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**SYSTEM ANALYSIS OF THE IMPACT OF MATERIAL REPETITION, SET  
OUT EARLIER, ON THE ASSIMILATION OF THE NEW  
СИСТЕМНИЙ АНАЛІЗ ВПЛИВУ ПОВТОРЕННЯ МАТЕРІАЛУ, ВИКЛАДЕНОГО  
РАНИШЕ, НА ЗАСВОЄННЯ НОВОГО**

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**Abstract.** *The article presents the results of the research, in which on the basis of a system approach using the author's mathematical models of the process of forgetting the knowledge obtained by the student at the lecture, it is mathematically substantiated that the realization of the thesis expressed more than half a century ago by Donetsk educator Viktor Shatalov, the substantive essence of which is the need to repeat more than once the concepts expressed by the lecturer at the lecture, for their reliable memorization by students, in the interpretation of this thesis in the form of repetition at the beginning of the current lecture key concepts new material that the lecturer informs them at this current lecture. It is shown how much more material the student will have in his memory on the eve of the exam, if, while reading each current lecture, the lecturer mentioned and detailed the basic concepts of the lecture course, set out by him in previous lectures.*

**Keywords:** *current lecture, previous lectures, repetition of concepts, process of forgetting, mathematical model, substantiation of necessity of repetitions, quantitative characteristic of residual knowledge.*

### **Introduction**

In the 60's of the last century, the experience of teaching disciplines was widely promoted by an educator from Donetsk (Ukraine) Viktor Shatalov, who at his first lecture at the Institute for Teacher Training every year in each stream put such an experiment –he introduced a number of terms that were unfamiliar to his audience, one of which he mentioned only once during the lecture, the second three times in different word combinations, and the third again five times in different word combinations. At the end of the lecture, Victor Shatalov invited his audience to remember what characterized each of these terms. It turned out that the description of the term, which sounded during the lecture only once, could give in different streams only 2 % to 5 % of listeners, the description of the term, which sounded three times, could give in different streams from 30 % to 40 % of listeners, and from 75 % to 90 % of listeners were able to describe the term, which sounded five times. And the conclusion is that the more often a lecturer mentions something in a lecture, the better this "something" is engraved in the students' memory. And this technique is quite justified if the lecturer gives 2 or 3 lectures, but there are doubts about its effectiveness, if, for example, the lecture course is read during a semester consisting of 18 weeks, with one lecture per week, because in each subsequent lecture it is impossible to restate the main points of all previous lectures even once. So what about the use of Shatalov's method in this case? This is the question we asked, and



realizing that according to Shatalov's method the efficiency of mastering the lecture material is higher than the traditional method of adapting new material in each subsequent lecture to the one already stated in the previous ones, we decided to try to mathematically substantiate the need and detail the amount of material presented students in previous lectures, and in what doses it is necessary to mention at the beginning of the lecture devoted to the presentation of new material [1].

### Initial prerequisites and problem statement

As initial preconditions we used, first, a step-by-step recovery model at the beginning of the current lecture of the material  $v^{pl}(t)$ , read in previous  $i$  lectures, where  $i = 1, 2, \dots, m$ , in the form [1]:

$$v^{pl}(t) = I_c + \sum_{i=1}^m v_i^{pl} \cdot 1\left(t - \sum_{k=1}^i t_k\right) \quad (1)$$

$$v_m^{pl} = \sum_{i=1}^m v_i^{pl}, \quad (2)$$

where  $v_i^{pl}$  is the dose of material from  $i$  – the previous lecture, the memory of which spends part of the time  $t_i$  of the current lecture;  $1(t - t_i)$  is a single function;  $I_c$  – is the amount of material from previous lectures, which is engraved in the student's memory so that does not require mention of him at the beginning of the current lecture.

Second, we used a linear model of loading into the student's brain during the current lecture for the time  $\Delta t$  remaining until its end after the procedure of mentioning the material of previous lectures, new material  $v(t)$  in the form [1]:

$$v(t) = (v_m^{pl} + I_c) + q \cdot \left\{ \left( t - \sum_{k=1}^m t_k \right) \cdot 1\left( t - \sum_{k=1}^m t_k \right) - \left( t - \sum_{k=1}^m t_k - t_\Delta \right) \cdot 1\left( t - \sum_{k=1}^m t_k - t_\Delta \right) \right\}, \quad (3)$$

$$t_\Delta = t_z - \sum_{k=1}^m t_k, \quad t \in [0, t_z], \quad (4)$$

$$v(t_z) = (v_m^{pl} + I_c) + q \cdot t_\Delta = I_0, \quad (5)$$

where  $t_z$  is the duration of the lecture, and  $q$  – is the coefficient of perception of new material by the student.

Thirdly, we used the mathematical model of the process of forgetting information  $I(t)$  by the student developed by us and published in [2], received at the lecture, in the inter-lecture period, in the form [1]:

$$x_{1(\%)}(\tau) = \bar{\phi}_{(\%)} + (100 - \bar{\phi}_{(\%)}) e^{-(1 - \alpha_{12} x_2) \tau}, \quad (6)$$

which we synthesized by combining in one design the structure of the model proposed in [3], with the structure of the model proposed in [4].

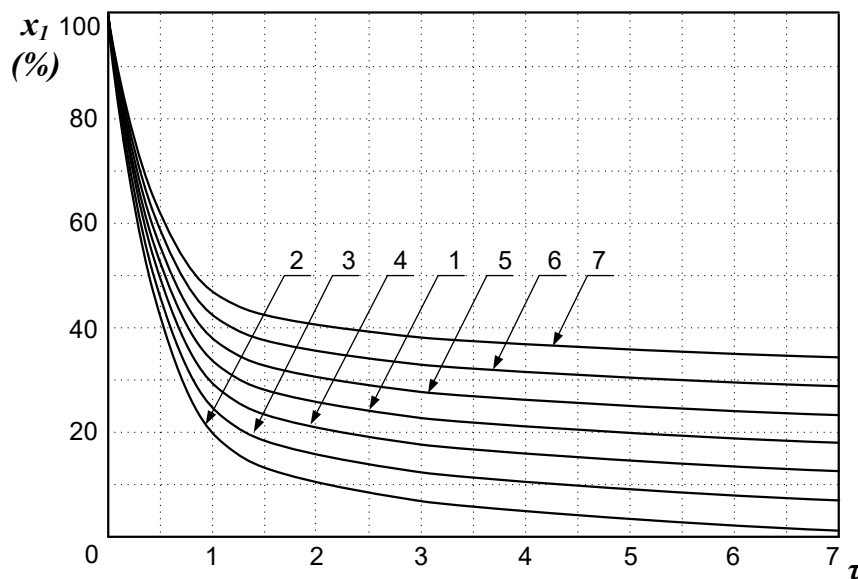
In the model (6):

$$x_{1(\%)} = 100 \frac{I}{I_0}, \quad \bar{\phi}_{(\%)} = \frac{I_c}{I_0}, \quad \tau = \frac{t}{T}, \quad (7)$$

and  $\alpha_{12} x_2$  – is a synergetic component, in which  $x_2$  – is the information generated by the student's brain on the topic of the lecture at a time when the information from the lecturer no longer comes and there is a process of forgetting, which is slowed down by the synergetic component.



And fourth, we used what was done for the first time in [5] using [6], [7] probabilistic extension of the well-known “forgetting curve” of G. Ebbinghaus [8] to dismembered “forgetting bands” which we specified in [2] and has the form shown in the Fig. 1.



**Fig. 1 Weekly graphs of the “forgetting curve” of G. Ebbinghaus (line 1) and “forgetting bands”, bounded by lines 2 and 7, 3 and 6, 4 and 5**

The essence of proposed algorithm is to sequentially fit the simulation results at different time intervals using the above mathematical models at different values of their parameters, tied to those shown in Fig. 1 “forgetting bands” [1]. At the same time, for the implementation of computational procedures, we used the optimization methods described in [9], which can be considered the best in this direction.

In [10] a successful replication of Ebbinghaus’ classic forgetting curve based on the method of savings was presented. It was investigated which mathematical equations correspond well to the Ebbinghaus forgetting curve and its repetition.

In [11] the results of the research in which students listened to a lecture on natural sciences and viewed it 1 or 8 days after study were presented. As a result of the research, it was found that 5 weeks later, students who repeated the material of the lecture 8 days after listening to it remembered better the material of both simpler and more complex level than those students who repeated the material only 1 day after the lecture.

In [12] retroactive interference model of forgetting was proposed. The main idea of this model is strength-dependent retroactive interference between the memories, so that only if a stronger memory is acquired after the weaker one, then the weaker one is erased [12].

The task we set in our research was to, first, prove that the repetition of the basic concepts expressed by the lecturer in previous lectures, at the beginning of the current lecture, leads to an increase in the level of knowledge that will remain in the memory of students after listening to the full course of lectures, and secondly, to obtain numerical characteristics of the degree of forgetting by the student of the information



received at the lecture from the lecturer, and numerical characteristics of the amount of information that will be stored in the student's memory in the inter-lecture period, depending on the frequency of repetition in the current lectures of the basic concepts from previous lectures.

### Presentation of the main results of the research

We will consider a lecture course, which will consist of ten one-hour lectures given to students of technical university by a lecturer once a week for ten weeks – such lecture courses take place in the second semester of the 4th year of bachelor, as this semester for 4th year students is shortened to ten weeks in connection with the need for undergraduate practice and diploma design before the end of the academic year. It is obvious that if these ten lectures are not one-hour, but two-hour (pairs), then the calculations and graphs will be identical and will differ from those given by us only in scale on the time axis.

Note, that we will consider the perception of lecture material only by a representative of the cohort of "E" and "D" students, ie those students whose memory is characterized by a point in the “forgetting band”, (Fig. 1), located between curves 4 and 5.

So, let the lecturer present the material in his 10 lectures, linearly increasing the amount of information during the time spent on each lecture, within one tenth (ie, within 10%) of that 100 percent volume, which must be presented to the lecturer in accordance with the program of the discipline, designed for 10 lectures – in Fig. 2, in which all 10 lectures are time-matched to each other, this process is shown as a straight line  $v_l(t^*)$  running from a point with coordinates (0; 0) to a point with coordinates (10; 100). And let a week after the student listens to the first lecture in his memory left only 2% of what he heard in this lecture from the lecturer, which is embedded in the simulation using a mathematical model (6) and corresponds to the point with coordinates (7; 20) on Fig. 1.

Then the student will start mastering the material of the second lecture of the lecturer from a point with coordinates (1; 2) on the graph shown in Fig. 2, a broken line, and in the case of a full understanding of the material presented by the lecturer in the second lecture, he will complete the process of mastering this material at a point with coordinates (2; 12). In general, this graph will show a broken line, which is described by a mathematical model  $v_s(t^*)$  in relative time, transformed from models (3), (6) to the form

$$v_s(t^*) = \sum_{m=0}^{N-1} v_{m+1}(t^*), \quad (8)$$

$$v_{m+1}(t^*) = m \cdot v^* + q \cdot \left\{ (t^* - m) \cdot (1(t^* - m) - 1(t^* - (m+1))) \right\}, \quad (9)$$

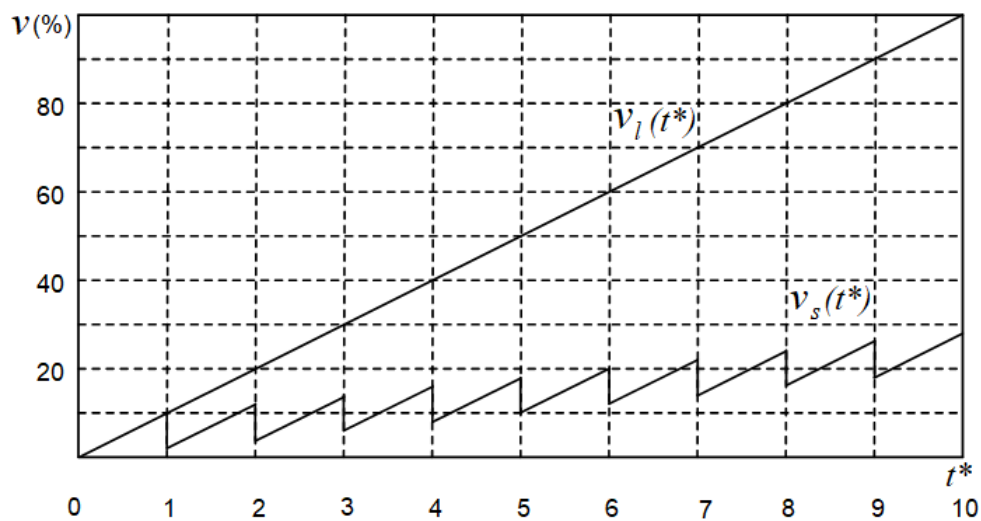
$$m = 0, 1, \dots, N-1; \quad v^* = x_{1(\%)}(\tau) \Big|_{\tau=7}; \quad t^* = \frac{t}{t_z}, \quad (10)$$

where  $N$  – is the number of lectures of the discipline, the length  $t_z$  of each.

From the graph of the function  $v_s(t^*)$  shown in Fig. 2, it is easy to see that a student from the category of "E" and "D" students, after listening to the full cycle of lectures, on the eve of the exam will retain less than 30 % of the total amount given by the lecturer in 10 lectures, which will not be enough to obtain admission to the



exam.



**Fig. 2 Graph of presentation of material by the lecturer in 10 lectures (continuous straight line from the point (0; 0) to the point (10; 100) and the graph of perception by the student from the category of "E" and "D" of this material (zigzag line from the point (0; 0) to points (10; 28))**

This is easy to see if we take into account that the amount of information  $I_l$  provided by the lecturer after reading 10 lectures is

$$I_l = \int_0^{10} v_l(t^*) dt^* = 500, \tag{11}$$

and the amount of information  $I_s$  that will be stored in the memory of a student in the category of "E" and "D" students, after he listens to these 10 lectures, is

$$I_s = \int_0^{10} v_s(t^*) dt^* = 130, \tag{12}$$

that is, on the eve of the exam, this student will keep in his memory only

$$\frac{I_s}{I_l} 100 = \frac{130}{500} 100 = 26\% \tag{13}$$

from the amount that he is obliged to know in accordance with the requirements of the educational program in this discipline and which is brought to him by the lecturer in lectures.

Now let's analyze how much of the information from the lecturer in 10 lectures will be remembered before the exam by the same student from the cohort of "E" and "D" students, if each current lecture the lecturer begins with a brief concentrated summary of the main provisions of previous lectures. Obviously, if the lecturer spends 2 minutes at the beginning of the next lecture to present the main points of the previous lecture, then he will have less time by 2 minutes to present new material at the second lecture, less by 4 minutes at the third lecture, and at the fourth lecture – for 6 minutes, for the fifth lecture – for 8 minutes, for the sixth lecture – for 10 minutes, for the seventh lecture – for 12 minutes, for the eighth lecture for 14 minutes, for the ninth lecture – for 16 minutes and for the tenth lectures – for 18 minutes. But due to



this there will be a deeper understanding and understanding by the student of the new material that the lecturer will convey to students in the current lecture, because bringing the key concepts of previous lectures by the lecturer at the beginning of each current lecture will help to transfer from the deep memory of the student to his memory all the material that was listened to by this student in previous lectures. And thanks to the students' full understanding of what the lecturer is teaching, he can increase the speed of submission of new material after teaching at the beginning of the lecture the key concepts of the material presented in previous lectures, and in this way to convey to students all the amount of knowledge that is determined by the educational program of this discipline.

To be able to model the above process and obtain numerical characteristics, we create an appropriate mathematical model. It is easy to see that in this case, being synthesized using expressions (1) – (5), the mathematical model of the process of mastering the material presented by the lecturer in 10 lectures, a student from a cohort of "E" and "D" students will have the form

$$v_s^*(t^*) = \sum_{m=1}^N \left\{ (m-1)I_c + \sum_{i=1}^{m-1} v_i^{pm} \cdot 1 \left( t^* - \sum_{k=1}^i t_k^* - i \right) + q_m \left( t^* - \sum_{k=1}^{m-1} t_k^* - (m-1) \right) \left\{ 1 \left( t^* - \sum_{k=1}^{m-1} t_k^* - (m-1) \right) - 1 \left( t^* - \sum_{k=1}^{m-1} t_k^* - m \right) \right\} \right\}, \quad (14)$$

where  $N$  is the number of lectures (in our case  $N=10$ );  $m$  is the number of the current lecture;  $i$  is the number of the previous lecture;  $v_i^{pm}$  – is the amount of material from the previous lecture by number  $i$ , which must be compensated at the beginning of the current lecture by number  $m$  by a short concentrated repetition of the basic concepts of the previous lecture;  $q_m$  – is the slope factor of the rectilinear segment of the graph of the function  $v_s^*(t^*)$ , which characterizes the rate of submission of material by the lecturer at the lecture by number  $m$  and which can be determined by the ratio

$$q_m = \frac{10}{1 - \sum_{k=1}^{m-1} t_k^*}, \quad (15)$$

and when  $m = 1$ , as required by formal logic, we assume that

$$\sum_{i=1}^{m-1} v_i^{pm} = \sum_{i=1}^0 v_i^{p0} = 0, \quad \sum_{k=1}^{m-1} t_k^* = \sum_{k=1}^0 t_k^* = 0 \quad (16)$$

The graph of the function represented by expression (14) is shown in Fig. 3.

It is obvious that in this case the amount of information  $I_s^*$  that will be stored in the memory of the student in the category of "E" and "D" students after he listens to these 10 lectures, can be found by the expression

$$I_s^* = \int_0^{10} v_s^*(t^*) dt^* = 437, \quad (17)$$

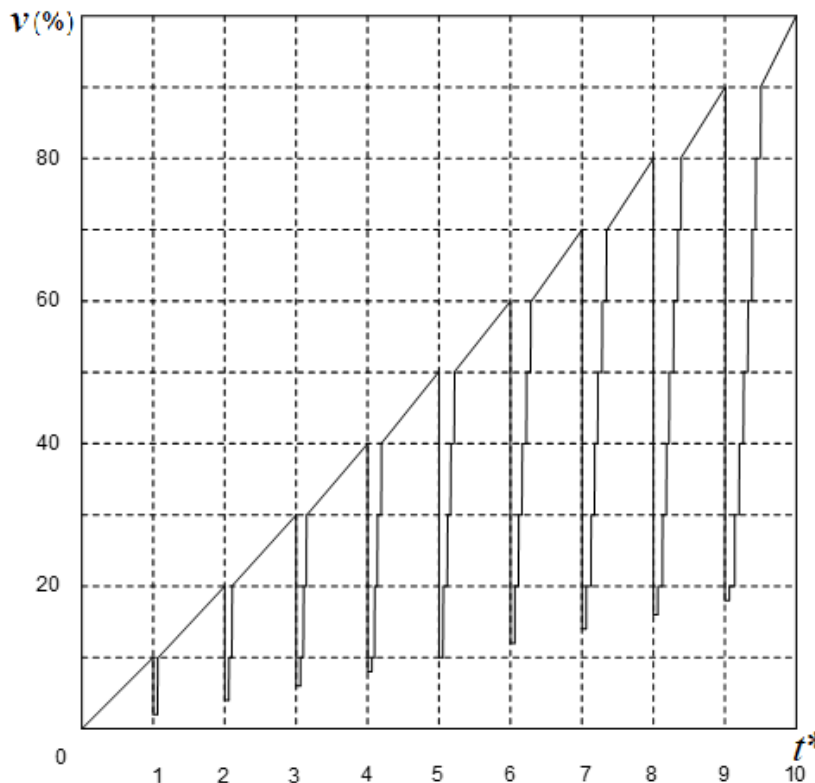
comparing the numerical value obtained from the eve of the exam, we can say that this student will keep in his memory





$$\frac{I_s}{I_l} 100 = \frac{437}{500} 100 = 87,4\% \tag{18}$$

from the amount brought to him by the lecturer in lectures.



**Fig. 3 Graph of a function that reflects the process of perception of information by a student from the category of "E" and "D" during ten lectures of the lecturer, who begins each current lecture with a short concentrated repetition of the main provisions of previous lectures**

Comparing the numerical characteristics obtained by expressions (13) and (18), we can say that the method of presenting the material in lectures with repetition at the beginning of the current lecture in a short concentrated form of the main provisions presented in previous lectures is much more effective than the method of linear building material from lecture to lecture, which is now dominant.

**Conclusions**

Based on a systematic approach, it is mathematically substantiated that the presentation at the beginning of each current lecture of key concepts of material delivered by the lecturer to students in previous lectures, significantly increases the degree of assimilation of new material that the lecturer informs them at this current lecture.

A new mathematical model has been synthesized, which describes the process of assimilation by the student the knowledge received at the lecture, at the beginning of which the lecturer restores in the students' memory in a concentrated form the main provisions of the previous lectures.

Using the mathematical model of the process of student acquisition of knowledge synthesized in this work, a method of teaching the material by the lecturer



in lectures was developed, which takes into account the main provisions of previous lectures when bringing new material to students at each of the next lectures.

Using numerical characteristics, it is shown how much more material the student will have in his memory before the exam, if, during each current lecture, the lecturer mentioned and detailed the basic concepts of the lecture course, set out in previous lectures.

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**Анотація.** В статті представлено результати дослідження, в якому на основі системного підходу з використанням авторських математичних моделей процесу забування знань, отриманих студентом на лекції, математично обґрунтовано, що реалізація тези, висловленої більше ніж півстоліття тому педагогом із Донецька Шаталовим, змістовною суттю якої є необхідність повторювання більше одного разу понять, висловлених лектором на лекції, для їх надійного запам'ятовування студентами, в інтерпретації цієї тези у вигляді повторення на початку поточної лекції ключових понять матеріалу, викладеного лектором студентам на попередніх лекціях, суттєво підвищує ступінь засвоєння ними нового матеріалу, який лектор доносить їм на цій поточній лекції. Показано, на скільки більшим буде обсяг матеріалу, який матиме у своїй пам'яті студент напередодні екзамену у разі, якщо, читаючи кожну поточну лекцію, лектор згадував і деталізував основні поняття лекційного курсу, викладені ним в попередніх лекціях.

**Ключові слова:** поточна лекція, попередні лекції, повтор понять, процес забування, математична модель, обґрунтування необхідності повторів, кількісна характеристика залишкових знань.