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ABSTRACT

It is well known that the application of the weight redundancy in self-calibrating DAC enables to lower considerably, the requirements regarding the accuracy of digits weight formation, particularly in multidigit code-current converters and the idea of laser adjustment of the resistor matrices nominals. It is also known, that the construction of ADC of the bit-wise balancing on the base of such DAC additionally enables to increase (by an order or more) the operation speed as a result of a considerable reduction of the balancing cycle duration.

Keywords: analog to digital converter, circuit of the resistive matrices, weight redundancy

1. INTRODUCTION

It should be noted that the improvement of the above-mentioned characteristics greatly depends on the level of weight redundancy of the calculus, used in these information converters. In analog-digital converters (ADC) it is expedient to spent the smaller part of this redundancy to lower the requirements, concerning the manufacturing technology of digital to analog converters (DAC), in particular, for the correction of the statistical errors and greater part– for the compensation of the dynamic errors of the I and II kinds to increase the operation speed[1-2]. It is worth mentioning that the selection of the calculus is the key issue. For convenience adaptability to manufacture of ADC and DAC, the system must be positional to provide the regularity of the structure[4]. Besides, it is necessary that the base of the calculus α(relation of the weights of the neighboring bits)

$$\alpha = \frac{Q_i}{Q_{i-1}},$$

(where $Q_i=\alpha^i \cdot Q_0$ – weight of the i-th bit, $Q_0 = 1$ – weight of the lower zero bit) was a constant number. This will give the possibility to have constant ratio between the weights of bits, especially discharge currents or voltages, in its turn, this will provide the regularity of the circuits of the resistive or capacitor arrays of DAC. Another requirement for DAC construction is minimization of the number of nominals of the discharge resistors or capacitors, depending on the method of the realization[5]. The above-method requires complex approach to the solution of the calculus or calculation of the calculus base[6,7]. In spite of the fact, that there exists scientific-engineering literature on this subject, the problem of the system selection of the calculus and calculation of the calculus base was not considered separately. In this connection the subject of the paper dealing with the problem of the selection of calculus system base for ADC and DAC with the weight redundancy is relevant.

The aim of the research is systematization of the calculus system selection and its base calculation for high performance ADC and DAC with weight redundancy. The task of the research are as follows:

- Propose the set of the characteristic polynomials of the k^{th} degree, solutions of which would enable to obtain the necessary value of α;
- systematization of the set of the α values obtained according to the level of the set weight redundancy;

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- assessing the level of the redundancy, depending on the value of the calculus system base α;
- providing the examples of ADC and DAC with weight redundancy construction.

2. THE METHOD

In the positional calculus system, the real number is accurately presented[1] in the form

$$D=\sum_{i=-\infty}^{n-1} a_i \cdot \alpha^i\,, \tag{1}$$

where α_i – bit coefficients, $a_i \in \{0,1\};\{\bar{1},1\};\{\bar{1},0,1\}$ – binary number in the i^{th} bit (bit coefficient); α – basic calculus system, i – the number of bit. In the systems with natural base (the set of bit weights) α is a constant number unlike the artificial base, where the value ofα can vary from bit to bit[8].

As such form of the presentation requires the usage of the infinitely long bit grid, for ADC and DAC it is not real. It is expedient to pass from the real numbers to natural numbers. It was shown[9] that if *a* is a fractional irrational number, for instance, golden p-proportion, then any natural number is presented in the form:

$$N=\sum_{i=-n}^{n-1} a_i \cdot \alpha_p^i \tag{3}$$

where $a_p^i = a_p^{i-1} + a_p^{i-p-1}$ denotes i^{th} degree of the golden p-proportion. It should be noted that in such calculus system each successive term of series (weight of i^{th} bit) equals the sum of the weights of the neighboring bit and the bit located in p. Such recurrent connection allows to obtain minimize in DAC the values of the nominal of elements (resistors or capacitors) which set the discharge currents or voltages by means of their parallel or serial connection, that is convenient in the process of microelectronic realization[10,4]. The value of golden *p*-proportion[11,12,16] is calculated as the real positive root of the polynomial

$$x^{p+1} - x^p - 1 = 0 \tag{3}$$

We will consider the form of the polynomial for certain specific p and calculate the corresponding α, in the form of real positive roots, i.e. $\alpha > 0$ (calculations are carried out in Mathcad 15). The values of α_p for the calculus system of the golden *p*-proportion are presented in the Table 1.

Table 1. Values of α_p for calculus system of the golden *p*-proportion.

p	0	1	2	3	4	5	6	…	∞
α_p	2	1.618	1.466	1.380	1.324	1.285	1.256	…	1

Apart from "golden" *p*-proportions, "golden" *s*-sections are also redundant. In golden *s*-proportions every i^{th} member is the sum of the antecedents $i-1$, $i-2$,…, $i-s$. In golden *s*-proportions real positive values of α_s are calculated as the root of the following polynomial:

$$x^{s+1} - \sum_{0}^{s} x^i = 0. \tag{4}$$

The values of α_s are shown in the Table 2.

Table 2. Values of α_s for NS of the golden *s*-section.

s	1	2	3	4	5	6	…	∞
α_s	1.618	1.839	1.928	1.966	1.984	1.992	…	2

It should be noted that the values of the golden *s*-proportions is the continuation of the numerical series from 1.618 to 2.00[17]. At the same time, besides the golden p-proportions and *s*-proportions additionally golden *s*-*p*-proportions can be used. In such proportions *s* is the number of neighboring weight bits after $(i-1)^{th}, (s=2,3,4,...,n)$; p – is the distance between the group of s bits and high of low order bits. In golden s-p-proportions real positive values of α_s are calculated from the following polynomial:

$$\sum_{p+1}^{s+p+1} x^i - 1 = 0. \tag{5}$$

The values of α_{s-p} are presented in Table 3.

Table 3. Values of α_{s-p} for the calculus system of the golden s-p-proportion for s = 2 and s = 3.

p	0	1	2	3	4	5	6	…	∞
s = 2									
α_{s-p}	1.8393	1.7549	1.7049	1.6737	1.6536	1.6407	1.6324	…	1
s = 3									
α_{s-p}	1.9276	1.88852	1.8668	1.85455	1.84772	1.84392	1.84182	…	1

It should be noted, that these calculus systems have natural redundancy and meet the requirements:

$$\alpha^i < \sum_{0}^{i-1} \alpha^j \tag{6}$$

For the i^{th} bit the absolute WR is set by the expression:

$$\Delta\tilde{Q}_i = \sum_{j=0}^{i-1} Q_j - Q_i. \tag{7}$$

Relative WR is considered as the ratio of the absolute WRt o the sum of the all bit weights from 0^{th} to i^{th} and is evaluated from the expression:

$$\delta Q_i * = \frac{\sum_{j=0}^{i-1} \alpha^j - \alpha^i}{\sum_{j=0}^{i} \alpha^j}. \tag{8}$$

For its simplification the following transformations can be made:

$$\sum_{j=0}^{i-1} \alpha^j = \alpha^{i-1} \cdot \frac{1 - \left(\frac{1}{\alpha}\right)^i}{1 - \frac{1}{\alpha}} = \alpha^{i-1} \cdot \frac{\alpha^i - 1}{\alpha^i} \cdot \frac{\alpha}{\alpha - 1} = \frac{\alpha^i - 1}{\alpha - 1}.$$

Thus,

$$\delta Q_i * = \frac{\frac{\alpha^i - 1}{\alpha - 1} - \alpha^i}{\frac{\alpha^{i+1} - 1}{\alpha - 1}} = \frac{\alpha^i - 1 - \alpha^{i+1} + \alpha^i}{\alpha - 1} \cdot \frac{\alpha - 1}{\alpha^{i+1} - 1} = \frac{2\alpha^i - \alpha^{i+1} - 1}{\alpha^{i+1} - 1}.$$

As α = const, it can be assumed that:

$$\delta Q_i * = \frac{2 - \alpha}{\alpha} \tag{9}$$

Such value of WR should be taken for the assessment of the possibility of the static error correction in the process of self-calibration. Relative WR reduced to the i^{th} bit is calculated in the form:

$$\delta Q_i ** = \frac{\sum_{j=0}^{i-1}\alpha^j - \alpha^i}{\alpha^i} = \sum_{j=0}^{i-1}\alpha^{j-i} - 1 = \alpha^{-1}\cdot\frac{1-\left(\frac{1}{\alpha}\right)^i}{1-\frac{1}{\alpha}} - 1 = \frac{\alpha^i - 1}{\alpha^i(\alpha-1)} - 1 = \frac{1}{\alpha-1} - \frac{\alpha^{-i}}{\alpha-1} - 1.$$

For the separate *p*, within the frame of the "golden" *p*-proportions, the relative WR corresponds to the values as shown in Table 4 and for the separate *s* in Table 5, respectively.

Table 4. Value of the weight redundancy for CS of golden *p*-proportions.

p	0	1	2	3	4	5	6	..	∞
α_p	2	1.618	1.466	1.380	1.324	1.285	1.256	..	1
δQ_s* (%)	0	23.61	36.43	44.93	51.06	55.64	59.24	..	1
δQ_s** (%)	0	61.81	114.59	163.16	208.64	250.88	290.62	..	∞

Table 5. Values of the weight redundancy for the calculus system of golden *s*-proportions.

s	1	2	3	4	5	6	..	∞
α_s	1.618	1.839	1.928	1.966	1.984	1.992	..	2
δQ_p* (%)	23.61	8.75	3.73	1.73	0.81	0.4	..	0
δQ_p** (%)	61.81	19.19	7.76	3.52	1.63	0.8	..	0

With the increase of *i*, the last term rapidly changes, that is why, we may assume

$$\delta Q_i ** = \frac{1}{\alpha-1} - 1 = \frac{2-\alpha}{\alpha-1}. \quad (10)$$

This index characterizes the level of WR. It is used for the calculation of the dynamic characteristics in the process of the accelerated bit-wise analog-to-digital conversion[13]. Solving the inverse problem we will find the value of the calculus system base α for the target δQ_i* and δQ_i**:

$$\alpha = \frac{2}{\delta Q_i * + 1}, \quad (11)$$

$$\alpha = \frac{2 + \delta Q_i **}{1 + \delta Q_i **}. \quad (12)$$

For "golden" s-proportions, if *s*=2 and 3 we have (table 6)

Table 6.Value of weight redundancy for the calculus system of golden *s*-*p* proportions.

P	0	1	2	3	4	5	6	…	∞
s = 2									
$\alpha_{s\text{-}p}$	1.8393	1.7549	1.7049	1.6737	1.6536	1.6407	1.6324	…	1
δQ_l* (%)	8.74	13.97	17.31	19.5	20.95	21.9	22.52	…	∞
δQ_l** (%)	19.15	32.47	41.86	48.43	53	56.08	58.13	…	∞
s = 3									
$\alpha_{s\text{-}p}$	1.928	1.8885	1.8668	1.85455	1.84772	1.84392	1.8418	…	1
δQ_l* (%)	3.76	5.9	7.14	7.84	8.24	8.46	8.59	…	∞
δQ_l** (%)	7.81	12.55	15.37	17.02	17.96	18.49	18.79	…	∞

3. PRACTICAL REALIZATION

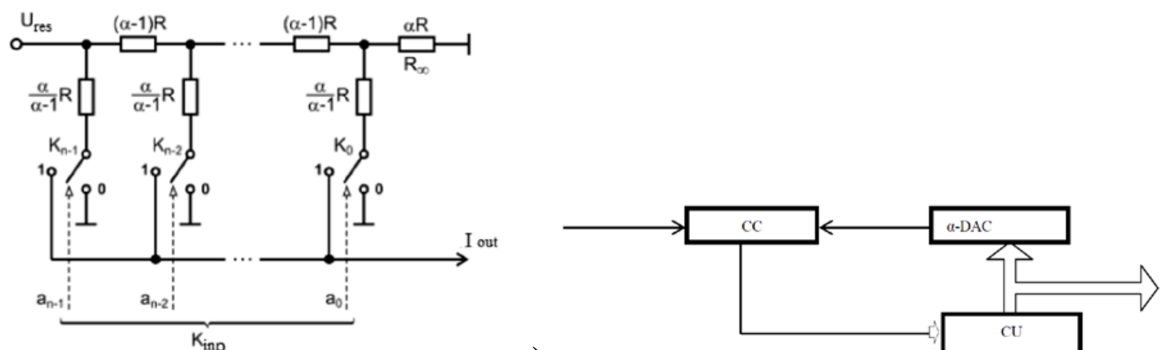
Weight redundancy gives the possibility to eliminate gaps in the characteristic of transformations. Application of the calculus system(CS) with WR can be demonstrated on the example of the resistive matrices, which are the part of DAC[3].The output signal of this code-current converter (CCC) is set by the input code:

$$I_{out} = I \cdot \sum_{i=0}^{n-1} a_i \cdot \alpha^{i-(n-1)} \qquad (13)$$

where α_i – bit coefficients K*inp*; $I = U_{res} / R$ – current of the high $(n-1)^{th}$ bit, which is for the circuit (Fig. 1b) U_{res} and $\frac{\alpha}{\alpha - 1} \cdot R$. Fig. 1a shows the diagram of *n*-bit code-current converter (CCC) on the base of the resistive matrix of ladder type in the inverse switching with α =1,839 for *s* = 2.

$$\frac{\alpha}{\alpha - 1} R = 2{,}192R\,;\ (\alpha - 1)R = 0{,}839R\,;\ \alpha R = 1{,}839R$$

where *R* – basic nominal of the matrix; $\alpha_i \in \{0{,}1\}$- bit coefficients.



a) b)

Figure 1. a) code-current converter on the base of the resistive matrix with the inverse switching; b) generalized structural diagram of ADC.

Table 6. Codes of the initial sequence of numbers for golden *s*-proportion for *α*=1,839, *s*=2.

bit order		6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
Weights of bits		α^6	α^5	α^4	α^3	α^2	α^1	α^0	α^{-1}	α^{-2}	α^{-3}	α^{-4}	α^{-5}	α^{-6}
Digital equivalent	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	1	0	0	0	0	0	0
		0	0	0	0	0	0	0	1	1	1	0	0	0
	2	0	0	0	0	0	0	1	1	1	1	0	0	0
		0	0	0	0	0	1	0	0	0	1	0	0	0
	3	0	0	0	0	0	1	1	0	0	1	0	0	0
		0	0	0	0	0	1	1	0	0	0	1	1	1
		0	0	0	0	0	1	0	1	1	1	1	1	1
	4	0	0	0	0	0	1	1	1	1	1	1	1	1

The diagram (Fig. 1a) contains the resistive matrix with the nominals $(\alpha - 1)R$, $\alpha R/(\alpha - 1)$ and αR, and the source of the reference voltage U_{res} and key elements $K_{n-1} - K_{n-1}$. During the serial supply of K_{inp} such correspondence of the conversion $K_{inp} \rightarrow A_{out}$ (I_{inp}) is formed. ADC contains the comparison circuit (CC) of the analog signals; α-DAC- digital analog converter on the base of CS with WR; CU – control unit.

In the process of the analog – digital conversion the input analog signal is sent to the first input of α-DAC. After the instruction of CU, α-DAC generates the compensation signal, that enters the second input of CC and balances A_{inp}. In the process of the conversion code K_{out} is formed at the conversion output, which is a digital equivalent of the input signal A_{inp} in the form[18,19]:

$$K_{out} = \alpha_{n-1}, \alpha_{n-2}, ..., \alpha_1, \alpha_0 ,$$

where $\alpha_i \in \{0,1\}$ digit coefficients of the code, formed at the input of α-DAC[14,20]. Operations of convolution and scanning can be performed over the bits of codes digits in the given system. Convolution is the replacement of zero in the i^{th} and ones in $(i - 1)^{th}$, $(i - 2)^{th}$ and $(i - 3)^{th}$ bits by their inversions. Scanning is an operation, inverse to convolution[15]. Presentation of the initial section of the natural numbers in golden calculus system is convenient to perform by means of convolution and scanning operations in the form as in Table 7[21,22].

4. CONCLUSIONS

The set of the characteristic polynomials of the k^{th} degree is suggested, their solution enables to obtain the value of α which is the base of CS, which correspond to the redundant "golden" p-proportion, s-proportions and s-p proportions. The set of the obtained values of α_p, α_s and α_{s-p} is systematized according to the level of the set weight redundancy if: $1 \le \alpha \le 2$. The level of the redundancy, depending on the value of the calculus system base α is determined. The examples of the construction of the code-current converter with weight redundancy of α=1,839 for s=2 on the base of the resistive matrix in the inverse switching and analog-digital converter, containing DAC with weight redundancy (WR) are given.

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