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Criterion of spatial resolution of imaging system

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ABSTRACT

The paper proposes the new criterion of spatial resolution of an imaging system. This criterion considers shape and dimensions of a central peak and side lobes of a point spread function, standard deviation of noise. As a result, it helps to reach the optimal balance between the characteristics of a central peak, side lobes and noise. It differs from the widely known full width at half maximum and Sparrow criterion that mainly consider only characteristics of the central peak. There are discussed the digital filter optimal according to the proposed criterion and the limitations in maximization of spatial resolution of imaging systems.

Keywords: spatial resolution, imaging system, FWHM, Sparrow criterion, digital filter, image processing.

1. INTRODUCTION

Imaging systems that produces digital images are widely used in smartphones, computers, security systems, optical microscopy, optical instruments in industry, material science, biology, medicine and other fields[1]. We consider the imaging system (IS) with the following elements[1]:

1. Optics that acts as a low-pass spatial filter due to diffraction and aberrations.
2. A focal plane array that contains a two-dimensional periodical structure of photosensitive cells. It is also a low-pass spatial filter due to signal integration throughout area of photosensitive cells. It also performs spatial sampling caused by discrete periodical structure of its photosensitive cells and noise generation.
3. A digital filter that compensates the distortions introduced by the optics and the focal plane array.
4. A restoration unit that is an optimal interpolator. It is a digital zoom unit.

Spatial resolution defines the ability to distinguish small objects in their images formed by an IS. A criterion of spatial resolution specifies the condition when small objects can be recognized as separate ones in their images. It is obvious that this condition depends on parameters and characteristics of an IS. The widely known criterions are based on analysis of a point spread function (PSF) as an IS impulse response[2,3,4]. Some of them like Rayleigh and Sparrow criterions evaluate the minimal resolvable distance between two point sources[2,5,6]. The other ones use multi bar or other test patterns, including the spatial random grayscale or binary patterns, fractal and other patterns[7,8,9].

2. THE PROPOSED CRITERION OF SPATIAL RESOLUTION

The well-known evaluations of spatial resolution are the full width at half-maximum (FWHM) and Sparrow criterion[4] (Fig. 1). Applications of Rayleigh criterion are limited by the cases when optics has a small numerical aperture and its PSF has shape of Airy plot[10,11]. These criterions do not consider amplitudes of side lobes and noise. However, high frequency (HF) digital filtering used to increase spatial resolution can magnify them. As a result, it decreases image quality and ability to distinguish small objects.

We are sure that the objective evaluation of spatial resolution has to take into account the influence of side lobes and noise[7,12,13]. We propose the following criterion: the evaluation of spatial resolution is defined as the distance between the centers of two-point sources when their contract in the image is equal to the sum of the amplitude of the highest side lobe and the confidence interval of noise (Fig. 1). The following expression describes the proposed criterion:

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$$x_R : \Delta u \geq \Delta u_{MIN} = u_S + u_N\left(\sigma_N, p_N\right) \tag{1}$$

where x_R – the evaluation or the limit of spatial resolution according to the proposed criterion; Δu_{MIN}, u_S – the minimal acceptable contract of two point source image and the amplitude of the highest side lobe, respectively

$$\Delta u = \Delta u(\Delta x) = u\left(\frac{\Delta x}{2}, 0, \Delta x\right) - u(0,0,\Delta x) =$$

$$= \left(h(-\Delta x, 0) + h(0,0)\right) - \left(h\left(-\frac{\Delta x}{2}, 0\right) + h\left(\frac{\Delta x}{2}, 0\right)\right)$$

$$u(x,y,\Delta x) = h\left(x - \frac{\Delta x}{2}, y\right) + h\left(x + \frac{\Delta x}{2}, y\right)$$

Δu – the contract in an image of two point sources in the point (0,0); $h(x, y)$ – the IS PSF recalculated for the object plane; u_N (p_N, σ_N) – the confidence interval of noise: noise values with the standard deviation σ_N are inside this confidence interval $[-u_N, u_N]$ with the confidence probability p_N, respectively. In case of the normal distribution the following expression links u_N , p_N and σ_N :

$$p_N = 2 \cdot \Phi\left(\frac{u_N}{\sigma_N}\right)$$

$\Phi(x)$ denotes the cumulative distribution function which is an integral of the Gaussian probability density function.

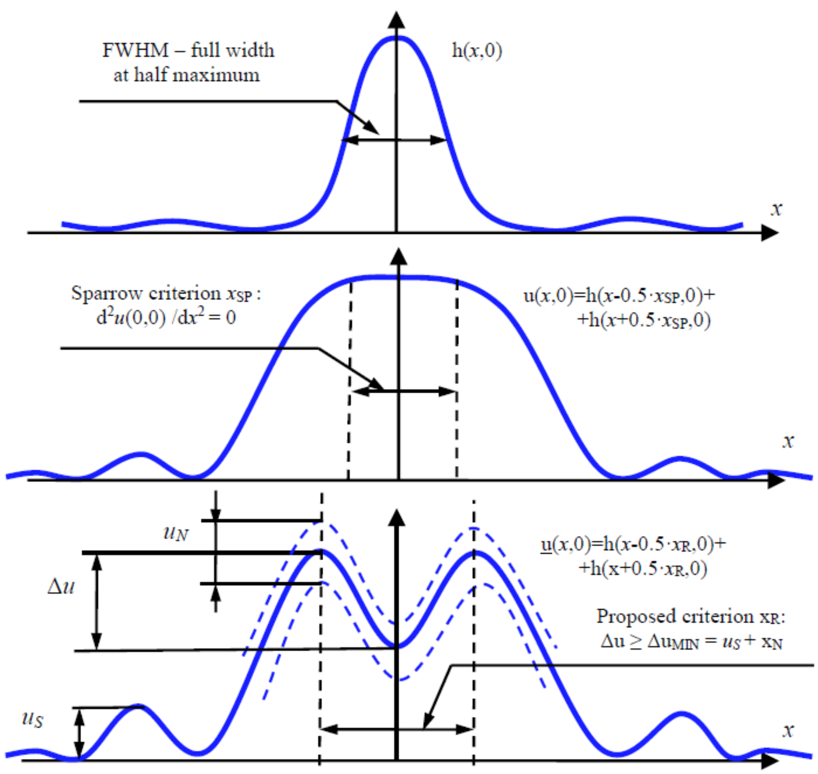


Figure 1. The illustrations of the criterions of spatial resolution: FWHM, Sparrow criterion and the proposed criterion.

There are several important features of the proposed criterion:

1. This criterion is stochastic: the limit xR is calculated only for a definite confidence probability (1).

2. Due to more strict conditions we have $x_R > x_{SP} >$ FWHM (Fig. 1). In case of absence of side lobes and noise the proposed criterion is equal to Sparrow one: when $u_S \to 0$ and $u_N(p_N, \sigma_N) \to 0$, $u_S + u_N(p_N, \sigma_N) \to 0$ and $x_R \to x_{SP}$.

3. HF filtering increases Δu, u_S and u_N proportionally. In some cases, it can even make spatial resolution lower when high side lobes and noise produced by HF filtering sufficiently reduce image quality. That is why this criterion is useful for the objective examinations of the techniques based linear or non-linear deconvolution.

4. The proposed criterion should not change the existed ones. It could be considered as the additional one that takes into account influence of side lobes and noise[14,15].

3. DIGITAL FILTER OPTIMAL FOR PROPOSED CRITERION

Now we design the digital filter that maximizes the spatial resolution using the proposed criterion (1). To simplify all the calculations, we accept the following assumption: an IS is a linear spatial filter[1,10]. The IS transfer function can be written in the following form[1,10]:

$$H_S(\upsilon_X,\upsilon_Y) = H_O(\upsilon_X,\upsilon_Y)\cdot H_D(\upsilon_X,\upsilon_Y)\cdot H_F(\upsilon_X,\upsilon_Y)\cdot H_R(\upsilon_X,\upsilon_Y) \quad (2)$$

where $H_S(\upsilon_X, \upsilon_Y)$, $H_O(\upsilon_X, \upsilon_Y)$, $H_D(\upsilon_X, \upsilon_Y)$, $H_F(\upsilon_X, \upsilon_Y)$, $H_R(\upsilon_X, \upsilon_Y)$ – the transfer functions of the IS, the optics, the focal plane array, the digital filter and the restoration unit, respectively; υ_X, υ_Y – the spatial frequencies along axis's *x* and *y*, respectively[16,17]. The transfer function (2) demonstrates the following properties[9,19]:
- it has a value 1 when $\upsilon_X = 0$, $\upsilon_Y = 0$;
- it has a value zero outside the spatial bandwidth;
- without digital filtering its absolute values are in the range [0 .. 1];
- it has symmetry properties in many cases [1,18];
- it is desirable that it would be differentiable and monotonous within its spatial bandwidth.

Now we present the optimal IS transfer function according to the proposed criterion (1) in the following parametric form[8,18]:

$$H_P(\upsilon_X,\upsilon_Y,p) = \begin{cases} 1-\left(\dfrac{\sqrt{\upsilon_X^2+\upsilon_Y^2}}{\upsilon_O}\right)^p, \sqrt{\upsilon_X^2+\upsilon_Y^2} \le \upsilon_O \\ 0, \sqrt{\upsilon_X^2+\upsilon_Y^2} > \upsilon_O \end{cases} \quad (3)$$

where $H_P(\upsilon_X, \upsilon_Y, p)$ – the IS transfer function with the parameter p in the range [0 .. 1], this parameter characterizes the degree of HF filtering: 0 means the maximal possible HF filtering, 1 – the effects of the digital filtering are minimal; υ_O – the cutoff frequency of the optics that limits the IS spatial bandwidth.

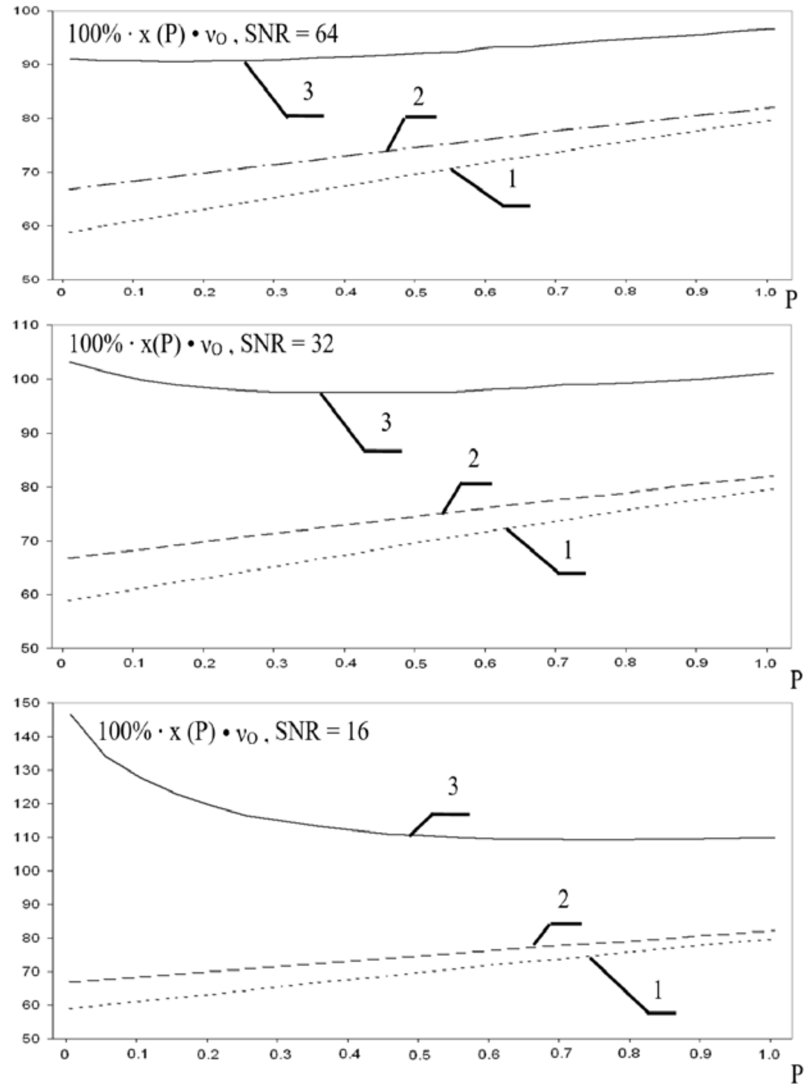
The corresponded transfer function of the HF digital filter can be written using (2), (3) as the following expression:

$$H_F(\upsilon_X,\upsilon_Y,p) = \begin{cases} \dfrac{H_P(\upsilon_X,\upsilon_Y,p)}{H_O(\upsilon_X,\upsilon_Y)\cdot H_D(\upsilon_X,\upsilon_Y)\cdot H_R(\upsilon_X,\upsilon_Y)}, \sqrt{\upsilon_X^2+\upsilon_Y^2} \le \upsilon_O \\ 0, \sqrt{\upsilon_X^2+\upsilon_Y^2} > \upsilon_O \end{cases} \quad (4)$$

where $H_F(\upsilon_X, \upsilon_Y, p)$ – the optimal transfer function of the HF digital filter with the parameter p. The inverse discrete Fourier transform of the transfer function (4) makes possible calculation of the HF digital filter coefficients[11]. It is possible to find the optimal value p (4) using the optimization algorithms for minimization of the distance Δx_R (1). Considering the symmetry properties of optics and a focal plane array we can replace the inverse discrete Fourier transform by the inverse cosine transform[11]:

The proposed mathematical apparatus (1)-(5) has been formalized as the design procedure of the optimal digital filter[11]. The results of application of this filter are demonstrated on Fig. 2. The IS has a diffraction limited optics, a focal plane array with $\upsilon_O \ge \upsilon_{NX}$ and $\upsilon_O \ge \upsilon_{NX}$ (3),(5) that generates white noise, the digital filter (5) and the optimal restoration unit[1,8,10]. It is obvious that efficiency of the HF digital filter depends on the signal to noise ratio (SNR): when in input images SNR $\geq$ 64 it is possible to reduce the value x_R (1) on 8 – 12 % (Fig. 2). When the SNR < 16 the HF digital filtering may cause reduction of spatial resolution.

Neglecting such important factors as noise and side lobe amplitudes in case of minimization of FWHM or Sparrow criterion leads to incorrect digital filter design: bigger degree of HF filtering always make a central PSF peak more narrow. As a result, it creates the illusion of higher spatial resolution. The influence of amplified noise and side lobes may overcome the influence of a narrow central PSF peak to spatial resolution. The proposed procedure (1)-(5) guarantees the optimal balance between influence of a central peak shape, side lobe amplitudes and noise in an output image that is necessary for reaching the maximal possible spatial resolution.



Illustrations how spatial resolution depends on the parameter p as the degree of HF digital filtering: 1 – FWHM; 2 – Sparrow criterion $x_{SP}=x_{SP}(P)$; 3 – the proposed criterion $x_R = x_R(P)$.

4. CONCLUSIONS

The new criterion of spatial resolution is proposed for ISs. It differs from Sparrow criterion and the FWHM because it takes into account the influence of side lobe amplitudes and noise characteristics.

The design procedure of the digital filter based on the proposed criterion is presented. This digital filter guarantees the optimal balance between influences of a central peak shape, side lobe amplitudes and noise in an output image that is necessary for reaching the maximal possible spatial resolution.

It is demonstrated that application of the HF digital filters designed by minimization of FWHM or Sparrow criterion may decrease spatial resolution: the influence of amplified noise and side lobes may overcome the influence of a narrow central PSF peak to spatial resolution.

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