http://doi.org/10.35784/iapgos.3504

received: 16.02.2023 | revised: 13.03.2023 | accepted: 29.04.2023 | available online: 30.06.2023

STATISTICAL METHODS FOR EVALUATING EXPERIMENTAL DATA ON THE USE OF MATHEMATICAL COMPETENCIES IN STUDY FOR A RESILIENT ECONOMY

Vira Petruk[1], Olena Prozor[1], Yuliia Sabadosh[1], Iryna Baranovska[2], Maksim Palii[3], Yevheniia Moroz[4], Saule Kumargazhanova[5], Dinara Mussayeva[6]

[1]Vinnytsia National Technical University, Vinnytsia, Ukraine, [2]Vinnytsia State Pedagogical University named after Mykhailo Kotsiubynsky, Vinnytsia, Ukraine, [3]Vasyl Stus Donetsk National University, Vinnytsia, Ukraine, [4]Taras Shevchenko National University, Kyiv, Ukraine, [5]D. Serikbayev East Kazakhstan Technical University, Ust-Kamenogorsk, Kazakhstan, [6]Al-Farabi Kazakh National University, Almaty, Kazakhstan

Abstract. *The description of the problems, solved within the statistics evaluation of the results of educational (pedagogical) experiment in resilient economy is given. Comparison of the two average samplings and hypothesis verification relative to dispersions of the two samplings are considered. It is shown the hypothesis that teaching and methodical training system of subjects of higher mathematics in technical universities, suggested on the base of the modern interactive technologies, significantly improves the quality of the base level of professional competence of future engineers. The methods for statistical processing of the obtained results, which very thoroughly research all the indexes which compose the suggested learning and methodical system for teaching higher mathematics in technical universities are used.*

Keywords: statistical methods, statistical analysis, abstract essence, elements of interaction, pedagogical experiment

STATYSTYCZNE METODY OCENY DANYCH EKSPERYMENTALNYCH DOTYCZĄCYCH WYKORZYSTANIA KOMPETENCJI MATEMATYCZNYCH W BADANIACH NA RZECZ ODPORNEJ GOSPODARKI

Streszczenie. *Podano opis problemów rozwiązywanych w ramach statystycznej oceny wyników eksperymentu edukacyjnego (pedagogicznego) w gospodarce odpornej. Rozważa się porównanie dwóch średnich prób i weryfikację hipotezy w odniesieniu do dyspersji dwóch prób. Wykazano hipotezę, że system nauczania i szkolenia metodycznego przedmiotów matematyki wyższej na uczelniach technicznych, zaproponowany w oparciu o nowoczesne technologie interaktywne, znacząco poprawia jakość podstawowego poziomu kompetencji zawodowych przyszłych inżynierów. Zastosowano metody statystycznego przetwarzania uzyskanych wyników, które bardzo dokładnie badają wszystkie wskaźniki składające się na proponowany system nauczania i metodycznego nauczania matematyki wyższej w uczelniach technicznych.*

Słowa kluczowe: metody statystyczne, analiza statystyczna, istota abstrakcyjna, elementy interakcji, eksperyment pedagogiczny

Introduction

Sustainable development and a resilient economy are impossible without the training of relevant personnel, because the quality of managerial decision-making depends on the professional training of specialists. Pedagogical experiments using elements of mathematical statistics are traditionally conducted in resilient economy. Mathematical statistics has a great variety of criteria, designed to verify the statistic hypothesis. The evaluation of manufacturing, medical, pedagogical and the like managerial decisions usually uses the hypothesis on numerical characteristics of the received figures. The most fully developed theory is the one for hypothesis verification relative to numerical characteristics of the normal law of distribution [1, 2, 10].

Let's consider an algorithm for verification of statistical hypothesis.

1. The first stage forms the matrix of the received figures:

$$X = \begin{pmatrix} x_{11} & \dots & x_{1m} \\ \dots & \dots & \dots \\ x_{n1} & \dots & x_{nm} \end{pmatrix}; \quad i = \overline{1,n}; \quad j = \overline{1,m} \tag{1}$$

2. For each parameter of X_j the hypothesis shall be tested for normality. If parameter X_j has distribution, different from the normal one, it then reconstructs into the parameter V_j with the normal law of distribution. The reconstruction shall be done as follows. The elements of the *j-th* matrix column (1) shall be arranged in ascending order:

$$X_1 \le X_2 \le \dots \le X_n, \tag{2}$$

frequencies m_k which correspond to the values X_K, are calculated for the statistical series with the further calculation of the accumulated frequencies

$$M_K = M_{K-1} + m_K, \;\; K = \overline{2,l}, \tag{3}$$

$M_1 = m_1$ and the value of the distribution function

$$F\left(X_K\right) = \frac{M_K}{n} \tag{4}$$

The value Z_{kj} of the normally distributed random variable $Z_j = V_j - m_{vj}$ with the zero mathematical expectation and unit variance, which correspond to the values of X_{kj} of the random variable X_j with the random distribution law, shall be found from the correlation

$$F\left(X_{kj}\right) = \mathrm{P}\left(X_j < X_{kj}\right) = \mathrm{P}\left(Z_j < Z_{kj}\right) = F_0\left(Z_{kj}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_{kj}} e^{\frac{Z^2}{2}} dZ \tag{5}$$

from which Z_{kj} shall be found as a function, inverse to the error function integral

$$Z_{kj} = F_0^{-1}\left(Z_{kj}\right) . \tag{6}$$

Equation (4.6) with the set accuracy may be solved by many methods. The approximate values of Z_{kj} with different stages of accuracy are found by the corresponding formulas. One of them:

$$Z_{kj} = \sqrt{\sum_{i=1}^{\infty} a_i \left(-\ln\left(1 - 4\left(F\left(X_K\right) - \frac{1}{2}\right)\right)^2\right)^i}. \tag{7}$$

The section of the series with the first four members provides for the necessary accuracy within the interval $0.03 < F(X_k) \le 0.97$, coefficients a_j equal: $a_1 = \frac{\pi}{2}$; a_2 = $0.37068870 \times 10^{-1}$; $a_3 = 0.8320945 \times 10^{-2}$; $a_4 = -\, 02323240 \times 10^{-3}$.

The second and more accurate formula:

$$Z_{kj} = t - \frac{\sum_{i=0}^{2} C_i \cdot t^i}{\sum_{i=0}^{3} d_i t^i} + \varepsilon\left(F\left(X_K\right)\right), \quad (8)$$

where $t = \sqrt{-2\ln\left(1 - F\left(X_k\right)\right)};\ \ 0.5 \le F\left(X_K\right) < 1;$

$|\varepsilon| < 4.5\times10^{-4}$; $C_0 = 2.515517$; $C_1 = 0.802853$; $C_2 = 0.010328$

With $0 \le F\left(X_K\right) < 0.5$ there shall be used the property of the function, inverse to the error function integral, $Z\left(F\left(Z\right)\right) + Z\left(1 - F\left(Z\right)\right) = 0,$ from which

$$Z_{kj}\left(F\left(X_k\right)\right) = Z_{kj}\left(1 - F\left(X_k\right)\right) \quad (9)$$

The value of Z_{kj}, calculated by the close formulas (7) and (8), may also be used as the initial during the solution of the equation (5). Evaluations of the mathematical expectations $\overline{V}_j$, dispersion S_{vj}, random variable V_j with the normal law of distribution may be found by solving the systems of the equations.

$$\begin{cases} Z_{kj} = \dfrac{V_{kj} - \overline{V_j}}{S_{vj}} \\ Z_{mj} = \dfrac{V_{mj} - \overline{V_j}}{S_{vj}} \end{cases} \quad (10)$$

in which

$$V_{kj} = X_{kj};\ V_{mj} = X_{mj};\ \ k \ne m;\ S_{vj} = \frac{X_{kj} - X_{mj}}{Z_{kj} - Z_{mj}};$$

$$\overline{V_j} = X_{kj} - S_{vj} Z_{kj}. \quad (11)$$

3. There shall be formed the hypothesis H_0 and an alternative hypothesis H_1. If hypothesis H_0 is simple, for example H_0: $\overline{x} = a$, it always may be reduced to H_0: $\overline{x} - a = 0$.
4. There shall be chosen the criteria K_n (critical statistics), which is a function of the observational results, $K_n = K\left(X_1, \ldots X_n\right)$. Criteria K_n, on condition of H_0 hypothesis validity, is a random quantity with the very well learned distribution law, which is set by the table in the kind of approximate formulas or programs for using a computer. Critical statistics, based upon the principle of correlation with the use of the Neumann-Pirson theorem is the most powerful.
5. There shall be set the level of significance by the table of critical quintiles; or using the approximation formula we find the critical quintiles $K_{1-\frac{a}{2}},\ \ K_{\frac{a}{2}}$.

In cases, when the hypothesis H_0 relates only to one-way deviation of the criteria K_n, that is, when we are interested in "extremely large" or "extremely small" values of the criteria K_n, there shall be found from K_a or K_{1-a} critical fractal.

6. Following the observational data $\left\{X_1, \ldots X_n\right\}$ there shall be calculated the values of the criteria K_n. If this value gets into the critical area, the hypothesis H_0 then deviates to an alternative hypothesis H_0. If the value K_n gets into the legitimate range, the hypothesis H_0 may then be considered as the one, which does not contradict to the research data. Test of hypothesis in production control, pedagogic, medicine etc. allows to solve the tasks for comparing the selected numeric characteristics (average, dispersion) with the corresponding desired values; numeric characteristics of two or some samplings between each other (test of hypothesis on belonging of these samplings to one set).

Comparison problems lie behind the efficiency evaluation of managerial decisions in many spheres. The most universal are the problems for checking hypothesis on correspondence with the empirical distribution with certain theoretical model or hypothesis on significance of deviation between the empirical laws of distribution. If the parameters under investigation with the random law of distribution are transferred into parameters which are distributed against the normal law of distribution during the solution of the specific problems, then there is no need to test the hypothesis in the form of law of distribution, since the normal law may totally be determined by its mathematical expectation and dispersion. Among various approaches to the evaluation of the efficiency of managerial decisions obtained wide development the evaluation of the importance of change in mathematical expectations of the parameters under research, which reflect the trajectories of the centers of their grouping. However, even nowadays the potential of the given approach is not fully implemented.

1. Literature review

The definition of resilience has received considerable attention in the economics literature, for instance in publication: [3] a basis for policy discussions of resilience at the local level is presented. Resilient economy is not a very widely used concept. More often, scientists use the concept of resilience. For instance, Mustonen T., Shadrin V., Mustonen K., Vasiliev V. attempted a synthesis framework for analyzing resilience from an economics viewpoint [6, 9, 11]. Generally, resilience can be defined as the ability of an economy, society, organization or individual to recover effectively from an unexpected shock. This rough definition of resilience raises two related questions: what it means to “recover” and what we mean by an effective recovery [4]. Perrings C. gives a definition of resilience as “the ability of the system to withstand either market or environmental shocks without losing the capacity to allocate resources efficiently or to deliver essential services” leaves the term of analysis indeterminate [8]. Walker B., Holling C., Carpenter S., Kinzig A. have defined resilience as “the capacity of a system to absorb disturbance and reorganize while undergoing change, so as to retain essentially the same function, structure, identity and feedbacks”. Hence, this definition encompasses the stability of a structure, rather than focus purely on the performance outcomes of that structure [7, 10].

It is in the conditions of a resilient economy that personnel who improve its results are especially valuable. The effectiveness of the system of educational and methodical training of personnel involved in the country's economy in institutions of higher education can be evaluated using various mathematical models and pedagogical experiments. The experience of pedagogical science research draws attention to the monotony of the application of mathematical statistics methods for analyzing the results of a pedagogical experiment. Also, quite often there are cases of only a percentage assessment of the data obtained in a pedagogical experiment without accompanying statistical analysis, which would scientifically confirm or refute the significance of the obtained positive changes. Works justifying the use of statistical criteria for processing and quality assessment of data for various types of pedagogical experiments are particularly relevant. Mathematical statistics includes a significant number of criteria designed to test various statistical hypotheses. As a rule, hypotheses concern either the law of distribution of data in the sample (normal, binomial, etc.) or the numerical characteristics of the sample (mean, variance, correlation, etc.) manual, which describes the use of criteria for the normal distribution law [5, 6].

As Gogol K., Brunner M., Martin R., Preckel F., Goetz T. [2] and Arens K. [1] truly state, that careful consideration is needed when selecting a specific model. Other core constructs in education research (e.g., academic anxiety, academic interest) have theoretical underpinnings that are similar to ASC as they can also be conceptualized as multidimensional and hierarchical in nature.

2. Material and methods

The aim of the research is the solution of problems of the statistics evaluation of the results of pedagogical experiment.

Objectives of the research:

1. Formulate and briefly describe the problems, solved by means of mathematical statistics for the assessing the efficiency of managerial decisions.
2. According to the suggested method and the presented algorithm check the efficiency of the teaching and methodical training system of subjects of higher mathematics in technical universities.

2.1. Comparison of the two average samplings

One of the main tasks during the evaluation of the efficiency of managerial decisions the evaluation of the importance of change in mathematical expectations of the parameters under research, which reflect the trajectories of the centers of their grouping. In the simplest case there shall be done the comparison of the averages in the two samplings, which correspond to the two types of managerial decisions. The task for comparing is formed as follows. There shall be researched the random quantities X_1, X_2, which have normal distributions $X_1 \in N(m_1\sigma_1),\ X_2 \in N(m_2\sigma_2)$.

Upon the results of the researches we receive the independent samplings $(X_{11},\dots X_{1n1})^T$, $(X_{21},\dots X_{2n2})^T$. For these data on formulas:

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{12}$$

$$S^2 = \frac{N}{N-1}\left(\overline{X^2} - (\overline{X})^2\right) \tag{13}$$

where $\overline{X^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2$, $i = \overline{1,n}$, there shall be calculated the point estimation $\overline{X}_1(n_1)$, $S_1(n_1)$ for the first sampling and $\overline{X}_2(n_2)$, $S_2(n_2)$ – for the second. It is necessary to check the hypothesis about the equity of mathematical expectations $m_1 = m_2$ of the random quantities X_1, X_2 against the alternative $m_1 \neq m_2$. There is a task to compare the mathematical expectation with the set value. To do this, we consider the random value $Y = X_1 - X_2$. It equals the difference of the two independent random values, which have the normal distribution, mathematical expectations m_1, m_2 and dispersions $\sigma^2{}_1$, $\sigma^2{}_2$. According to the theorem of adding the numerical characteristics of independent random values we receive $m_y = m_{X_1} - m_{X_2}$; $\sigma_y^2 = \sigma_1^2 + \sigma_2^2$.

The evaluation of the mathematical expectation of the random value Y is an average

$$\overline{Y} = \overline{X_1} - \overline{X_2} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_{i_1} - \frac{1}{n_2}\sum_{i=1}^{n_2} X_{i_2} \tag{14}$$

and the evaluation of the dispersion of this statistic is the sampling dispersion

$$S_{\bar{y}}^2 = \left(\frac{1}{n_1} + \frac{1}{n_2}\right)\frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)S_1^2 + (n_{2-1})S_2^2\right) \tag{15}$$

So the task of comparing the averages of the two samplings may be formed as follows.

Characteristics of the output data:

$Y \in N(0,\sigma_y)$, $Y = X_1 - X_2$, $X_1(n) = (X_{11},\dots X_{1n1})^T$, $X_2(n) = (X_{21},\dots X_{2n2})^T$; the known valuations $\overline{X_1}$, S_1^2, $\overline{X_2}$, S_2^2, $\overline{Y} = \overline{X_1} - \overline{X_2}$, $S_y^2 = \left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)\frac{n_1 + n_2}{n_1 + n_2 - 2}$.

Hypothesis H_0: $m_y = 0$. Alternative hypothesis H_1: $m_y \neq 0$. Level of significance α. Criteria (critical statistics):

$$t = \frac{\left|\overline{Y} - 0\right|}{S_{\bar{y}}} = \frac{\left|\overline{X_1} - \overline{X_2}\right|}{S_{\bar{y}}} \tag{16}$$

where $\overline{X_1}$, $\overline{X_2}$, S_1, S_2 shall be determined on the formulas (13), (14), and – $S_{\bar{y}}$ – on (15).

Solution. If the dispersions of the samplings to be compared are the same

$$\sigma_1^2 = \sigma_2^2 \text{ i } |t| \geq t_{\frac{a}{2},(n_1+n_2-2)} \tag{17}$$

Then the zero hypothesis H_0 deviates to the alternative one, in the other case H_0 shall be coordinated with the experimental data. For the realization of the algorithm for comparing the averages of the two samplings, received on the base of manufacturing researches, the critical statistics $t_{\left(1-\frac{a}{2},(n_1+n_2-2)\right)}$ of Student's distribution is appropriate to calculate on the approximation formulas:

$$t_{\beta',n} \approx \sum_{i=0}^{4} q_i\left(Z_{\beta'}\right)/v^i \tag{18}$$

where $\beta' = 1 - \frac{a}{2}$; $n = n_1 + n_2 - 2$;

$q_0\left(Z_{\beta'}\right) = Z_{\beta'} \approx \sum a_i\left(-\ln\left(1 - 2(1-a)\right)^2\right)^i$; $a_1 = \frac{\pi}{2}$;

$a_2 = 0.37068870 \times 10^{-1}$; $a_3 = 0.8320945 \times 10^{-2}$;
$a_4 = -0.2323240 \times 10^{-3}$;

$q_1\left(Z_{\beta'}\right) = \frac{1}{4}\left(Z_{\beta'}^3 + Z_{\beta'}\right)$;

$q_2\left(Z_{\beta'}\right) = \frac{1}{96}\left(5Z_{\beta'}^5 + 16Z_{\beta'}^3 + 3Z_{\beta'}\right)$;

$q_3\left(Z_{\beta'}\right) = \frac{1}{384}\left(3Z_{\beta'}^7 + 19Z_{\beta'}^5 + 17Z_{\beta'}^3 - 15Z_{\beta'}\right)$;

$q_4\left(Z_{\beta'}\right) = \frac{1}{92160}\left(79Z_{\beta'}^9 + 776Z_{\beta'}^7 + 1482Z_{\beta'}^5 - 1920Z_{\beta'}^3 - 945Z_{\beta'}\right)$;

$v = n - 1$ – number of degrees of freedom.

If H_0 deviates,one must be extremely careful in case of «extremely large» criteria value $t(n_1 + n_2 - 2)$, which cause the equation gain (17). The causes for this may be either the wide disagreements of the averages, that is, hypothesis H_0 non-execution, or big disagreements between the dispersions of the under research samplings.

That is why the research of the causes of heterogeneity of the manufacturing data samplings under consideration requires to verify the hypothesis of equality of dispersions $H_0: \ \sigma_1^2 = \sigma_2^2$ against an alternative $H_1: \ \sigma_1^2 \neq \sigma_2^2$. If during the verification of the hypothesis H_0 of the equality of the averages there appears the significant disagreement between the dispersions, the statistics shall be used as a criterion:

$$t = \frac{\left|\overline{X}_1 - \overline{X}_2\right|}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \tag{19}$$

The values of the critical statistics $\tilde{t}_{\frac{a}{2}}\left(v_1, v_2, c\right)$ are presented by the table of supplements in the textbooks [1]. These tables present $\tilde{t}_{\frac{a}{2}}\left(v_1, v_2, c\right)$ as *V(v₁,v₂,C,Q)*, where $v_j = n_j - 1$, *J*=1.2; *Q* = α / 2; α = 2.5 %, 5%,

$$C = \frac{S_1^2 / n_1}{S_1^2 / n_1 + S_2^2 / n_2} \tag{20}$$

In is known that the statistics (19) has distribution which is close to *t* – Student's distribution with the number of degrees of freedom

$$\tilde{v} = \frac{\left(S_1^2 / n_1 + S_2^2 / n_2\right)^2}{\frac{\left(S_1^2 / n_1\right)^2}{n_1 - 1} + \frac{\left(S_2^2 / n_2\right)^2}{n_2 - 1}} \tag{21}$$

Value $\tilde{v}$ lies between the smallest from $\left(n_1 - 1\right)$ and $\left(n_2 - 1\right)$ and their sum $\left(n_1 + n_2 - 2\right)$. So, the decision on the equality of the averages shall be made depending upon the results of hypothesis verification of the dispersion equality in the under research samplings. During the equality of the dispersions there shall be applied the rule, based on the inequality (18). If dispersions to be compared turn to be equal $\sigma_1^2 \neq \sigma_2^2$, the zero hypothesis then $H_0: \ m_1 = m_2$ deviates to the alternative one H_1: $m_1 \neq m_2$, if the inequalities come true

$$\left|\tilde{t}\right| > t_{\frac{a}{2}}\left(\tilde{v}\right) \text{ or } \left|\tilde{t}\right| > \tilde{t}_{\frac{a}{2}}\left(v_1, v_2, C\right) \tag{22}$$

The closest value of the critical fractile $t_{\frac{a}{2}}\left(v\right)$ may be received by expanding into series by negative exponent *v* as in (19).

2.2. Hypothesis verification relative to dispersions of the two samplings

Dispersion characterizes the stability of any process, including the learning one. Any managerial decisions are directed to the decreases in dispersion. Hence, the test of the hypothesis about the dispersion change significance is an important task which is formulated as follows: there are two normally distributed random values $X_j \in N\left(m_j, \sigma_j\right), \ \ j = 1, 2$ for which the sampling dispersions $S_1^2 \neq S_2^2$ as well as samples size $n_1, \ n_2$ are known. It is necessary to make a decision on the significance of disagreements between the dispersions σ_1^2 and σ_2^2. This decision may be made upon the verification results of the zero hypothesis $H_0: \ \sigma_1^2 = \sigma_2^2$ against an alternative $H_1: \ \sigma_1^2 \neq \sigma_2^2$. Hypothesis H_0 is verified by the schema. The characteristics of the output data: $X_j \in N\left(m_j, \sigma_j\right), \ j = 1, 2$, the known sample dispersions σ_j^2, sample size n_j.

1. Hypothesis $H_0: \ \sigma_1^2 = \sigma_2^2$.
2. Alternative hypothesis $H_1: \ \sigma_1^2 \neq \sigma_2^2$.
3. Significance level α.
4. Criteria (critical statistics): $F = \frac{S_A^2}{S_B^2}$, where S_A^2, S_B^2 – correspondingly the bigger and the smaller of the dispersions to be compared S_j^2.

This statistics has *F*- Fisher's distribution with the density:

$$P\left(F\right) = \begin{cases} \frac{\Gamma\left(\frac{V_A + V_B}{2}\right)}{\Gamma\left(\frac{V_A}{2}\right)\Gamma\left(\frac{V_B}{2}\right)}\left(\frac{V_A}{V_B}\right)^{\frac{V_A}{2}} \frac{F^{\frac{V_A}{2} - 1}}{\left(1 + \frac{V_A}{V_B}F\right)^{\frac{V_A + V_B}{2}}} & for \ \ F \geq 0 \\ 0 \quad for \ \ F < 0 \end{cases} \tag{23}$$

where $V_A = n_A - 1$ – numerator degree of freedom; $V_B = n_B - 1$ – denominator degree of freedom.

$$\text{If } F_{\left(v_A, v_B\right), 1 - \frac{a}{2}} \leq F \leq F_{\left(v_A, v_B\right), \frac{a}{2}} \tag{24}$$

the hypothesis H_0 of the equality of the dispersions does not deviate, in the opposite case – deviates. Criteria *F* (24) is the most powerful one.

During the realization of the algorithm for hypothesis verification H_0 with the use of computer for the calculation of the critical statistics $F_{v_A, v_B, \frac{a}{2}}$ it is possible to use an approximation formula [11]:

$$F_{\left(v_A, v_B\right), \frac{a}{2}} \approx e^{2W} \tag{25}$$

where:

$$W = \frac{Z_p\left(h + \lambda\right)^{\frac{1}{2}}}{n} - \left(\frac{1}{2b - 1} - \frac{1}{2a - 1}\right)\left(\lambda + \frac{5}{6} - \frac{2}{3h}\right)$$

$$h = 2\left(\frac{1}{2a - 1} - \frac{1}{2b - 1}\right); \lambda = \frac{Z_p^2 - 3}{6}; b = \frac{v_A}{2}; a = \frac{v_B}{2}$$

Z_p shall be determined by one of the formulas (17), (18) when $P = 1 - \frac{\alpha}{2} = F\left(X_K\right)$ for double sided, $P = 1 - \alpha = F\left(X_K\right)$ – for one sided reliance area. The tables present the values $F_{\left(v_A, v_B\right), \frac{a}{2}}$. The second critical area region boundary shall be determined by correlation

$$F_{\left(v_A, v_B\right), 1 - \frac{a}{2}} = \left(F_{\left(v_A, v_B\right), \frac{a}{2}}\right)^{-1} \tag{26}$$

and is used for the determination of reliable boundaries of root-mean-square deviation.

According to the suggested method [10] and the presented algorithm we set up a hypothesis and test it H_0: teaching and methodical training system of subjects of higher mathematics in technical universities, suggested on the base of the modern interactive technologies, *significantly* improves the quality of the base level of professional competence of future engineers [10]. For pedagogical experiment we select a screening group – $K\left(\overline{Z}, S_z^{\ 2}\right)$, group for adjustable experiment $E\left(\overline{Y}, S_y^{\ 2}\right)$

and a group for the formative experiment $\Phi\left(\bar{X}, S_x^{\ 2}\right)$, which are homogeneous in structure, which is proved by statistical methods. Let's present the reliable probability $P = 1-\alpha = 0.95$ and determine the formation of components (motivational, cognitive and creative, self-educational, communicative competence) of the basic level of professional competence.

2.3. Formation of motivational competence

For the statistical analysis of the indexes of the level of awareness of motivational competence we have composed a table of significance of their disagreements the students experience when they study parts of higher mathematics.

The results obtained allow to make a decision that the index of attitude to the chosen speciality remains stable. That is, the deviation percentage of the index of the respond to the question: "If you had to enter the university again, would you choose: *a*) same university, same speciality; *b*) same university, but the other speciality; *c*) other university?" in table 1 are insignificant.

The other indexes in the screening group – $K\left(\bar{Z}, S_z^{\ 2}\right)$, where the training process followed the traditional methods and the experimental groups of adjustable – $E\left(\bar{Y}, S_y^{\ 2}\right)$ and formative – $\Phi\left(\bar{X}, S_x^{\ 2}\right)$ experiments, in which training followed the suggested methodical system (an adjusting experiment disclosed the drawbacks, which were corrected prior to the formative experiment) are significant This allows to state, that the suggested teaching and methodical training system of subjects of higher mathematics in technical universities significantly increases the level of students' motivational competence during the first training year.

The improvement of teaching methods caused positive changes in indexes of conceptions as for the application of the knowledge obtained in writing course and bachelor papers. The difference in statistical averages of the formative and adjusting experiments is significant (column 1), but in dispersions (column 4) – it is not. It means that the index remains stable and its deviation from the statistical variable is insignificant.

Table 1. Significance of disagreements in indexes of numerical characteristics of development of motivational competence ("1" – significant deviation, "0" – insignificant deviation)

No	Indexes	1	2	3	4	5	6
		$\bar{X}$	$\bar{X}$	$\bar{Y}$	$S_x^{\ 2}$	$S_x^{\ 2}$	$S_y^{\ 2}$
	Self-estimation (adequacy factor considered)	$\bar{Y}$	$\bar{Z}$	$\bar{Z}$	$S_y^{\ 2}$	$S_z^{\ 2}$	$S_z^{\ 2}$
1	Conceptions as for the chosen speciality	0	1	1	0	1	1
2	Attitude to speciality choice	0	0	0	0	0	0
3	Conceptions as for the application of the knowledge obtained in:						
3.1	learning special subjects	0	1	1	0	1	1
3.2	writing course papers	1	1	1	0	1	1
3.3	work on speciality	1	1	1	0	1	1
4	Ability to apply the knowledge obtained in:						
4.1	learning special subjects	0	1	1	0	1	1
4.2	writing course papers	0	1	1	0	1	1
4.3	work on speciality	0	1	1	0	1	1

2.4. Improvement of the formation level of components of cognitive and creative competence – level of the knowledge and skills obtained in higher mathematics

Results of comparison of the theoretical knowledge and skills in solution of usual and applied tasks, making decisions on all the topics following the results of the test papers and exams are presented in table 2.

Analysis of the results of statistical research of significances of differences in average and dispersions between the groups show that the suggested system for learning mathematical subjects in technical universities greatly increases the level of application of the obtained knowledge and skills for the solution of tasks of any content. This is shown by the data in columns 2 and 5.

Index "1" points to the significant difference in students' knowledge of all the topics towards the favor of the suggested teaching and methodical system. Really, the statistical average from all the topics is higher in the experimental group of the formative experiment. After the statistical processing of the results of the adjustable stage of an experiment we see that despite the higher test results and better examination answers in the experimental group "E" in comparison with the screening group "K", the difference of indexes was not that significantly stable (columns 3 and 6).

This means that the suggested part of the teaching system does not influence the results we would like to obtain. The methods of lecture-wise and practical classes in vector algebra, analytical geometry, multiple integral, theory of series, function of complex variety and operational calculus had been improved.

The results of statistical analysis after the formational experiment proved the positive shift in the results of knowledge obtained after the correction of the teaching methods, which is proved by data in columns 1 and 4 as well as 2 and 6.

This may be explained by the fact that the study of these sections following the curricula requires more hours. To change the situation, we issued study books aimed at organization of independent learning some sections of higher mathematics.

Table 2. Significance of differences in indexes of numerical characteristics of development of components of cognitive and creative competences ("1" – significant deviation, "0" – insignificant deviation)

No	Indexes	1	2	3	4	5	6
		$\overline{X}$	$\overline{X}$	$\overline{Y}$	$S_x^{\ 2}$	$S_x^{\ 2}$	$S_y^{\ 2}$
	Section	$\overline{Y}$	$\overline{Z}$	$\overline{Z}$	$S_y^{\ 2}$	$S_z^{\ 2}$	$S_z^{\ 2}$
1	Linear Algebra	0	1	1	0	1	1
2	Vektor Algebra	1	1	1	0	1	0
3	Analytik geometry	0	1	1	0	1	0
4	Indirect differentiation	0	1	1	0	1	1
5	Definition integral	1	1	0	1	1	0
6	Multiple integral	1	1	1	0	1	1
7	Differential equations	0	1	1	0	1	1
8	Theory of numerical and functional series	1	1	1	1	1	0
9	Function of complex variable	1	1	0	1	0	0
10	Operational calculus	0	1	0	0	1	0
11	Probability theory and mathematical statistics	0	1	1	1	1	1
Levels of residual knowledge of higher mathematics (4-th year)							
high		1	1	1	1	1	1
sufficient		0	1	1	0	1	1
satisfactory		1	1	1	0	1	1
Low		1	1	1	1	1	1

2.5. Formed components of self-education and communicative competence learning higher mathematics

The results of analysis of statistical indexes of self-instruction skills levels, strivings to an advanced learning of the subject, self-education, gaining skills of public speaking, research work are presented in table 3.

Results of comparing the indexes of acquiring skills for self-dependent work and strivings to self-education prove our hypothesis on the reliability level of 0.95, this means that the suggested learning and methodical system significantly increases the level of self-dependent work (rows 1–6), striving to self–education in comparison with traditional training methods. This is shown in columns 2 and 5, as well as 3 and 6. Throughout the indexes the difference of the statistical average and dispersions of the coefficient of availability of the knowledge of self–dependent work, strivings to an advanced learning of material, to self–education is significant.

The data of the columns 1 and 4 show that the correction of the methods for giving lectures and the additional developments of interactive methods for practical classes aimed at improvement of the research indexes after the adjusting experiment was a success. The Indicators of differences of the statistical averages and dispersions of parameters, which determine the skills for pedagogical and research activities show that the suggested learning methods for higher mathematics significantly increases the level of skills in the experimental groups Φ and E in comparison with the screening group K (columns 2 and 5, 3 and 6). Improvement of methods after the adjusting experiment made great positive changes in indicators of public speech skills, research activities and partially – abilities in self–education work with textbooks and independent solution of applied tasks in experimental groups of formative and adjusting experiments (columns 1 and 4).

For the statistical analysis of the indexes of level of formation of communicative competence we have considered the questionnaire data as well as quantitive evaluation of levels of awareness of this competence. The results of comparing indexes of levels of awareness of communicative competence are presented in item 11 of table 3 and item 3 of table 4.

Analyses of the results of significance in differences of the indexes of numerical characteristics of awareness show that there is a significant difference between the suggested teaching and methodical system and a traditional one. Differences between the values in the screening group of the formative (Φ) and adjusting (E) experiments show that the decision to test it by the adjusting experiment with further formative one was correct.

So, the results of statistical analysis of comparison of the averages and dispersions proved our hypothesis H_0: the suggested teaching and methodical system increases the formation of the basic level of professional competence of future specialists with technical education.

To process the obtained results in pedagogical experiment we used some methods for analysis. The quantitative assessment of the level of knowledge was the known index of progress. For obviousness and convenience of numerical indexes there had been built the bar graphs and diagrams, but the percentage analysis may not always answer the questions of efficiency of the suggested methods, therefore we used the methods for statistical processing of the obtained results, which very thoroughly researches all the indexes which compose the suggested learning and methodical system for teaching higher mathematics in technical Universities.

Table 3. Significance of differences in indexes of numerical characteristics of development of components of self-educational and communicative competence ("1" - significant deviation, "0" – insignificant deviation)

No	Indexes	1	2	3	4	5	6
		$\overline{X}$	$\overline{X}$	$\overline{Y}$	S_x^2	S_x^2	S_y^2
	Components of self-educational and communicative competence	$\overline{Y}$	$\overline{Z}$	$\overline{Z}$	S_y^2	S_z^2	S_z^2
1	Independent work with textbook and manuals (electronic resource)	1	1	1	0	1	1
2	Independent work on composing reference outlines for practical classes	0	1	1	0	1	1
3	Independent solution of problems per sample	0	1	1	0	1	1
4	Making algorithm for solving problems on speciality	0	1	1	0	1	1
5	Independent solution of applied tasks	1	1	1	0	1	1
6	Making decisions from professional point of view	0	1	1	0	1	1
7	Striving for advanced learning	0	1	1	0	1	1
8	Striving for self-education	0	1	1	0	1	1
9	Public speech skills	1	1	1	1	1	1
10	Skills for research work	1	1	1	1	1	1
11	First year students adaptation	1	1	1	1	1	1

Table 4. Significance of differences in indexes of numerical characteristics of development of basic professional competences for future specialists with technical education

No	Indicators	1	2	3	4	5	6
		$\overline{X}$	$\overline{X}$	$\overline{Y}$	S_x^2	S_x^2	S_y^2
		$\overline{Y}$	$\overline{Z}$	$\overline{Z}$	S_y^2	S_z^2	S_z^2
Levels							
1	Levels of formation of motivational competence						
1.1	IV (high)	1	1	1	0	1	1
1.2	III (sufficient)	0	1	1	0	1	1
1.3	II (satisfactory)	1	1	1	1	1	1
1.4	I (low)	1	1	1	1	1	1
2	Levels of formation of cognitive and creative competence						
2.1	High	0	1	1	0	1	1
2.2	Sufficient	1	1	1	0	1	1
2.3	Satisfactory	0	1	1	0	1	1
2.4	Low	0	1	1	1	1	1
	Levels of formation of communicational competence						
3.1	High	0	1	1	1	1	0
3.2	Sufficient	1	1	1	1	1	1
3.3	Satisfactory	1	1	1	0	1	1
3.4	Low	1	1	1	1	1	1

3. Conclusions

In the conditions of a resilient economy that personnel who improve its results are especially valuable. The effectiveness of the system of educational and methodical training of personnel involved in the country's economy in institutions of higher education can be evaluated using various mathematical models and pedagogical experiments. The experience shows that students of technical universities find it difficult to learn mathematical subjects. Academic course of parts and some subjects of higher mathematics is not always useful for future engineers. The development and the introduction of the new interactive training methods which rest on tasks with technical content considering the speciality, in particular RPG's and business games, project method, simulation, «brain storming colloquium», «scientific and technical seminar», «scientific and technical conference», «optimal project» etc. is a complex process for the fundamental subjects in technical universities, which requires interdisciplinary approach and teachers' wish to give students knowledge and form the basic level of professional competence. However, future graduates of environmental specialties must possess mathematical apparatus, in particular comparative analysis of the obtained statistical data, which is the basis for evaluating the effectiveness of management decisions. In our opinion, the gaps in the knowledge of the use of mathematical apparatus are due to the uniformity of the application of teaching methods to sections of higher mathematics and the filling of mathematical textbooks with general examples. As our long–term experience shows, the application of modern innovative teaching methods and the use of textbooks for sections of higher mathematics with tasks of applied content significantly increase not only knowledge from them, but also the ability of future ecologists to make the right management decisions.

The advantage of the suggested algorithm for the statistical analysis of the results of pedagogical experiment lies in the fact that after receiving the differences between the parameters under research it is possible to make a decision on significance of difference of numerical characteristics of formation of the indexes received. The difference in percentage does not always respond to the efficient results. Correlation analysis, Pierson's criteria and other methods for the evaluation of the results of pedagogical experiment do not present such a strict, significant difference as the correlation of the statistical average and dispersion of the two samplings.

References

[1] Arens K.: The Structure of Academic Self–Concept: A Methodological Review and Empirical Illustration of Central Models. The Review of Educational Research 91(1), 2020, 34–72 [http://doi.org/10.3102/0034654320972186].

[2] Gogol K. et al.: Affect and motivation within and between school subjects: Development and validation of an integrative structural model of academic self–concept, interest, and anxiety. Contemporary Educational Psychology 49, 2017, 46–65 [http://doi.org/10.1016/j.cedpsych.2016.11.003].

[3] Grechanovska O., Petruk V.: Methods of formation of intercultural competence by means of art in students of technical universities. Scientific Bulletin of Kremenets Regional Humanitarian and Pedagogical Institute 2, 2013, 36–40.

[4] Kovtun V., Kovtun O., Semenov A.: Entropy–Argumentative Concept of Computational Phonetic Analysis of Speech Taking into Account Dialect and Individuality of Phonation. Entropy 24(7), 2022, 1006 [http://doi.org/10.3390/e24071006].

[5] Lytvynenko V. et al.: International Scientific and Technical Conference on Computer Sciences and Information Technologies 1, 2020, 36–39.

[6] Mustonen T. et al.: Songs of the Kolyma tundra co–production and perpetuation of knowledge concerning ecology and weather in the indigenous communities of Nizhnikolyma, Republic of Sakha (Yakutia), Mimeo, University of Essex Perrings C Resilience and sustainable development. Environment and Development Economics 11, 2006, 417–427.

[7] Petruk V., Kashkanova G.: Possible and statistical models and statistical evaluation of the decision, 2–nd edition. 2006.

[8] Petruk V., Sabadosh Y.: Development of self–educational competence of students of technical universities during foreign language teaching. Modern information technologies and innovative teaching methods in training: methodology, theory, experience, problems 50, 2018, 57–63.

[9] Régibeau P., Rockett K.: Economic analysis of resilience: a framework for local policy response based on new case studies. Journal of Innovation Economics & Management 2013 [http://www.cairn.info/revue–journal–of–innovation–economics–2013–1–page–107.htm].

[10] Walker B. et al.: Resilience, adaptability and transformability in social–ecological systems. Ecology and Society 9(2), 2004, 5.

[11] Zabolotna N. et al.: Diagnostics of pathologically changed birefringent networks by means of phase Mueller matrix tomography. Proc SPIE 8698, 2013, 86980E.

D.Sc. Vira Petruk
e–mail: petruk-va@ukr.net

Doctor of Pedagogical Sciences, professor, Department of Higher Mathematics, Vinnytsia National Technical University. Research Interests: formation of the basic level of professional competence in future specialists of technical specialties by means of interactive technologies, digitization of mathematics education in technical institutions, methods of statistical analysis of experimental data.

http://orcid.org/0000-0001-7588-6721

Ph.D. Olena Prozor
e–mail: prozor@vntu.edu.ua

Candidate of Pedagogical Sciences, associate professor, Department of Higher Mathematics, Vinnytsia National Technical University. Research Interests: professional pedagogy, theoretical foundations of technical education, mathematical modeling and computational methods.

http://orcid.org/0000-0003-1454-8352

Ph.D. Yuliia Sabadosh
e–mail: avataric87@gmail.com

Candidate of Science (Pedagogy), senior lecturer of the Department of Foreign Languages Vinnytsia National Technical University. Research interests are humanitarian integrated knowledge of future information technology specialists, the use of interactive technologies in the educational process.

http://orcid.org/0000-0002-2850-1224

Ph.D. Iryna Baranovska
e–mail: bdn993@gmail.com

Ph.D., associate professor. Vinnytsia State Pedagogical University named after Mykhailo Kotsiubynsky.
Research Interests: formation of the basic level of professional competence in future specialists of technical specialties by means of interactive technologies, digitization of mathematics education in technical institutions, methods of statistical analysis of experimental data.

http://orcid.org/0000-0003-0033-4425

Ph.D. Maksim Palii
e–mail: paliymv@ukr.net

Candidate of Legal Sciences, associate professor. Department of Constitutional, International and Criminal Law, Vasyl Stus Donetsk National University. Research Interests: criminal law, criminology, criminology of religion, crime prevention, statistical analysis of experimental data.

http://orcid.org/0000-0002-1113-6355

Ph.D. Yevheniia Moroz
e–mail: morozeo@knu.ua

Ph.D. associate professor. Department of Theory and History of Sociology, Taras Shevchenko National University of Kyiv. Scientific interests: history of sociology, modern sociological theories, theories of cultural capital, post–structuralist approach, urban studies, data analysis.

http://orcid.org/0000-0002-2618-3541

Ph.D. Saule Kumargazhanova
e–mail: SKumargazhanova@gmail.com

She is currently the dean of the Department of Information Technologies and Intelligent Systems of D. Serikbayev East Kazakhstan Technical University. She is a co–author over 50 papers in journals and conference proceedings.
Her professional interests are software engineering, data processing and analysis.

http://orcid.org/0000-0002-6744-4023

M.Sc. Dinara Mussayeva
e–mail: d_i_n_mus@mail.ru

Scientific Secretary and Research Associate of the IE. Institute of Economics CS MES RK, Almaty, Kazakhstan.
She is a master of Al–Farabi Kazakh National University, Ph.D. student of the Institute of Economics of the CS MES of the RK, specialty– "World Economy". She started her career at the Institute of Economics as a junior researcher. As a responsible executive, she actively participates in conducting fundamental and applied research in priority areas for the state, such as digitalization of the economy, knowledge–based economy, etc.

http://orcid.org/0000-0002-8349-213X