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THE COMPARATIVE ANALYSIS OF SCALAR AND TENSOR MODELS OF DAMAGE ACCUMULATION ON TWO-STAGE COLD DEFORMATION EXAMPLE

BY

MIKHALEVICH V.M., KRAEVSKY V.A. AND KOZLOV K.E.

Abstract: *In this work the comparative analyses linear and nonlinear, scalar and tensor models by comparison to experimental data on two-stage deformation is executed . The method of the tensor model parameters definition based on creep curves is offered. In the given method all parameters of damage accumulation tensor model are determined under condition of a stationary stressing.*
Keywords: *damage tensor, nonmonotonic deformation, ductility, creeping, rupture strength*

The theory of deformability has been widely used for the analysis of materials shapig processes. V. L. Kolmogorov /1/ has offered the first damage accumulation model as a functional within the framework of the given approach

$$\psi(t) = \int_0^t E(t-\tau) \cdot B(\tau) \cdot \frac{\dot{\varepsilon}_u(\tau)}{\varepsilon_{*c}[\eta(t)]} \cdot d\tau, \tag{1}$$

where ψ characterizes damage measure of a material and varies from 0 in an initial state up to 1 at the fracture moment; t,τ is the time; $E(t-\tau)$ is a coefficient considering the damage "healing" at high temperature; B is a coefficient considering a history of deformation; ε_{*c} is the strain accumulated up to fracture under stationary deformation; η is the parameter of stress state.

But the principles of functions B and E definition were not offered. Therefore V. L. Kolmogorov's simplest model expressing a linear principle of damage accumulation received wide application in practice

$$\psi(\varepsilon_u) = \int_0^{\varepsilon_u} \frac{d\varepsilon_u}{\varepsilon_{*c}[\eta(\tau)]}, \tag{2}$$

where ε_u is the accumulated strain.

Major advantage of the present model (2) is its simplicity. But many experimental data were not stacked in frameworks of a linear principle (2). It caused the appearance of nonlinear models. These models were offered by G. D. Dell', V. A. Ogorodnikov etc. /2,3,4/.

Let's analyze scalar nonlinear model with a power kernel

$$\psi(\varepsilon_u) = \int_0^{\varepsilon_u} n \cdot \frac{\varepsilon_u^{n-1}}{\varepsilon_{*c}^n(\eta(\varepsilon_u))} \cdot d\varepsilon_u . \tag{3}$$

where n is a material constant.

The new stage of fracture model construction opens with appearance of the G. D. Dell's /5/ and A. A. Kiiko's /6/ works, in which the variants of tenzor model were offered. The basic premises used to construct tensor models of damage accumulation were developed by A. A. Il'yushin /7/. Within these premises G. D. Dell' has defined components of a tensor-deviator as follows

$$\psi_{ij}(\varepsilon_u) = \int_0^{\varepsilon_u} F(\varepsilon_u, \eta, \mu_\sigma) \cdot \beta_{ij} \cdot d\varepsilon_u, \qquad (4)$$

where μ_σ is the Lode's parameter; β_{ij} is the strain increment guide tensor.

The essential development of the damage accumulation tensor theory is presented in work /8/. In particular, the nonlinear tensor model for cold deformation is offered

$$\psi_{ij}(\varepsilon_u) = \int_0^{\varepsilon_u} F(\varepsilon_u^*, \eta, \mu_\sigma) \cdot \left\{ a \cdot \beta_{ij}(\varepsilon_u^*) + b\left[\beta_{ij}(\varepsilon_u^*) \cdot \beta_{ij}(\varepsilon_u^*) - \frac{1}{3} \cdot \delta_{ij} \right] \right\} \cdot d\varepsilon_u^* \qquad (5)$$

where δ_{ij} is the unit tensor, $\delta_{ij} = 1$ at $i = j$, $\delta_{ij} = 0$ at $i \neq j$; $a = a(\eta, D)$, $b = b(\eta, D)$ are parameters of model, which should be defined.

Let's define adequacy of offered scalar (3) and tensor (5) models to experimental data of metals two-stage cold deformation. At such scheme of deformation within the limits of each stage we have stationary deformation. When $0 \le \varepsilon_u \le \varepsilon_u^{(1)}$ then $\beta_{ij}(\varepsilon_u) = \beta_{ij}^{(1)} = const$, $\eta(\varepsilon_u) = \eta^{(1)} = const$, when $\varepsilon_u^{(1)} \le \varepsilon_u \le \varepsilon_u^{(2)}$ - $\beta_{ij}(\varepsilon_u) = \beta_{ij}^{(2)} = const$, $\eta(\varepsilon_u) = \eta^{(2)} = const$, $\mu_\sigma(\varepsilon_u) = \mu_\sigma^{(2)} = = const$. In this case from a scalar model (3) we obtain the following criterial relation

$$\psi_{*2} = \left[1 - \psi_1^{n_1} + (\psi_1 \alpha_{12})^{n_2} \right]^{\frac{1}{n_2}}, \qquad (6)$$

where ψ_i is the spent resource of a ductility on i-th stage (i=1,2). Here

$$\psi_1 = \frac{\varepsilon_u^{(1)}}{\varepsilon_{*c}^{(1)}}, \psi_{*2} = \frac{\varepsilon_u^{(2)} - \varepsilon_u^{(1)}}{\varepsilon_{*c}^{(2)}}, \alpha_{kl} = \frac{\varepsilon_{*c}^{(k)}}{\varepsilon_{*c}^{(l)}},$$

where α_{kl} is the parameter, characterizing the order of strain conditions alternation. If harder conditions are changed to softer ($\varepsilon_{*c}^{(1)} < \varepsilon_{*c}^{(2)}$), then $\alpha_{12} < 1$.

If n=1, model (3) is converted to (2), the expression (6) can be written in the form reflecting linear principle of damage accumulation

$$\psi_{*2} = 1 - \psi_1, \qquad (7)$$

From the model (5) for two-stage deformation we have

$$\psi_{*2} = \left[(\psi_1 \cdot \alpha_{12})^{n_2} - \psi_1 \cdot I_{12} + \sqrt{\psi_1^{2n_1} \cdot (I_{12}^2 - 1) + 1} \right]^{\frac{1}{n_2}} - \psi_1 \cdot \alpha_{12}. \qquad (8)$$

For nonlinear tenzor model

$$I_{12} = k_{12} \cdot a^{(1)} \cdot a^{(2)} + I_1 \cdot a^{(1)} \cdot b^{(2)} + I_2 \cdot a^{(2)} \cdot b^{(1)} + (I_3 - \frac{1}{3}) \cdot b^{(1)} \cdot b^{(2)}, \qquad (9)$$

for linear tenzor model

$$I_{12} = k_{12}, \qquad (10)$$

where $k_{12} = \beta_{ij}^{(1)} \cdot \beta_{ij}^{(2)}$ is cosine of the strain trajectory sharp angle; I_1, I_2, I_3 are the invariants of tensors product

$$I_1 = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(2)} \cdot \beta_{ki}^{(2)}, I_{21} = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(1)} \cdot \beta_{ki}^{(2)}, I_1 = \beta_{ij}^{(1)} \cdot \beta_{jk}^{(1)} \cdot \beta_{kl}^{(2)} \cdot \beta_{li}^{(2)}.$$

The parameters n_1 and n_2 included in criteria relations which we received from scalar and tenzor (linear and nonlinear) models for a case nonmonotone two-stage deformation were found by a method of least-squares. For this purpose the program was developed in mathematical application MathCad. This program automatically calculates parameters of models and draws the graphs of dependence of a ductility spent resource from resource used at the first deformation stage after introduction of experimental data . The experimental data nonmonotone two-stage deformation were obtained from the literature /5/. The parameters of the model were found for two cases: $n_1 \neq n_2$ (thick lines) and $n_1 = n_2$ (thin lines). The results of calculation are presented in fig. 1.

In tab. 1. the values of the difference quadrates sum S_{sum} of experimental data and results of calculation by scalar, linear tensor and nonlinear tensor models are presented. From fig. 1 and tab. 1 it can be seen that the tensor-nonlinear model at $n_1 \neq n_2$ most accurately describes experimental data (S_{sum} has a minimum value). In accordance with analyses of tab. 1 data the introduction in models additional parameter n_2 reduces in a raise of their exactitude (diminution S_{sum}).

Table 1 – Compare of the difference quadrates sum S_{sum} of experimental data and results of calculation after scalar (3) and tensor (5) models

	9XC	P6M5	45	$\sum S_{sum}$
Scalar model when $n_1 = n_2$	3,849	3.886	3,45	11,185
Scalar model when $n_1 \neq n_2$	1.396	3,508	0,944	5,848
Linear tenzor model when $n_1 = n_2$	0,751	0,432	0.239	1,422
Linear tenzor model when $n_1 \neq n_2$	0,701	0,336	0.021	1,058
Nonlinear tenzor model when $n_1 = n_2$	0,717	0,379	0,0094	1,1054
Nonlinear tenzor model when $n_1 \neq n_2$	0,627	0,329	0.0044	0,9604

When $n_1 = n_2$, $\alpha_{12} = 1$ (fig. 1, and, c, h, i) the relation (6), which follows from a scalar model, converted to damage accumulation linear principle form (7). In this case great qualitative discordance of model (3) with experimental data is observed. We should also point out dead-level of a relation (6) with two independent parameters ($n_1 \neq n_2$) during exposition of different two-stage deformation processes of the same material. It once again testifies that the scalar model does not take into account the ductility anisotropy of deformable metal.

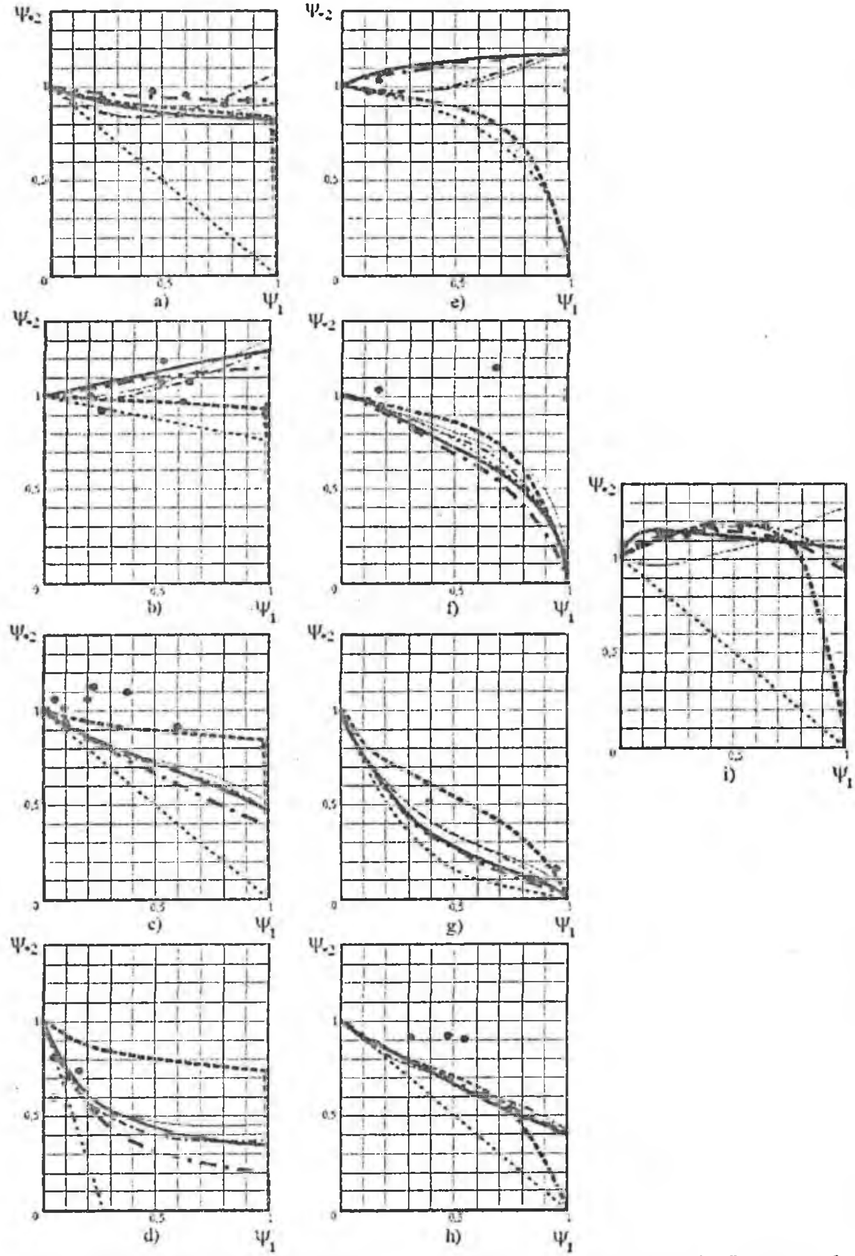

Fig. 1 - Dependence of the ductility spent resource from resource used at the first stage of deformation: a) steel 9XC, alternating torsion; b) steel 9XC, axial tension - squeezing; c) steel 9XC, d) steel 9XC, axial squeezing - tension; e) steel P6□5, axial tension- squeezing; f) steel P6□5, torsion - tension; g) steel P6□5, torsion - tension; h) steel P6□5, axial squeezing - orthogonal squeezing; i) steel 45, alternating torsion; $n_1 = n_2$ - thin lines; $n_1 \neq n_2$ - thick lines;

...... calculation from relation (6); ______ - calculationon from relation (8) with allowance for (10);

– · – - calculationon from relation (8) with allowance for (9); ◎ - experimental data.

In spite of satisfactory correspondence to experimental data, use of tensor model (5) for prediction of the extreme condition is problematic, as there are no physical principles of parameters n_1 and n_2 definition. During definition of rupture-strength tenzor model parameters the same problem has arised. This model is represented in the form /8/

$$\psi_{ij}(t) = \int_0^t [1-\rho(I(\tau))] \cdot \varphi_1(t-\tau; \eta(\tau), D(\tau)] \cdot \beta_{ij}(\tau) \cdot f[I(\tau)] \cdot d\tau +$$

$$+ \int_0^{\varepsilon_*(t)} \rho(I(\tau)) \cdot \varphi_2\left(\frac{\varepsilon_u^{\cdot}}{\varepsilon_{*c}(\varepsilon_u^{\cdot})}\right) \cdot \frac{1}{\varepsilon_{*c}} \cdot \beta_{ij}(\varepsilon_u^{\cdot}) \cdot d\varepsilon_u^{\cdot}, \qquad (11)$$

For a solution of this problem we offer the method of the tensor model parameters definition based on creep curves /9/. Analyzing a series of creep curves approximations /10,11,12,13,14/, and also during analyses experimental data /15,16/, we received very interesting result: the dependence of creep strain on time reduced to relative values $\frac{\varepsilon}{\varepsilon_{*c}} = \frac{\varepsilon}{\varepsilon_{*c}}\left(\frac{t}{t_{*c}}\right)$ is invariant from loading. The form of creep curves for different materials and temperatures constructed in absolute and relative coordinates is shown on a fig. 2.

For stationary deformation the model (11) takes the form

$$\psi_u(t) = (1-\rho) \cdot \psi^n + \rho \cdot \psi^p, \qquad (12)$$

Differentiating the equation (10) we obtain

$$\frac{d\psi_u}{d\psi} = (1-\rho) \cdot n \cdot \psi^{n-1} + \rho \cdot p \cdot \psi^{p-1}$$

$$\frac{d^2\psi_u}{d\psi^2} = (1-\rho) \cdot n \cdot (n-1) \cdot \psi^{n-2} + \rho \cdot p \cdot (p-1) \cdot \psi^{p-2}.$$

From the analyses of two last relations it is clear, that $\frac{d\psi_u}{d\psi} \geq 0$, and the second order derivative can accept both positive and negative values. Therefore the damage accumulation curve crown in coordinates ψ-ψ_u can vary. So, for example, for values ρ=0.7, n=0.1, p=1/n=10 the damage accumulation curve has three characteristic regions, which are similar to creep curves.

It is natural to assume, that the process of damage accumulation reflects regularities of a creep strain. Taking into account that the character of creep curves coincides with damage accumulation curves (the characteristic regions both on curve accumulation of damages and on creep curves are observed), we offer to find parameters of model (11) based on the creep curves, which are constructed in relative coordinates.

We used experimental data of a two-stage loading of alloy □□826 at temperature 800°C /17/ for verification of the offered approach of model parameters definition (11). On the first step all specimens were loaded by stress 200 MPa during 2800 h. On the second step stress varied from 180 up to 350 MPa for different specimens. As a result of approximation of □□826 creep curves we obtain following

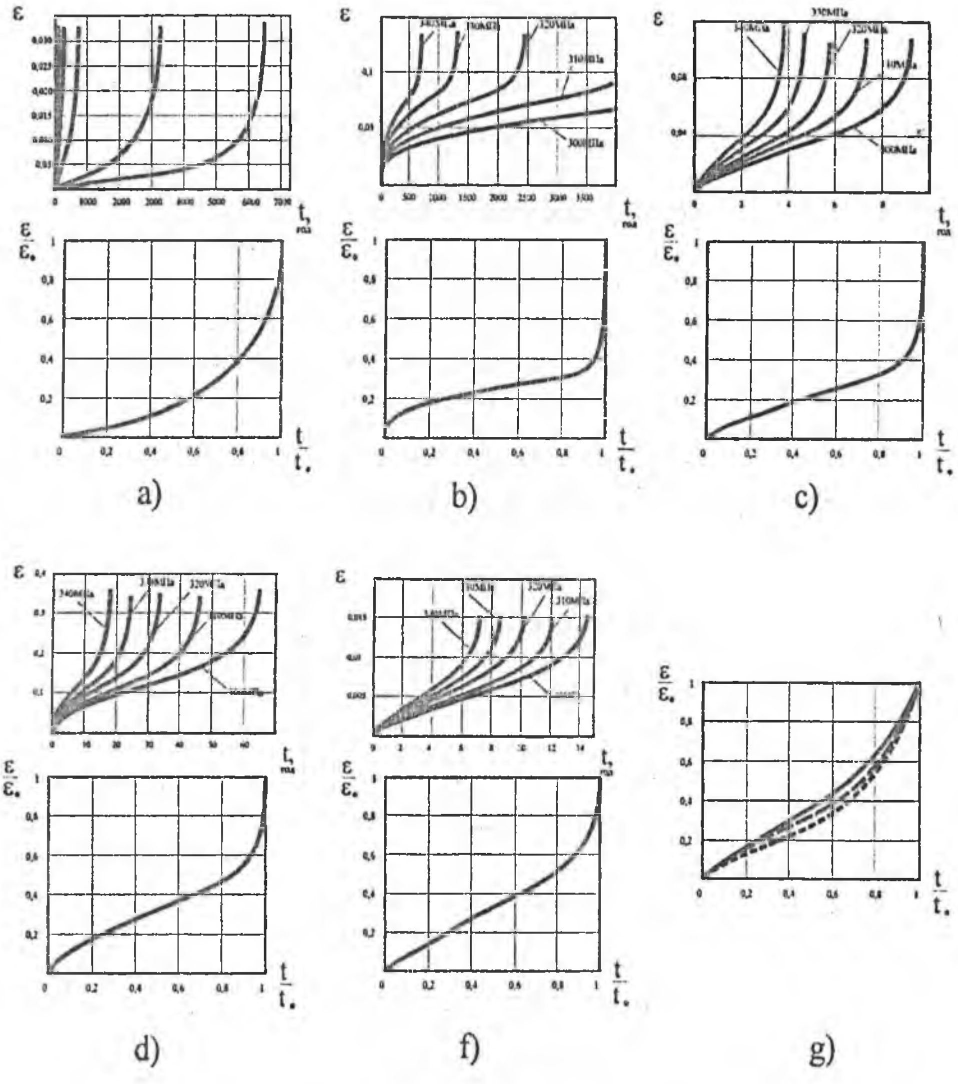

Fig. 2. Creep curves in absolute and relative coordinates:
a) - alloy □□826 at 800°C; b) - alloy 03X20□45□4□P□ at 550°C; c) - alloy 03X20□45□4□P□ at 700°C; d) - alloy 03X20□45□4□□ at 650°C; e) - alloy 03X20□45□4□□ at 700°C; f) - molybdenum at1000°C.

values of parameters : $n = 1$, $p = 20.94278$, $\rho = 0.613368$. Result of experiment and calculation from the model (11) are shown on a fig. 3.

The offered approach of parameters definition of damage accumulation model will allow to use a considerable quantity of experimental data, accumulated in the literature, that will give an opportunity to forecast longevity of materials without additional experiments.

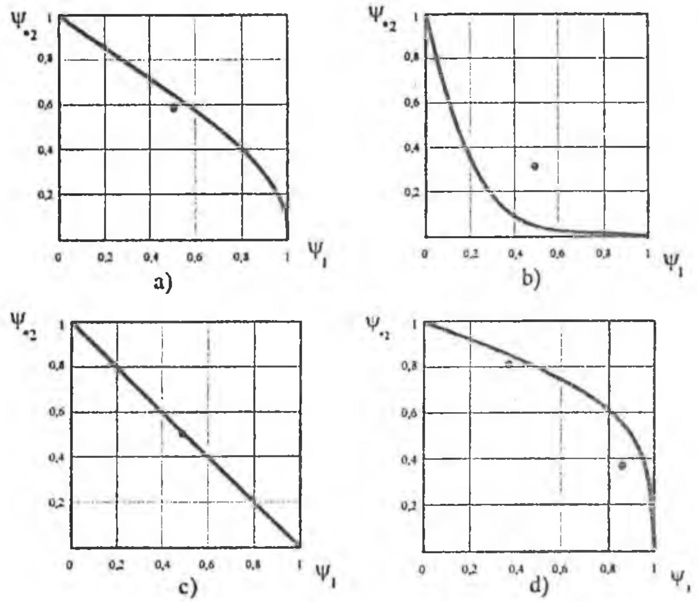

Fig. 3. Dependence between residual and used by rupture-strength resources during the two-stage stressing:

a) - alloy □□826 under 800°C ($\alpha_{21} = 1.3$); b) - alloy □□826 under 800°C ($\alpha_{21} = 0.3$); c) - alloy □□826 under 800°C ($\alpha_{21} = 1$); d) - alloy 03X20H45□4□□ under 700°C ($\alpha_{21} = 2.6$);

(• - experiment; – - calculation based on (11)).

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ANALIZA COMPARATIVA A MODELELOR TENSORIAL SI SCALAR ALE ACUMULARII DEFECTELOR PE UN EXEMPLU DE DEFORMARE LA RECE IN DOUA FAZE

REZUMAT: Lucrarea prezinta comparativ analiza liniara si neliniara, modelele tensorial si scalar pe date experimentale obtinute in urma unei deformatii in doua faze. Este data metoda de definire a parametrilor tensoruluipe baza curbelor de dislocatii. In metoda data sunt determinati toti parametrii tensorului acumularii defectelor in conditii de solicitare stationare.