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# THE SPECTRAL METHOD OF JITTER ESTIMATION IN FIBER OPTICS TRANSMISSION SYSTEMS

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## ABSTRACT

The spectral method of the jitter parameters estimation in fiber optics transmission systems is being suggested. The suggested method is featured with high accuracy at the expense of the statistical stability improvement of the results, obtained during the implementation of overlapping fast Fourier transform.

**Keywords:** jitter, overlapping fast Fourier transform, fiber optics transmission system

## 1. INTRODUCTION

Digital means of information (data) transmission are being more frequently used in telecommunication sphere. Digital methods of telecommunication having their advantages based on the new telecommunication technologies cause the list of problems which appear during the signals transmission in digital form<sup>1</sup>. One of these problems is the problem of synchronization in fiber optics transmission systems (FOTS). The main parameter which characterizes synchronization signal is the jitter, that is the phase jitter of the signal in input unit of the system<sup>2</sup>. The jitter estimation enables it to detect the reasons of its appearance and therefor to reduce its impact on the performance of FOTS. Two classes of methods of jitter estimation are applied in modern means of measurement – according phase terms and frequency terms. Taking into consideration that phase and frequency parameters are connected due to a simple relation and uniquely determined, these two classes may be considered to be equivalent. In the end, the jitter estimation is depicted as the frequency shift of a received signal. In this particular case the main parameters of the jitter are its amplitude and frequency.

## 2. PUBLICATIONS ANALYSIS

While analyzing the general methodology of the jitter estimation it should be mentioned that the methodology itself is on the formation stage at the moment<sup>3,12</sup>. Thus, the parameters of the errors in digital paths can be easily calculated. At the same time the processes of jitter formation and transmission in FOTS are not completely studied. The complication of the methodology of the jitter estimation lowers the probability to locate the reasons of digital flow degradation and causes essential stability reduction of FOTS functioning. Finally, one of the features of the accumulated jitter in a complex FOTS is that its impact on the system parameters remains hidden for a long time. And the light jitter growth causes abrupt decrease of telecommunication quality. The most applied method of jitter estimation is the spectral analysis of jitter which uses the frequency selective receiver<sup>3,12</sup>. The given method is featured with low accuracy and realization complexity because of analog methods of signals processing. The methods based on two stages of the jitter estimation which differ in high and low frequencies filters have become widely applied in the practice of operating measurements<sup>4,13</sup>. In spite of the simplicity of realization, the given methods are featured with low accuracy and restricted quantity of controllable parameters of jitter. The spectral method of jitter estimation is depicted in the work<sup>4</sup>. This method is featured with high accuracy in narrow frequency band and can be applied for parameters estimation of cable telecommunication systems only. So, there is the necessity to develop the method of jitter estimation which would ensure the high accuracy of jitter parameters determination in FOTS in broad frequencies band.

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### 3. RESEARCH OBJECTIVES AND TASKS

The research objective is to improve the jitter estimation accuracy in fiber optics transmission systems thus enabling better telecommunication quality in digital networks. To obtain the mentioned objective, the following tasks are necessary to solve:

- to select and base the principal criteria of the phase estimation accuracy in FOTS;
- to suggest the method of jitter spectral estimation on the basis of digital methodology of signals processing;
- to analyze the accuracy of the suggested method.

### 4. BASIC RESEARCH MATERIAL

The method of spectral analysis based on the discrete Fourier transform (DFT) is the most exact according to the quantity of controllable jitter parameters<sup>6</sup>. It is easy to estimate the jitter parameters using digital spectral analysis based on the algorithms of fast Fourier transform (FFT) with windows weighting. The principal objective of the spectral analysis of the jitter on the transmission system output is the power spectrum (PS) determination of digital equivalents of jitter<sup>7</sup>:

$$S(k) = \left| \sum_{n=0}^{N-1} x(n)W_N^{nk} \right|^2, \quad (1)$$

where  $N$  – the quantity of the discrete values of the signal;  $W_N = e^{-j(2\pi/N)}$  – multipliers of DFT;  $x(n)$  – the signal selection in time domain.

The main factor which greatly defines the effectiveness of the jitter parameters estimation is the accuracy. The accuracy criterion depends on the errors values which include spectrum leakage and the dispersion of PS estimation. The general reason for the appearance of the outlined errors is the character of discrete harmonic functions, determined in limited interval, in other words the difference between the integrated Fourier transform and its technical implementation in the form of the finite DFT.

The method of spectral analysis based on the overlapping FFT with optimal windows weighting is suggested to improve the accuracy of jitter parameters estimation. To do that, the data array of the under study signal, which includes  $x(0), x(1), \dots, x(N-1)$  of the marking-offs, are divided into subsequences  $x_P(n)$  with the length of  $M$  marking-offs in each of these subsequences with the shift between the adjacent segments on  $B$  marking-offs. According to the spectrum sliding analysis there is the relation that defines the offset coefficient of adjacent segments of processed data<sup>7,14,16</sup>

$$L = \frac{B}{M}. \quad (2)$$

Depending on the overlapping level of under study selective values of the signal, three modes of the spectral analysis can be determined: when  $L \geq 1$  the absence of the data segments overlap is observed, which is typical for selective mode of spectral values; when  $L = 1/M$ , that is the minimal shift  $B = 1$ , sliding mode is ensured; when  $1 > L > 1/M$  the sliding mode transforms into overlap mode.

In the overlapping DFT mode the quantity of research intervals can be defined according to the formula which interrelates in one subsequence the total volume of  $N$ , processed realization, the shift value  $B$  and the data level:

$$P = \frac{N - M}{B} + 1. \quad (3)$$

The subsequences, shifted relatively to each other on  $B$  marking-offs, is connected with the input data volume in the following ratio:

$$x_P(n) = x[n + B(P-1)]. \quad (4)$$

The DFT coefficients for the isolated research intervals can be calculated by analogy with direct DFT

$$X_P(k) = \sum_{n=0}^{M-1} x_P(n)e^{-j2\pi nk/M}. \quad (5)$$

The isolated data subsequences are processed with the weighting function to enable the control of side lobes of digital signal spectrum. Thus, the expression (5) considering the windows weighting, assumes the following form:

$$X_{pW}(k) = \frac{1}{U} \sum_{n=0}^{M-1} x_p(n)w(n)e^{-j2\pi nk/M}, \quad (6)$$

where  $U$  – the energy of the weighting function.

The power spectrum of the isolated weighted data segment can be derived according to the following formula

$$S_p(k) = |X_{pW}(k)|^2. \quad (7)$$

In doing so, the  $P$  values of PS of the isolated subsequences  $x_p(n)$  are derived as a result of the processing of the under study data volume according to (4) – (7).

The procedure of spectral averaging of weighted overlapping data segments is suggested to obtain statistically stable values of PS. The finite value of PS of under study jitter signal considering the averaging of spectral components assumes the following form

$$S(k) = \frac{1}{P} \sum_{p=1}^P S_p(k) = \frac{1}{P} \sum_{p=1}^P |X_{pW}(k)|^2. \quad (8)$$

The quantity of the analyzed subsequences for the given input realization volume in comparison with other ways of smoothing can be multiplied due to the overlapping of the data segments. As a result, the dispersion of the resulting PS decreases.

The procedure of the optimal “window” synthesis means the solution of the mathematical task to search the bounded function whose DFT in the best way approximates frequency-limited PS signal, that is has minimal energy outside the given data array. Kaiser-Bessel and Nutoll functions are considered to be good examples of the known optimal “windows”<sup>8</sup>. These functions are quite complicated and there are no exact analytical expressions to determine their frequency characteristics. Besides, the practical use of these functions in the means of jitter estimation are undesirable because of the complexity of practical realization.

The family of continuous cosine weighting functions are frequently applied in the technique of the spectral analysis. The mentioned functions are described with the finite trigonometric series and assume the following form<sup>8</sup>:

$$W_C(t) = \sum_{r=0}^R a_r \cos\left(\frac{2\pi r t}{T}\right). \quad (9)$$

After Fourier transform the equation (9) assumes the following form

$$W_C(\omega) = \frac{\sin(\omega T/2)}{\omega T/2} \sum_{m=0}^{\infty} \frac{1}{T/2\omega^{2m}} \sum_{r=0}^R (-1)^r r^{2m} a_r. \quad (10)$$

The second sum in the equation (10) defines the asymptotic representation of the function  $W_C(\omega)$ , whose frequency spectrum features depend on the form of the window function and its paired terms. The following two figures are applied to evaluate the characteristics of side lobes of the function. The figure  $V_S$ , is asymptotic descent rate of the side lobes level, which characterizes the spectrum leakage. The other figure is side lobes level  $A_S$ , which is measured in decibels relative to the level of the main lobe. The higher  $V_S$  and the lower  $A_S$  are, the weaker are the main components distortions of the spectrum relative to the side lobes. The analysis of the expression (10) shows that the more terms of series (the second sum in the expression) on the window edge are equal to 0, the higher is the speed of the side lobes fall. That is the determinant in the process of the weighting function synthesis. The expression (6) shows that the descent rate of side lobes is proportional to the amount of cosine terms in the window function:

$$V_S = V_R(2l - 1), \quad (11)$$

where  $V_R = -6 \text{ dB/okt}$  – asymptotic descent rate for rectangle weighting function.

The continuous window function is transformed into the discrete form with the help of quantization and the function shift to the starting point. The AFC of discrete analog of the weighting function does not differ much from its prototype, excepting that the main lobe of the characteristic repeats with the period multiple of discretization frequency. The discrete cosine function generally assumes the following form

$$W_C(n) = \sum_{r=0}^R (-1)^r a_r \cos\left(\frac{2\pi n r}{M}\right), \quad (12)$$

where  $r$  – the term number of trigonometric series.

The task of the given weighting function synthesis is to calculate the coefficients  $a_r$ , which ensure maximal asymptotic descent rate of the side lobes. Besides, taking into consideration further PS processing in order to define particular jitter parameters, it is necessary to normalize the coefficients in such a way, that their sum is equal to 1. Thus, it is necessary to solve the  $R+1$  expressions system to define the weighting function coefficients depending on the window  $R$  order

$$\begin{cases} \sum_{r=0}^R a_r = 1 \\ \sum_{r=0}^R (-1)^r a_r = 0 \\ \dots \\ \sum_{r=0}^R (-1)^r r^{2R-2} a_r = 0 \end{cases} \quad (13)$$

To ensure the jitter parameters estimation in the frequency band 0,01 – 100 MHz with the accuracy no less than 1% it is necessary to synthesize the weighting function with  $V_S = -40 \text{ dB/okt}$  and  $A_S = -60 \text{ dB}$ . To this purpose, having increased the window expressions system up to  $R=3$ , four expressions system (13) must be solved and the window function coefficients are found:  $a_0 = 0,3125$ ;  $a_1 = 0,46875$ ;  $a_2 = 0,1875$ ;  $a_3 = 0,03125$ .

Having substituted the found numerical values  $a_r$  into the expression (12), the optimal weighting cosine “window” is defined:

$$W_{0C}(n) = 0,3125 - 0,46875 \cos\left(\frac{2\pi n}{M}\right) + 0,1875 \cos\left(\frac{4\pi n}{M}\right) - 0,03125 \cos\left(\frac{6\pi n}{M}\right). \quad (14)$$

The frequency characteristic of the defined functioned can be found if to take into consideration that it is depicted in the form of the product of rectangular “window” and the given weighting functions for all  $n$  values:

$$W_{0C}(n) = W_R(n) \left[ \sum_{r=0}^3 (-1)^r a_r \cos\left(\frac{2\pi n r}{M}\right) \right]. \quad (15)$$

In consideration of equivalency between the multiplications in time domain and the fold in frequency domain, it is possible to define the general frequency characteristics as the fold of the rectangular “window” with the sequence of cosine impulses:

$$W_{0C}(\omega) = W_R(\omega) \otimes \left\{ \sum_{r=0}^3 (-1)^r [U_0(\omega - 2\pi r / M) + U_0(\omega + 2\pi r / M)] \right\}. \quad (16)$$

Here, having substituted numerical values of the coefficients, the resulting formula of the optimal cosine weighting function in the frequency domain assumes the following form:

$$\begin{aligned} W_C(\omega) = & W_R(\omega) - 0,75[W_R(\omega - 2\pi / M) + W_R(\omega + 2\pi / M)] + \\ & + 0,3[W_R(\omega - 4\pi / M) + W_R(\omega + 4\pi / M)] - 0,5[W_R(\omega - 6\pi / M) + W_R(\omega + 6\pi / M)]. \end{aligned} \quad (17)$$

The frequency characteristic of the weighting function in the frequency domain is depicted in figure 1.

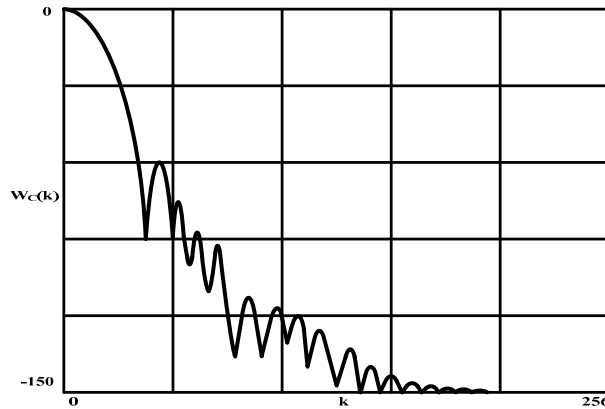


Figure 1. The power spectrum of the weighting function.

It is observed in the figure, that the side lobes level of PS of the given function is lower than  $-60$  dB. The asymptotic descent rate of the side lobes level according to (11) is equal to  $-42$  dB/okt. Thus, the defined weighting function meets the requirements of the spectral jitter estimation in FOTS<sup>9,17,18</sup>.

The analysis of the statistical stability of PS of jitter signal, weighted with optimal “cosine” window  $W_{0C}(n)$ , can be fulfilled with the appliance of the formula for PS dispersion calculation [1]:

$$D[\hat{S}(k)] = \frac{[S(k)]^2}{P} \left[ 1 + 2 \sum_{d=1}^{P-1} \frac{P-d}{P} C(d) \right], \quad (18)$$

where  $C(d)$  – the overlap coefficient of the segments for  $d = 1, 2, \dots, M-1$ .

The minimal value of the  $D[\hat{S}(k)]$  function for various shift sizes is to be found. In so doing, the expression (18), as well as the following features of the algorithm of overlapping FFT and limited technical realization of the jitter estimation means have to be taken into account:

$$\begin{aligned} M &= 2^i, & i &= 6, 7, \dots, 20; \\ B &= \frac{M}{2^l}, & l &= 1, 2, 3, \dots, M/2. \end{aligned} \quad (19)$$

The minimal dispersion of PS in case of given conditions is obtained for  $B = M/4$ . The efficiency coefficient of overlapping FFT for such shift value considering the expression (18) can be calculated according to the formula

$$Q_{ES} = \frac{P_{0,75}}{P_0 \left[ 1 + 1,5C\left(\frac{M}{4}\right) + C\left(\frac{M}{2}\right) + 0,5C\left(\frac{3M}{4}\right) \right]}. \quad (20)$$

The dependences of the jitter estimation effectiveness on the intervals division number in case of the suggested method (graph 1) and the classical method of periodograms are depicted in figure 2.

The advantage of the suggested spectral method of jitter parameters estimation is the improvement of measurement accuracy at the expense of the statistical stability improvement in 1,3 – 3,4 times of the results, derived during the implementation of the weighted overlapping FFT.

High resolution is characterized by the proposed method with 75% -s overlap, and for  $P_0 = 4$  criterion  $\beta_{f_{0,75}}$  reaches values of  $2 \cdot 10^{-4}$ , and for the  $P_0 = 32$  criteria  $\beta_{f_{0,75}} = 0,002$ .

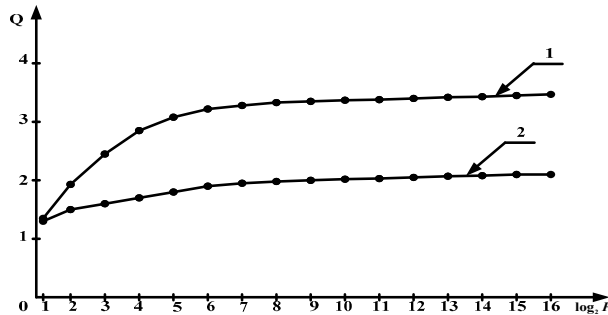


Figure 2. The effectiveness of methods of spectral jitter estimation.

By increasing the number of separate sequences ability decreases. This is due to a decrease in the amount of data in one segment  $M$  while increasing the total number of segments.

The generalized indicator of the effectiveness of phase jitter (PJ) estimation in the frequency domain is introduced for this which is defined by known values of the coefficient of efficiency of the signal spectral estimation PJ  $Q_S$  and frequency resolution  $\beta_f$  :

$$Q_d = Q_S \cdot \frac{1}{\beta_f} . \quad (21)$$

This criterion determines the effectiveness of the relationship between the two fundamental parameters of digital signal processing<sup>10,11,15</sup>. These parameters are determined by the spectral characteristics evaluation signal jitter, namely the sampling frequency  $f_S$ , the volume of signal samples  $N$ , the number of processed data segments  $P$ .

The classical method of assessment is characterized by the volume of sales for the  $N=16384$  sample generalized measure of effectiveness  $Q_{d_0} \approx 1250$ . At the same time, the proposed method for the synthesis performance indicator  $Q_{d_{0,75}} \approx 14000$ , which is more than an order of magnitude greater than the qualitative indicators of the classical approach in the evaluation of PJ. With an increasing number of processed segments  $P_0=4$  to  $P_0=32$  at a constant value input sample, generic performance indicators of both methods are reduced to the classical values of  $Q_{d_0}=125$ , and offers – to a value of  $Q_{d_{0,75}}=1700$ . This reduction  $Q_d$  is attributed to a decrease in the number of samples within a single segment,  $M$  which inevitably leads to deterioration of the frequency resolution. But in this mode of signal processing PJ effectiveness of the proposed method is almost 14 times higher than for classic method of estimation PJ.

## 5. CONCLUSIONS

The spectral method of the jitter estimation in FOTS is being suggested. Its main idea means the data division into overlapping segments. As opposed to the present method based on the direct FFT the suggested one makes it possible to lower PS dispersion. The segments overlapping results in the equalizing of separate marking-offs values, which are further processed with the optimal window function. That is the main advantage of the suggested method relative to the analysis method of sample spectrums, where the use of window functions heightens one frequency marking-offs and lowers other ones, as well as worsens the separative power without any dispersion value improvement of the jitter PS estimation. The suggested method is featured with high accuracy at the expense of the estimation dispersion reduction of jitter PS and the effective suppression of the parasite side lobes of jitter signal in the frequency domain.

Analysis of the resolution showed that the spectral estimation according to the proposed method is carried out with a frequency resolution, which is 4 times higher than for the classical method. Analysis of the generalized criterion of quality proves that the method of spectral estimation is characterized by a generalized measure of effectiveness, which is more than an order of magnitude higher than the efficiency of the existing classical method of estimation PJ.

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