentonite solutions for trenchless communications <b>Dmytriieva N. V., Agafonova I. P.</b> .....	102
Formula development of the nutritional paste of a high biological value <b>Kolesnichenko S. L., Dzyuba N. A.</b> .....	120
Перспективні тенденції формування сучасних міських утворень на основі низьковуглецевої стратегії розвитку <b>Крижановська Н. Я., Смірнова О. В.</b> .....	135
Use of pea concentrate in mixed fodder's production <b>Макарунска А. В., Турпурова Т. М.</b> .....	152
Анализ и выбор концептуальных моделей процессов развития производственных дефектов и отображения информации о техническом состоянии РЭА <b>Невлюдова В. В., Моспан Д. В.</b> .....	170
Математична модель віброагрегата з ланцюгово-розгалуженим способом з'єднання твердих тіл <b>Нісонський В. П., Незамай Б. С.</b> .....	189

# THE METHOD OF TRANSLATION THE ADDITIVE AND MULTIPLICATIVE ERROR OF A TORQUE MEASURING MEANS IN THE INSTRUMENTAL COMPONENT OF MEASUREMENT UNCERTAINTY

Vasilevskyi O. M., Didych V. M.

## INTRODUCTION

This research was conducted into the basic metrological characteristics displayed when developing new designs of measurement means for the evaluation of physical quantities, which is seen to be a crucial issue in science, the solution helping to ensure the uniformity of measurements and the identification of core and additional errors (uncertainties) in the measurement of a given type of measurement means.

As has been demonstrated in the relevant literature, the basic characteristics of measurement means include conversion equations<sup>1</sup>, static characteristics<sup>2</sup>, sensitivity<sup>3</sup>, additive and multiplicative errors<sup>4</sup>, the measurement range<sup>5</sup>, and the basic and additional errors in the Measurement Means<sup>6</sup> (MM). These characteristics are sufficient to establish standards for the accuracy of the MM. This requires that the mathematical tools used in research study of the MM in static mode enable us to easily obtain functions of the metrological characteristics enumerated above. The difficulty is that the equations describing measurement conversion describe different physical processes for different MMs, which relate to a variety of disciplines and are functions of numerous variables. It would be valuable to produce a generalised method of research, independent of the physical phenomena underlying the design of MMs. This research aims to achieve this, by

---

<sup>1</sup> ISO/IEC 17025:2005, General requirements for the competence of testing and calibration laboratories (Geneva, Switzerland, ISO, 2005), 28 p.

<sup>2</sup> Röske D. et al. Metrological characterization of a 1 N m torque standard machine at PTB, *Metrologia* 51, 87 (2014).

<sup>3</sup> Vasilevskyi O.M. Metrological characteristics of the torque measurement of electric motors, *International Journal of Metrology and Quality Engineering*, 8, 7 (2017).

<sup>4</sup> Vasilevskyi O.M., Kucheruk V.Y., Bogachuk V.V., Gromaszek K., Wójcik W., Smailova S., Askarova N. The method of translation additive and multiplicative error in the instrumental component of the measurement uncertainty, *Proc. SPIE 10031*, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments, 2016, 1003127.

<sup>5</sup> Schlegel C., Kahmann H., Kumme R. MN·m torque calibration for nacelle test benches using transfer standards, *ACTA IMEKO* 5, 12 (2016).

<sup>6</sup> ISO 5725-1:1994/Cor 1:1998, Accuracy (trueness and precision) of measurement methods and results – Part 1: General principles and definitions (Geneva, Switzerland, ISO, 1994/1998).

appraising a series of derived conversion equations using the Taylor's Series method to study static metrological characteristics by mathematical means.

As a result of the above, the aim of this paper is to obtain mathematical models of basic static metrological characteristics, enabling us to explore and analyse the characteristics of their change and synthesise MM for torque using predetermined standardised metrological characteristics and to develop methods of recalculating additive and multiplicative components in MM errors in an instrumental component of the measurement uncertainty.

### 1. Model equations for torque measurement means

A structural diagram of the MM was developed which is shown in Figure 1 to measure the torque of EMs. The components of the means for measuring the torque of motors include a drive EM (DM) that is connected to the test motor (MT) via the coupling conjugation (CC), a measuring transducer for angular speeds in voltage (v/U), a torque converter into an angular position (a measuring lever) (MP/f), a rotation angle converter (f/Q), an energy converter in voltage (Q/U), scale converters (SC1, SC2), a microcontroller with built-in analogue-to-digital converter (ADC) (MC), electronic voltage variators (EVV1, EVV2) and a personal computer<sup>7</sup>.

The essence of the method of measuring the torque of the test motor is to use a drive motor to generate a slow rotation of the shaft of the test motor with an equivalent frequency corresponding to 0.2–0.3 of the nominal rotational speed of the test motor<sup>8</sup>. Under these conditions, the test motor is provided with nominal voltage and frequency. This design allows the dependence of the minimum and the maximum torque of the test motor to be obtained, depending on the angular position of the rotor<sup>9</sup>.

Using the analytical expression of the conversion function of a MM of the torque of electric motors in static mode and the standard equation for conversion by an analogue-to-digital converter (ADC), we obtain this equation for the conversion of a torque MM, which we may use to obtain basic metrological characteristics

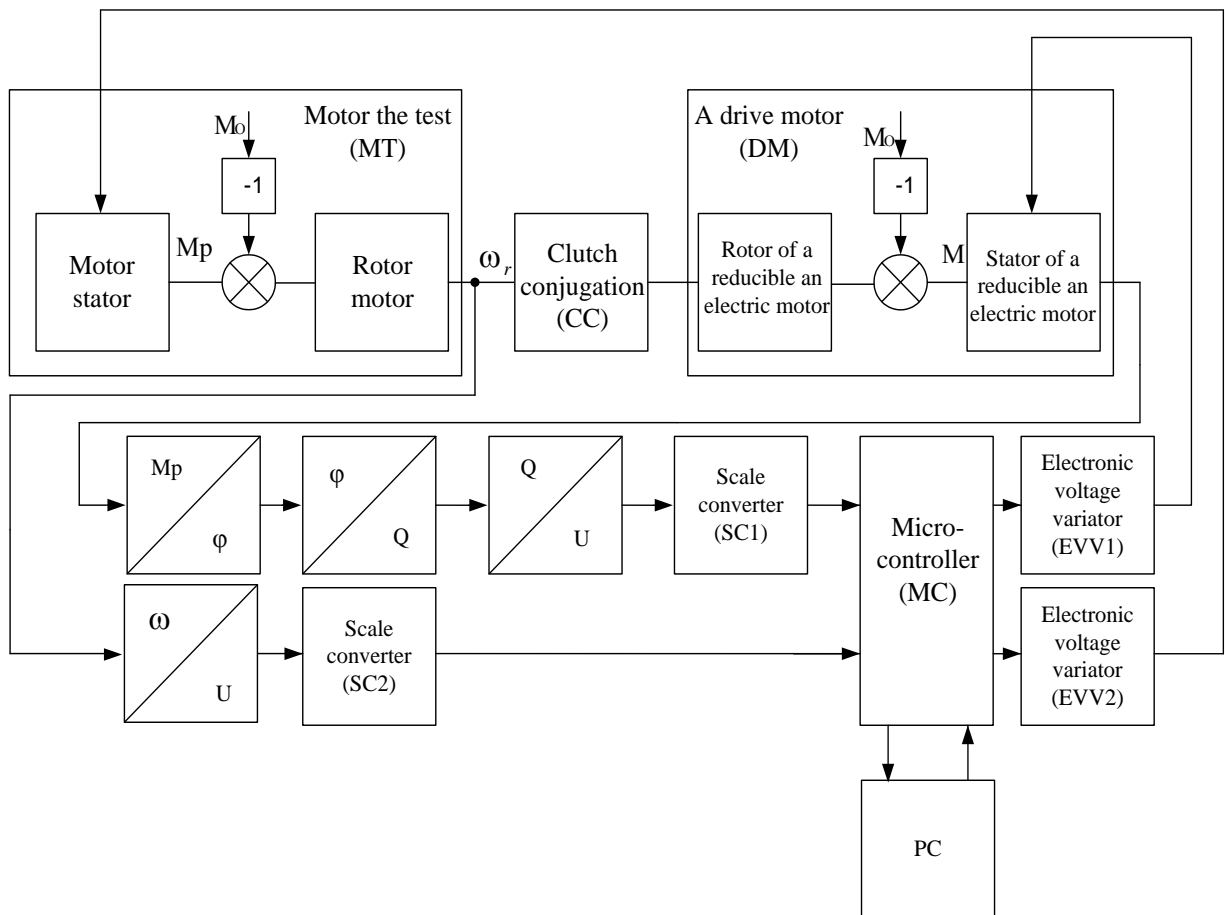
$$N_{My} = 0,25U_p 2^m g^{-1} l^{-1} R^{-1} S_C^{-1} U_o^{-1} \left( M_p - M_{MS} + K1 \left( S + (M_p - M_{MS}) \frac{T_s P}{i \eta J \omega_r} \right) \left( \gamma - \frac{T_s P}{J \omega_r} \right)^{-1} J^{-1} \omega^{-2} \right), \quad (1)$$

<sup>7</sup> Vasilevskiy O.M. Advanced mathematical model of measuring the starting torque motors. *Tekhnichna elektrodinamika*. 2013. Vol. 6. P. 76–81.

<sup>8</sup> Subrahmanyam V. *Electric Drives* (McGraw-Hill, USA, 1996), 715 p.

<sup>9</sup> Vasilevskiy O.M., Kulakov P.I., Ovchynnykov K.V., Didych V.M. Evaluation of dynamic measurement uncertainty in the time domain in the application to high speed rotating machinery. *International Journal of Metrology and Quality Engineering*. 2017. Volume 8. Article Number 25.

where  $U_p$  is the ADC supply voltage;  $U_O$  is the ADC reference voltage;  $m$  is the ADC digital capacity;  $g$  is the acceleration of gravity;  $l$  is the length of the measurement arm;  $R$  is resistance value of the tensoresistive bridge;  $S_C$  is the value of the sensitivity of the effort sensor;  $\omega$  is the free natural frequency (undamped) of the fluctuations of the transformer;  $K_1$  is the value of the coefficient of the conversion of the tensoresistive transformer;  $S$  is the slip;  $J$  is the moment of inertia of the moving part of the electric motor (EM);  $p$  is the number of pole pairs of the EM;  $i$  is the transmission value of the reductor;  $\eta$  is the efficiency coefficient of the reductor;  $\omega_r$  is the angular speed of the EM rotor;  $M_{MS}$  is the moment of eccentricity of the clutch coupling;  $T_s$  is the electromagnetic time constant of the stator;  $\gamma$  is the value of the coefficient of the slope of the slip characteristics;  $M_P$  is the torque of the electric motor (measured value).



**Fig. 1. Block diagram of the torque MM**

## 2. A method to obtain model metrological characteristics of the accuracy of the Measurement Means

Under the working conditions of the MM, the torque provides functional conversion of the informative (useful) parameter  $M_p$  in the source code  $N_{My}$ . In addition to the informative signal, it is also influenced by impact values which include all the other parameters that have a direct relationship with the original value and causing uninformative (stray) components of the conversion<sup>10</sup>. Analysis of the derived conversion equation (1) demonstrates that the process of the conversion of informative parameter in the output signal is influenced by a large number of parameters (impact values). But due to the fact that almost all the values are unchanged, and one of the most important parameters when testing the EM in addition is the torque is the moment of inertia, we will explore the impact of the moment of inertia as an impact value on the informative parameter.

To obtain analytical dependences of the basic static metrological characteristics of the MM of the torque we may use Taylor's Series with our conversion equation (1) to obtain.

$$N_{My} = N_{My0} + \left[ \frac{\partial N_{My}}{\partial M_p} \right]_0 M_p + \frac{1}{2!} \left[ \frac{\partial^2 N_{My}}{\partial M_p^2} \right]_0 M_p^2 + \frac{1}{3!} \left[ \frac{\partial^3 N_{My}}{\partial M_p^3} \right]_0 M_p^3 + \dots + \left[ \frac{\partial^2 N_{My}}{\partial M_p \partial J} \right]_0 M_p \Delta J + \left[ \frac{\partial N_{My}}{\partial J} \right]_0 \Delta J + \frac{1}{2!} \left[ \frac{\partial^2 N_{My}}{\partial J^2} \right]_0 \Delta J^2 + \dots \quad (2)$$

The analytical expression of the sensitivity of the torque MM has the form

$$S_{M_p} = \frac{\partial N_{My}}{\partial M_p} = \frac{U_p 2^n \left( \frac{K1T_s p}{i\eta J^2 \omega_r \left( \gamma - \frac{T_s p}{J\omega_r} \right) \omega^2} + 1 \right)}{4glRS_c U_o} \quad (3)$$

Changes in sensitivity across the range of conversion of the informative parameter  $M_p$  is a derivative of the second order equation of the conversion (1) where  $S'_{M_p} = \frac{\partial^2 N_{My}}{2\partial M_p^2}$  in this case is zero.

The coefficients of the influence of the impact value i.e the moment of inertia  $J$  on the output parameter of the measurement means of the torque, may be described by analytical expressions such as:

---

<sup>10</sup> Tegtmeier F. et al. Investigation of transfer standards in the highest range up to 50MN within EMRP project Sib 63, XXI IMEKOWorld Congress, Prague, Czech Republic, 2015 (2015).

$$\beta_{0J} = \frac{\partial N_{My}}{\partial J} = \frac{U_p 2^m \left[ \frac{K1T_s p \left( S + \frac{T_s p (M_P - M_{MS})}{i\eta J \omega_r} \right)}{\left( \gamma - \frac{T_s p}{J \omega_r} \right)^2 J^3 \omega^2 \omega_r} - \frac{K1 \left( S + \frac{T_s p (M_P - M_{MS})}{i\eta J \omega_r} \right)}{\left( \gamma - \frac{T_s p}{J \omega_r} \right) J^2 \omega^2} \right]}{4glRS_C U_o} - \frac{U_p 2^m K1T_s p (M_P - M_{MS})}{4glRS_C U_o i\eta \left( \gamma - \frac{T_s p}{J \omega_r} \right) J^3 \omega^2 \omega_r}; \quad (4)$$

$$\beta_{0J}' = \frac{\partial^2 N_{My}}{2\partial J^2} = \frac{U_p 2^m \left[ \frac{2K1T_s^2 p^2 \left( S + \frac{T_s p (M_P - M_{MS})}{i\eta J \omega_r} \right)}{\left( \gamma - \frac{T_s p}{J \omega_r} \right)^3 J^5 \omega^2 \omega_r^2} + \frac{4K1T_s p \left( S + \frac{T_s p (M_P - M_{MS})}{i\eta J \omega_r} \right)}{\left( \gamma - \frac{T_s p}{J \omega_r} \right)^2 J^4 \omega^2 \omega_r} \right]}{8glRS_C U_o} + \frac{U_p 2^m \left[ \frac{2K1T_s^2 p^2 (M_P - M_{MS})}{i\eta \left( \gamma - \frac{T_s p}{J \omega_r} \right)^2 J^5 \omega^2 \omega_r^2} + \frac{2K1 \left( S + \frac{T_s p (M_P - M_{MS})}{i\eta J \omega_r} \right)}{\left( \gamma - \frac{T_s p}{J \omega_r} \right) J^3 \omega^2} + \frac{4K1T_s p (M_P - M_{MS})}{i\eta \left( \gamma - \frac{T_s p}{J \omega_r} \right) J^4 \omega^2 \omega_r} \right]}{8glRS_C U_o}. \quad (5)$$

The coefficient of joint influence of the informative parameter and the impact value – that is the moment of inertia  $J$  on the nominal sensitivity  $S_{M_P}$  of the torque  $MM$ , may be described by an analytical expression with the form

$$\alpha_{0J} = \frac{\partial^2 N_{My}}{\partial M_P \partial J} = \frac{U_p 2^m \left[ \frac{K1T_s^2 p^2}{\left( \gamma - \frac{T_s p}{J \omega_r} \right)^2 J^4 \omega^2 i\eta \omega_r^2} - \frac{2K1T_s p}{\left( \gamma - \frac{T_s p}{J \omega_r} \right) J^3 \omega^2 i\eta \omega_r} \right]}{4glRS_C U_o}. \quad (6)$$

The analytical expression of the nominal function of the conversion of the torque  $MM$  taking into account equation (3) may now be stated in the form

$$N_N = S_{M_P} M_P = \frac{U_p 2^m \left( \frac{K1T_s p}{i\eta J^2 \omega_r \left( \gamma - \frac{T_s p}{J \omega_r} \right) \omega^2} + 1 \right)}{4glRS_C U_o} M_P. \quad (7)$$



The analytical expression which describes the absolute multiplicative error of the torque MM  $\Delta N_{MJ}$  under the condition of the change of the moment of inertia J (as an impact value) on the value where  $\Delta J = J - J_N$ , now takes the form

$$\Delta N_{MJ} = \alpha_{0J} M_p \Delta J = \frac{U_p 2^m \left( -\frac{K1T_s^2 p^2}{\left(\gamma - \frac{T_s p}{J\omega_r}\right)^2 J^4 \omega^2 \omega_r^2 i \eta} - \frac{2K1T_s p}{\left(\gamma - \frac{T_s p}{J\omega_r}\right) J^3 \omega^2 \omega_r i \eta} \right) M_p \Delta J}{4glRS_c U_o}. \quad (8)$$

The analytical expression describing the absolute additive error of the torque MM  $\Delta N_{aJ}$  under the conditions where the change in the moment of inertia J (as an impact value) on the value of  $\Delta J$ , may now be expressed as

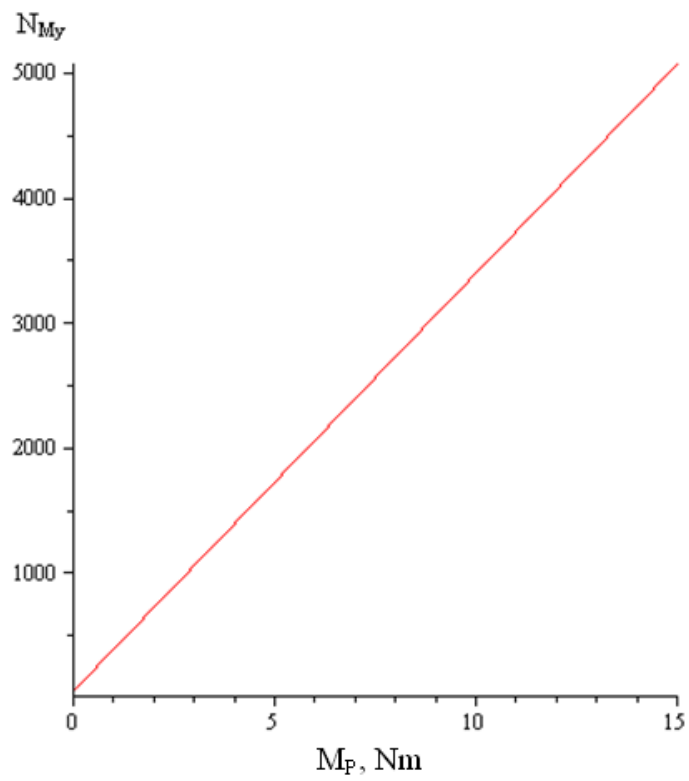
$$\begin{aligned} \Delta N_{aJ} &= \beta_{0J} (J - J_N) + \beta'_{0J} (J - J_N)^2 = \\ &= -\frac{U_p 2^m K1Si\eta J^3 \omega_r \gamma (\omega_r^2 \gamma J_N - T_s p) \Delta J}{4glRS_c U_o (\gamma J \omega_r - T_s p)^3 J^3 \omega^2 i \eta} + \\ &+ U_p 2^m K1\Delta J \frac{(M_p - M_{MS}) (T_s^3 p^3 J_N - T_s p \gamma^2 J^3 \omega_r^2 - 3T_s^2 p^2 J_N \gamma J \omega_r + 3T_s p J_N \gamma^2 J^2 \omega_r^2)}{4glRS_c U_o (\gamma J \omega_r - T_s p)^3 J^3 \omega^2 i \eta}. \end{aligned} \quad (9)$$

The characteristics of change in the obtained metrological models of static characteristics models (1), (3) – (9), of the torque measurement means at nominal values for the impact values, are shown in Fig. 2–8.

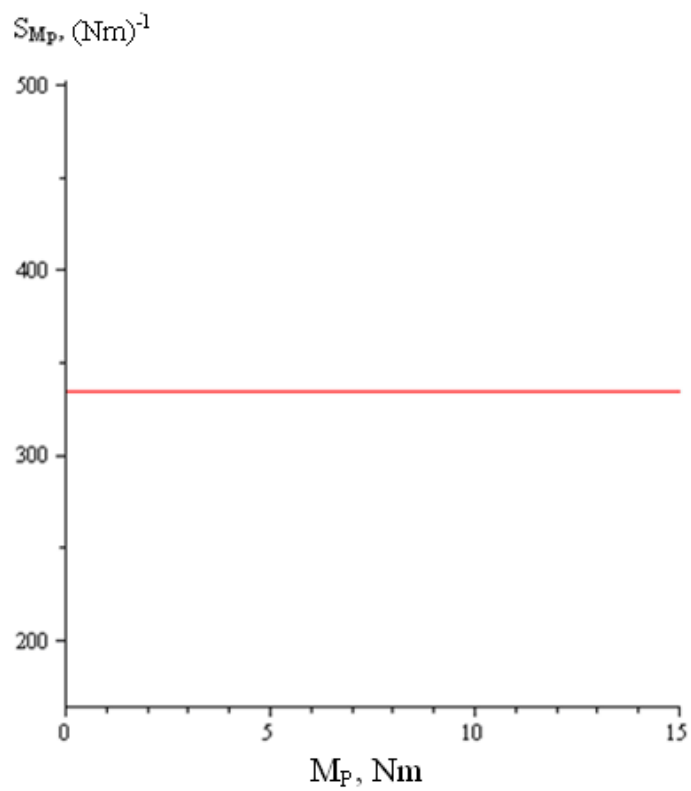
As may be seen in Fig. 2, the static characteristics of conversion equation (1) of the torque MM are linear over the entire range of measurement. The sensitivity of the MM (Fig. 3) is constant, and therefore the scale of such a MM is even.

It may be seen in Fig. 5 that the absolute multiplicative error of the torque MM under the conditions of deviation of the impact value J will acquire the highest values when measuring the torque at the upper limits of the measuring range. The range of fluctuation in the multiplicative error under the conditions of deviation of the impact value J, from its nominal value, is 500 denary pulses at the value of the maximum output code of 5000 pulses, that equals 10%.

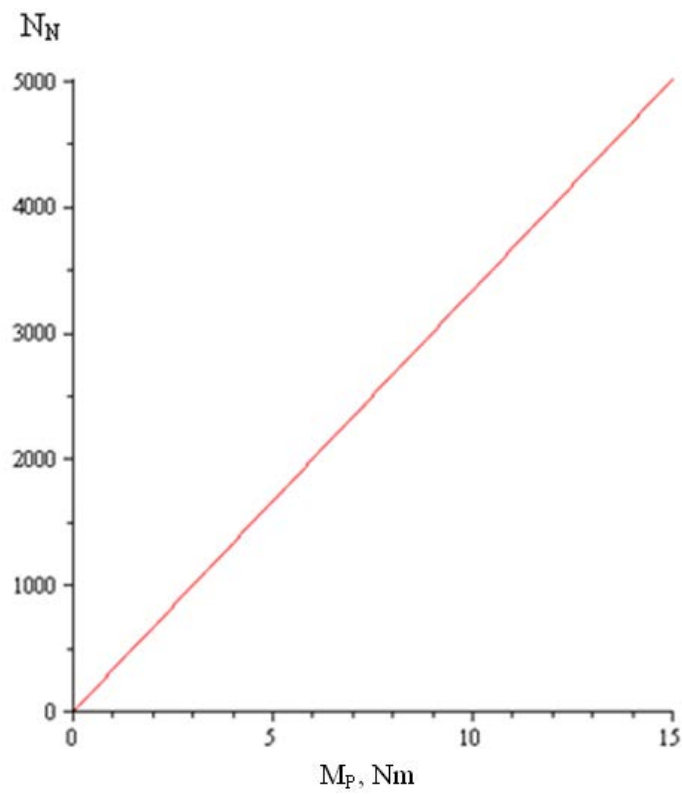
Therefore, it is feasible that the nominal value of the moment of inertia of the electric motor when undertaking measurements of the torque of the electric motor may take into account the above nominal value when calculating the torque in a digital measurement transformer of the torque MM.



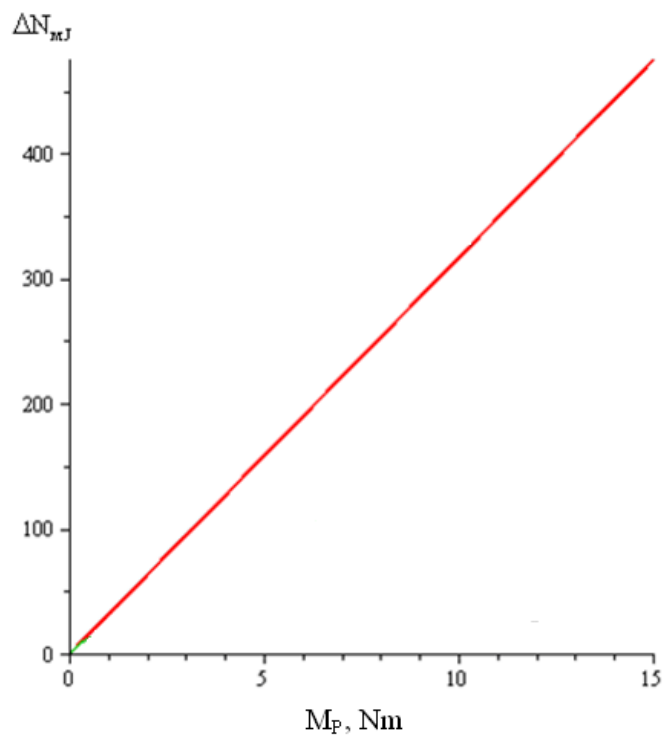
**Fig. 2. Static characteristics of the torque MM**



**Fig. 3. MM sensitivity**



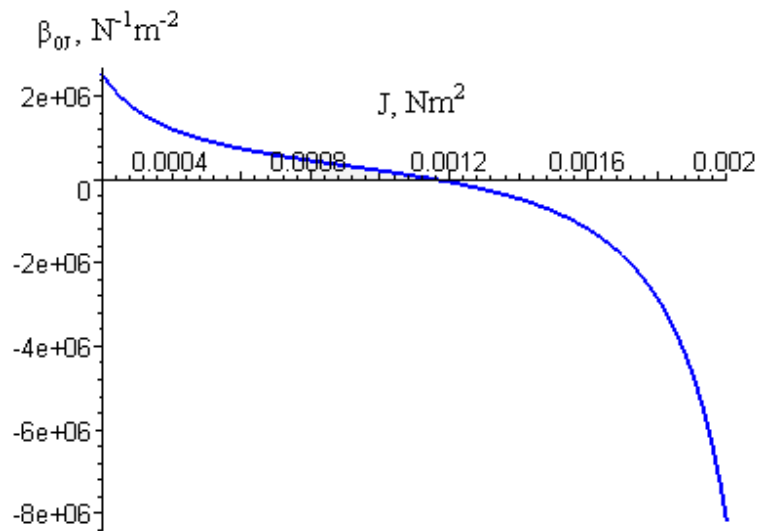
**Fig. 4. Description of change in the nominal function of the torque MM**



**Fig. 5. Description of change in the absolute multiplicative error of the torque MM under the conditions of deviation from the impact value on  $\Delta J$**

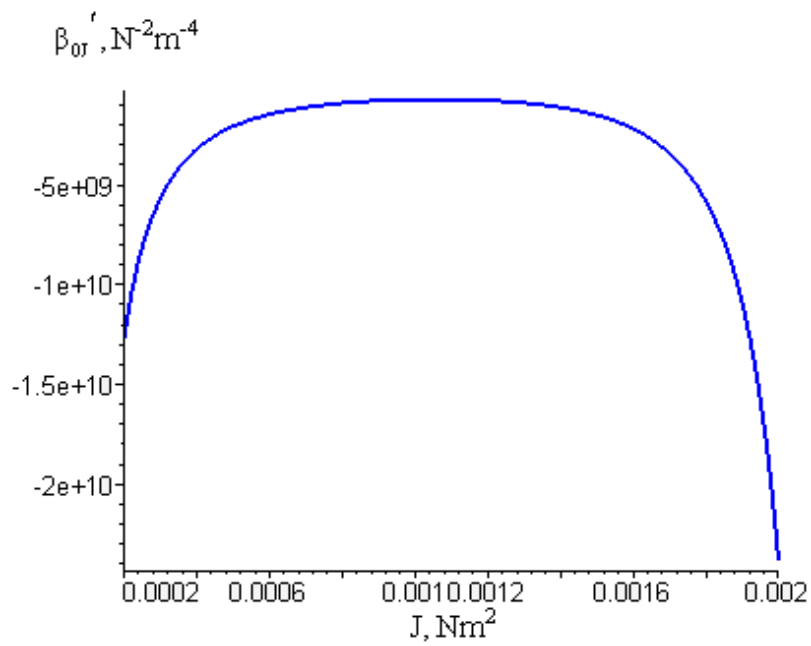
In terms of deviation of the impact value  $J$  (moment of inertia of the electric motor) from its nominal value by the amount  $\Delta J$ , the value of the absolute additive error of the torque  $MM$  increases when the value of the moment of inertia of the electric motor is more than  $0,0014 \text{ Nm}^2$  (Fig. 9). And as the additive error is independent of the measured value, the determination of its value is problematic.

Characteristics of the change of the nominal function of the conversion (Fig. 4) of the torque  $MM$  are analogous to the theoretical static characteristics described by analytical expression (1). The characteristics which demonstrate the difference between theoretical characteristics of the torque  $MM$  (1) and the nominal function of the conversion (7) are shown in Fig. 10.

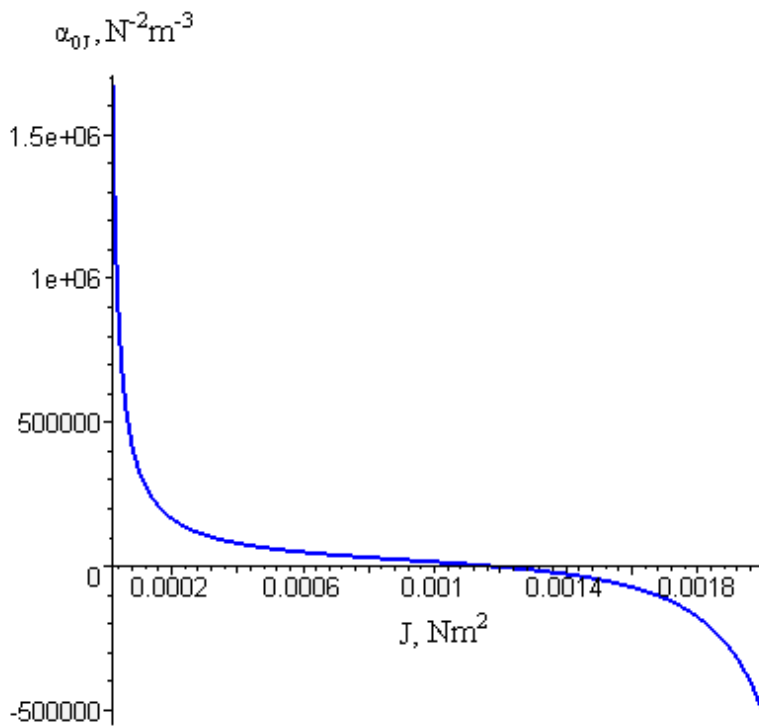


**Fig. 6. The coefficient affecting the impact value  $J$  on the output value**

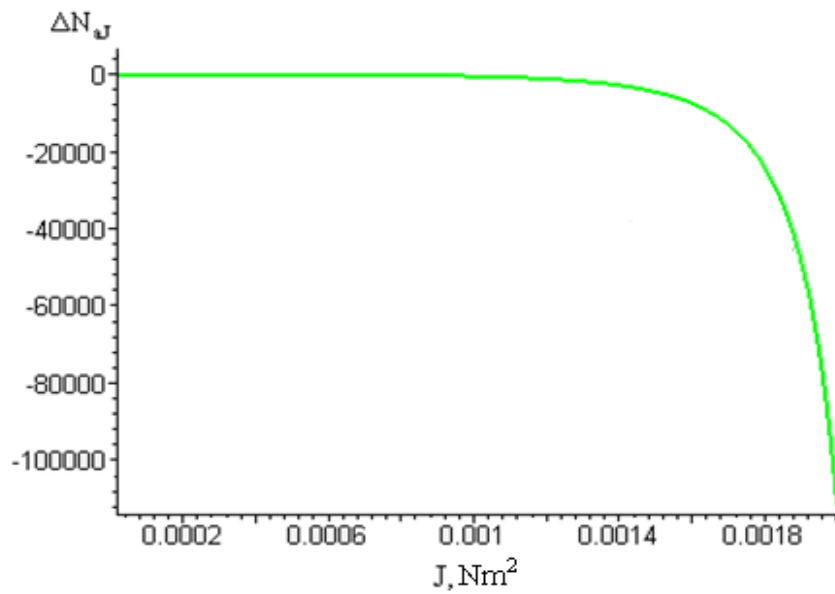
As may be seen in Fig. 10, the discrepancy in the values between the theoretical characteristics and the characteristics of the change in the nominal function of conversion across the entire range of torque measurement differ only by a constant absolute value of the decimal base  $\Delta N_{Mp}$  not exceeding 57 pulses. Taking into account that in the range of the conversion of the measurement means from 0 to 15 Nm, the maximum value of the output value in the decimal code  $N_{My}$  corresponds to 5000 pulses (Fig. 1), the difference of 57 denary pulses will match the decimal value of the torque of 0.17 Nm. This difference may be taken into account when processing the measurement results by a modification to the specified value of the decimal base.



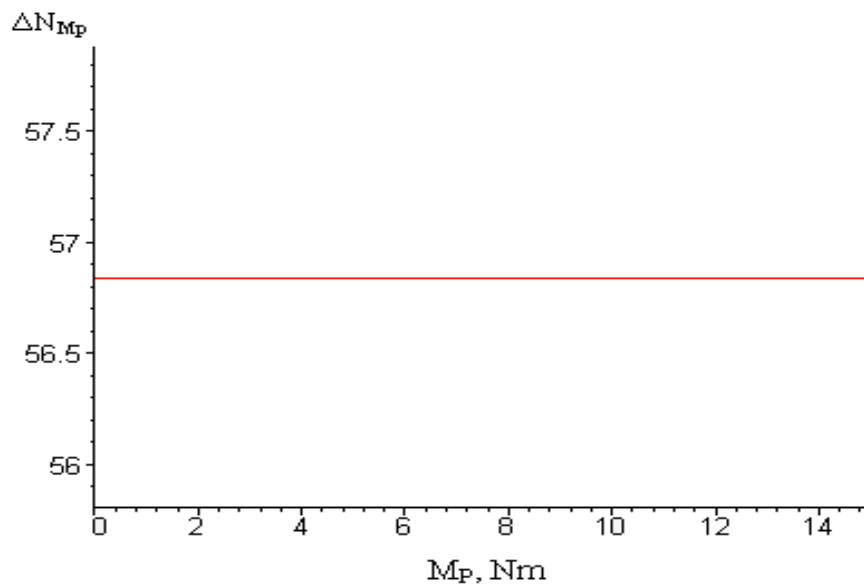
**Fig. 7. The speed of change of the impact coefficient of the impact value J on the output value**



**Fig. 8. Characteristics of the change in the coefficient of the cumulative impact of the informative parameter and the impact value J on the value  $\Delta J$**



**Fig. 9. The boundary of the change in the absolute additive error of the torque MM under the conditions of the deviation of the impact value on  $\Delta J$**



**Fig. 10. The difference between theoretical characteristics and the characteristics of the change in the nominal function of conversion**

To represent the obtained accuracy characteristics in accordance with international standards of quality evaluation measurements<sup>11</sup> the method of recalculation

<sup>11</sup> Evaluation of measurement data. *Guide to the expression of uncertainty in measurement*, JCGM 100, GUM 1995 with minor corrections, 1st edn. (2008).

of the additive and multiplicative errors in the measurement means in the instrumental component<sup>12</sup> of the uncertainty is set out below<sup>13</sup>.

### 3. The methodology of recalculating the components of the additive and multiplicative errors of the measurement means in the instrumental component of uncertainty

When using the above proposed methods to obtain metrological models for additive and multiplicative errors there is an issue to be considered of how to recalculate the components of such errors in the instrumental component of uncertainty, in order to represent the characteristics relating to the precision of measuring instruments, in accordance with international standards for evaluating the quality of measurements – the concept of uncertainty.

As a result of applying the model equation of the conversion of the measurement means to the Taylor series, we may obtain the additive  $\Delta N_a$  and multiplicative  $\Delta N_m$  components of error in the measurement means, which may in general be described by the expressions:

$$\Delta N_a = \frac{\partial N}{\partial \eta}(\eta - \eta_0) + \frac{\partial^2 N}{2\partial \eta^2}(\eta - \eta_0)^2 = \frac{\partial N}{\partial \eta} \Delta \eta + \frac{\partial^2 N}{2\partial \eta^2} \Delta \eta^2; \quad (10)$$

$$\Delta N_m = \frac{\partial^2 N}{\partial x \partial \eta} (x - x_0)(\eta - \eta_0) = \frac{\partial^2 N}{\partial x \partial \eta} \Delta x \Delta \eta, \quad (11)$$

where N is the output value of the measurement means (MM); x is the measured input value;  $x_0$  is the nominal value of the input value (the value that under normal conditions meets the requirements for calibration of the MM);  $\eta$  is the impact value; and  $\eta_0$  is the nominal value of the impact value.

In order to recalculate the additive and multiplicative components of the error of the measurement means in the instrumental component of uncertainty necessary to determine the uncertainty of the measured input value  $u(\Delta x)$  and the impact value  $u(\Delta \eta)$  for the upper and lower boundaries ( $[x_-; x_+]$  i  $[\eta_-; \eta_+]$ ), having assumed a possible interpretation of the law, we may assess the distribution of error within the limits<sup>14</sup>.

<sup>12</sup> Sysoev Yu.S., Tikhomirova A.I. Analysis of the operational stability of measurement instruments by stochastic prediction of drift in their metrological characteristics, Meas. Tech. 6, 613–621 (2012).

<sup>13</sup> Evaluation of measurement data. *An introduction to the Guide to the expression of uncertainty in measurement and related documents*, JCGM 104 (2009).

<sup>14</sup> Vasilevskiy O.M., Calibration method to assess the accuracy of measurement devices using the theory of uncertainty, International Journal of Metrology and Quality Engineering, 2014, 5.04: 403.

If nothing is known about the interpretation of the law of the distribution of the error within the limits, in accordance with international recommendations on the evaluation of measurement uncertainty, we should assume that there is equal probability that such an error may have any value with the accepted range, and hence we may adopt an even probability distribution<sup>15</sup>. If the form of the distribution law of error within the boundaries of the studied values is known, then the uncertainty may be determined by taking into account the form of the distribution law based on the known formulae<sup>16</sup>.

We may now consider the case where information about the nature of the distribution is absent, and in this case we should initially determine the uncertainty of the input and output values in accordance with the formulae:

$$u(\Delta x) = \Delta x / \sqrt{12}; \quad u(\Delta \eta) = \Delta \eta / \sqrt{12}. \quad (12)$$

We may then perform the recalculation of the additive and multiplicative components of the error in the measurement means in the instrumental component of uncertainty according to the formula

$$\begin{aligned} u_{\text{Binst}} &= \sqrt{\left(\frac{\partial N}{\partial \eta}\right)^2 u^2(\Delta \eta) + \left(\frac{\partial^2 N}{2\partial \eta^2}\right)^2 (2\Delta \eta u(\Delta \eta))^2 + \left(\frac{\partial^2 N}{\partial x \partial \eta}\right)^2 u^2(\Delta x) u^2(\Delta \eta)} = \\ &= \sqrt{\left[\left(\frac{\partial N}{\partial \eta}\right)^2 + \left(\frac{\partial^2 N}{\partial \eta^2} \Delta \eta\right)^2\right] \frac{\Delta \eta^2}{12} + \left(\frac{\partial^2 N}{12 \partial x \partial \eta} \Delta x \Delta \eta\right)^2}. \end{aligned} \quad (13)$$

Let us now introduce the following factors for the influence  $\bar{\eta}$  of the impact values on the output parameter value N:

$$\sum_{i=1}^n \frac{\partial N}{\partial \eta_i} = \sum_{i=1}^n \beta_{0i}; \quad (14)$$

$$\sum_{i=1}^n \frac{\partial^2 N}{2\partial \eta_i^2} = \sum_{i=1}^n \beta'_{0i}, \quad (15)$$

and the coefficient of the influence of the impact values on the nominal sensitivity of the measurement means

$$\sum_{i=1}^n \frac{\partial^2 N}{\partial x \partial \eta_i} = \sum_{i=1}^n \alpha_{0i}, \quad (16)$$

<sup>15</sup> A new approach to assessing the dynamic uncertainty of measuring devices / O. Vasilevskyi, P. Kulakov, D. Kompanets, O. Lysenko, V. Prysyazhnyuk, W. Wójcik, Doszhon Baitussupov. Proceedings Volume 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018, 108082E.

<sup>16</sup> Vasilevskyi O.M. Methods of determining the recalibration interval measurement tools based on the concept of uncertainty, *Tekhnichna Elektrodynamika* 6: 81–88, 2014.



The model equation that will determine the instrumental component of the variance at the impact values (more than one impact value) takes the form

$$u_{Binst}^2 = \sum_{i=1}^n \beta_{0i}^2 u^2(\Delta\eta_i) + 4 \sum_{i=1}^n \beta_{0i}'^2 \Delta\eta_i^2 u^2(\Delta\eta_i) + \sum_{i=1}^n \alpha_{0i}^2 u^2(\Delta x) u^2(\Delta\eta_i). \quad (17)$$

When we consider equation (12), the analytical expression (17) for n impact values will take the form

$$u_{Binst}^2 = \frac{\sum_{i=1}^n (\beta_{0i} \Delta\eta_i)^2}{12} + \frac{\sum_{i=1}^n (\beta_{0i}' \Delta\eta_i^2)^2}{3} + \frac{\sum_{i=1}^n (\alpha_{0i} \Delta x \Delta\eta_i)^2}{12^2}. \quad (18)$$

For a single impact value the expression (18) takes the form

$$u_{Binst}^2 = \frac{(\beta_0 \Delta\eta)^2}{12} + \frac{(\beta_0' \Delta\eta^2)^2}{3} + \left( \frac{\alpha_0 \Delta x \Delta\eta}{12} \right)^2. \quad (19)$$

Thus, the method for recalculating the additive and multiplicative components of the error in the measurement means is reduced to the determination of the measured and impact values based on the a priori information on uncertainty of type B and the calculation of the formula of the instrumental component of variance (18).

This proposed method for the recalculation of the component of the instrumental error of the torque measurement means (8, 9) in the instrumental component of uncertainty may now tested with practical calculations.

In as much as it has been established (Fig. 9) that the difference between the theoretical equation of the conversion (1) of the torque MM and the obtained nominal function of the conversion (7) across the whole measurement range of the torque is 0.17 Nm. Then the standard uncertainty of type B, that is introduced by the differences between the nominal function of the conversion and the model equation of the measurement may be determined by the expression (12) thus  $u(\Delta M) = 0.17/\sqrt{12} = 0.05$  Nm.

Uncertainty of type B, that is introduced by the deviation by the moment of inertia of the rotor of the EM  $\Delta J = J - J_N = 10^{-5} Nm^2$ , as an impact value under the assumption of evenness in the distribution law, is  $u(\Delta J) = 10^{-5}/\sqrt{12} = 2,89 \cdot 10^{-6} Nm^2$ . In order to recalculate the additive and multiplicative components obtained for the error in the torque MM in the instrumental component of t uncertainty in the measurement, let us introduce the obtained values of the uncertainties  $u(\Delta M)$  and  $u(\Delta J)$ , and maximum values of influence factors (4) – (6) at nominal parameters of the impact values shown in Fig. 5 – Fig. 7 to the analytical expression (17). The result is the value of the instrumental component of the variance in the torque measurement means, shown by the expression

$$\begin{aligned}
u_{\text{Binst}}^2(\Delta M, \Delta J) &= \beta_{0J}^2 u^2(\Delta J) + 4\beta_{0J}'^2 \Delta J^2 u^2(\Delta J) + \alpha_{0J}^2 u^2(\Delta M) u^2(\Delta J) = \\
&= (-8 \cdot 10^6 \text{ N}^{-1} \text{ m}^{-2})^2 (2,89 \cdot 10^{-6} \text{ Nm}^2)^2 + \\
&+ 4(-2,5 \cdot 10^{10} \text{ N}^{-2} \text{ m}^{-4})^2 (0,5 \text{ Nm}^2)^2 (2,89 \cdot 10^{-6} \text{ Nm}^2)^2 + \\
&+ (1,65 \cdot 10^6 \text{ N}^{-2} \text{ m}^{-3})^2 (0,05 \text{ Nm})^2 (2,89 \cdot 10^{-6} \text{ Nm}^2)^2 = 536,68. \quad (20)
\end{aligned}$$

Taking the square root of the calculated value of the variance (20), we obtain an instrumental component of uncertainty that is 23.17 of the denary pulses of the output signal of the torque measurement means within the range of measurement from 0 to 15 Nm at a maximum value of decimal pulses at the upper boundary of measurements at 5.000 (Fig. 1). Having recalculated the obtained value of uncertainty for the denary pulses into an absolute value of the measured value ( $15 \text{ Nm} \cdot 23.17 / 5000 = 0.07 \text{ Nm}$ ), we obtain the value of instrumental component of uncertainty, which is 0.07 Nm under the conditions where the deviation of the moment of inertia of the rotor of the electric motor from its nominal value by  $10^{-5} \text{ Nm}^2$ .

Thus, as a result of our study, we have shown that the instrumental component of uncertainty of the torque measurement means is 0.07 Nm within a measurement from 0 to 15 Nm with an acceptable deviation of the impact value (moment of inertia) of its nominal value by  $10^{-5} \text{ Nm}^2$ .

The next stage is the evaluation of the standard uncertainty of type B, which is determined by available information about the discarded remainder of repeatable effects that in theory, together with a justified degree of certainty, can be manifested in the process of measurement<sup>17</sup>. In doing so, one should rely on information derived from prior measurements, acceptable working conditions for the means the measurement, the physical properties of the measured value, technical documentation data for the measurement device or means of reference data<sup>18</sup>. After evaluating the theoretically possible components of standard uncertainties of type B, it is necessary to calculate the combined total standard uncertainty of type B  $u_{\text{CB}}$  based on the known forms of representation of combined uncertainty.

After calculating the total measurement uncertainty of type B, it is necessary to calculate the total uncertainty of the measurement result based on the standard

---

<sup>17</sup> Method of evaluating the level of confidence based on metrological risks for determining the coverage factor in the concept of uncertainty / O. Vasilevskiy, V. Didych, A. Kravchenko, M. Yakovlev, I. Andrikevych, D. Kompanets, Ye. Danylyuk, W. Wójcik, A. Nurmakhambetov. Proceedings Volume 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018, 108082C.

<sup>18</sup> Vasilevskiy O.M. A frequency method for dynamic uncertainty evaluation of measurement during modes of dynamic operation, International Journal of Metrology and Quality Engineering, 6.2 (2015): 202.

uncertainty of type A. The equation for calculating the total uncertainty of the measurement result in the absence of correlation is:

$$u_c^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i), \quad (21)$$

where  $\frac{\partial f}{\partial x_i} = c_i$  are the sensitivity coefficients for the equation of transformation of measurement;  $u(x_i)$  – standard uncertainty, assessed by type A or type B.

## CONCLUSIONS

To represent standard uncertainty of type B, which is caused by the presence of additive and multiplicative errors in the MM, mathematical expressions are proposed that allow recalculation of these errors to standard uncertainty. The mathematical expressions were tested using the example of the recalculation of additive and multiplicative errors of the MM of the torque in the standard uncertainty of type B. These analytical expressions of additive and multiplicative errors were derived from the decomposition of the transformation equation of the MM of the torque into a Taylor's Series. The characteristics of the change in the additive and multiplicative errors are presented in Figures 5 and 9, respectively. As a result of the recalculation of these type B instrument uncertainties using formula (20) we determined that in the measurement range from 0 to 15 N m that the standard uncertainty was 0.07 N m. The proposed technique of the recalculation of the additive and multiplicative errors in the standard uncertainty of type B allows the research results to represent the characteristics of the precision of measuring means in accordance with international standards for evaluating the uniformity of measurements.

Thus, the resulting mathematical tools and characteristics of change of the basic static metrological MM for torque give us the opportunity to identify the causes of increase in error in the measurement of torque in electric motors, to explore how to diminish or eliminate them, and also to synthesise torque MM with standardised metrological characteristics.

## SUMMARY

This paper proposes a method of determination of the basic metrological characteristics of measurement means (MM) in order to measure the torque of electric motors (EM), such as sensitivity, nominal conversion functions, and additive and multiplicative errors. The nature of change of metrological characteristics was researched using the Measurand Model derived, in the conversion measuring range studied in this paper. Summarised values for additive and multiplicative errors were established for the MM of torque, under the

conditions of deviation in terms of the impact value from its nominal value. In addition, the methodology was analysed to enable recalculation of additive and multiplicative errors related to the standard uncertainty of type B. This was undertaken to establish the quality of measurement according to international standards. The research was founded on Measurand Model obtained for multiplicative and additive components in the errors inherent in torque MM.

## REFERENCES

1. ISO/IEC 17025:2005, General requirements for the competence of testing and calibration laboratories (Geneva, Switzerland, ISO, 2005), 28 p.
2. Röske D. et al. Metrological characterization of a 1 N m torque standard machine at PTB, *Metrologia* 51, 87 (2014).
3. Vasilevskyi O.M. Metrological characteristics of the torque measurement of electric motors, *International Journal of Metrology and Quality Engineering*, 8, 7 (2017).
4. Vasilevskyi O.M., Kucheruk V.Y., Bogachuk V.V., Gromaszek K., Wójcik W., Smailova S., Askarova N. The method of translation additive and multiplicative error in the instrumental component of the measurement uncertainty, *Proc. SPIE* 10031, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments, 2016, 1003127.
5. Schlegel C., Kahmann H., Kümme R. MN·m torque calibration for nacelle test benches using transfer standards, *ACTA IMEKO* 5, 12 (2016).
6. ISO 5725-1:1994/Cor 1:1998, Accuracy (trueness and precision) of measurement methods and results – Part 1: *General principles and definitions* (Geneva, Switzerland, ISO, 1994/1998).
7. Vasilevskyi O.M. Advanced mathematical model of measuring the starting torque motors. *Tekhnichna elektrodinamika*. 2013. Vol. 6. P. 76–81.
8. Subrahmanyam V. *Electric Drives* (McGraw-Hill, USA, 1996), 715 p.
9. Vasilevskyi O.M., Kulakov P.I., Ovchynnykov K.V., Didych V.M. Evaluation of dynamic measurement uncertainty in the time domain in the application to high speed rotating machinery. *International Journal of Metrology and Quality Engineering*. 2017. Volume 8. Article Number 25.
10. Tegtmeier F. et al. Investigation of transfer standards in the highest range up to 50MN within EMRP project Sib 63, XXI IMEKOWorld Congress, Prague, Czech Republic, 2015 (2015).
11. Evaluation of measurement data. Guide to the expression of uncertainty in measurement, JCGM 100, GUM 1995 with minor corrections, 1st edn. (2008).
12. Sysoev Yu.S., Tikhomirova A.I. Analysis of the operational stability of measurement instruments by stochastic prediction of drift in their metrological characteristics, *Meas. Tech.* 6, 613–621 (2012).

13. Evaluation of measurement data. An introduction to the Guide to the expression of uncertainty in measurement and related documents, JCGM 104 (2009).

14. Vasilevskyi O.M. Calibration method to assess the accuracy of measurement devices using the theory of uncertainty, *International Journal of Metrology and Quality Engineering*, 2014, 5.04: 403.

15. A new approach to assessing the dynamic uncertainty of measuring devices / O. Vasilevskyi, P. Kulakov, D. Kompanets, O. Lysenko, V. Prysyzhnyuk, W. Wójcik, Doszhon Baitussupov. *Proceedings Volume 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018*, 108082E.

16. Vasilevskyi O.M. Methods of determining the recalibration interval measurement tools based on the concept of uncertainty, *Tekhnichna Elektrodynamika* 6:81–88, 2014.

17. Method of evaluating the level of confidence based on metrological risks for determining the coverage factor in the concept of uncertainty / O. Vasilevskyi, V. Didych, A. Kravchenko, M. Yakovlev, I. Andrikevych, D. Kompanets, Ye. Danylyuk, W. Wójcik, A. Nurmakhambetov. *Proceedings Volume 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018*, 108082C.

18. Vasilevskyi O.M. A frequency method for dynamic uncertainty evaluation of measurement during modes of dynamic operation, *International Journal of Metrology and Quality Engineering*, 6.2 (2015): 202.

**Information about authors:**

**Vasilevskyi O. M.,**

Doctor of Engineering Sciences,

Professor of Department of Metrology and Industrial Automation,

First Vice-Rector

Vinnitsia National Technical University

95, Khmelnytskoye Shose, Vinnitsya, 21021, Ukraine

**Didych V. M.,**

Candidate of Technical Sciences,

Associate Professor of Department of Biophysics,

Informatics and Medical Equipment

Vinnitsia National Medical University named after M. I. Pirogov

56, Pirogova, Vinnitsa, 21018, Ukraine