MODAL ANALYSIS OF THE SPREADER BOOM AS A SYSTEM OF ARTICULATED TIMOSHENKO BEAMS

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Abstract: *This paper proposes a mathematical model and algorithm for calculating the transverse vibrations of a mechanical system consisting of pivotally jointed beams with varying elastic-inertial characteristics. In accordance with the theory of S.P. Timoshenko, shear deformation and rotary inertia movement of cross sections must be taken into consideration for structural elements. The algorithm for calculating the harmonic oscillation uses the method of initial parameters. Construction features of transition matrices to building areas and connecting knots.*

Keywords: *rod system, articulated connection, modal analysis,Timoshenko beams*

1. Introduction

Modern mechanical engineering widely uses bearing steel structures consisting of several sections made in the form of rods or trusses that are pivotally interconnected. Such mechanical systems include spreader arrows, spraying rods, frame conveyors and more. In most cases, elastic-inertial characteristics of connecting elements (sections) depend on the longitudinal coordinate [2, 4, 6]. The rational model of such elements is considered a solid straight shaft which, in accordance with the Tymoshenko theory of beams, takes into account shear deformation and rotary inertia movement of cross sections.

Bearing metal constructions are predominantly under the influence of stationary dynamic loads. Therefore, prevention of resonance reactions and a comprehensive analysis of established forced oscillations of these systems are of particular practical importance.

The method of initial parameters [1, 3, 5, 7] allows for high efficiency in the calculation of harmonic oscillations of multispan rods. However, the application of the given method has certain specifics in circumstances where hinged section connectors are used.

2 Mathematical model of oscillatory processes

We will construct a generalized mathematical model and algorithm for calculating free vibrations of a mechanical system shown in Fig. 1, where l_1, l_2, ..., ll_{n-1} are the lengths of the composite metal sections and m_1, m_2, ..., m_n are the mass of solids that are tightly attached to the connecting hinge axes; c_{y1}, c_{y2}, ..., c_{yn} are the stiffness of resilient supports in the vertical direction; x_1, x_2, ..., x_{n-1} are the longitudinal axes of the elements of the system with the starting points, located on the left edge of the corresponding sections; w_1, w_2 , ..., w_{n-1} are the deflections of section design.

For each of the sections, area A_i, axial moment of inertia of the the cross section $I_{i,}$, coefficient κ_i which is used for calculating shear deformation, the average density of the material ρ_i and axial compressive force P_i, we assume continuous functions relative longitudinal coordinates $\xi_i = x_i / l_{i.}$

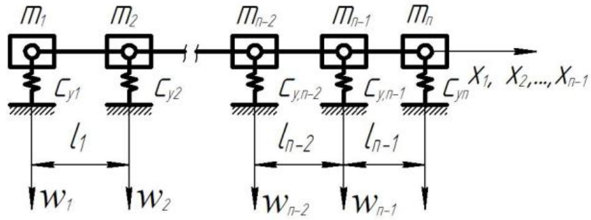

Figure 1: – Design circuit of the boom structure.

We record the following differential equations of lateral oscillation sections given bending and shear deformations [8]:

$$\frac{\partial w_i}{\partial \xi_i} = \frac{l_i \kappa_i G A_i}{\kappa_i G A_i - P_i} \varphi_i + \frac{l_i}{\kappa_i G A_i - P_i} Q_i ,$$

$$\frac{\partial \varphi_i}{\partial \xi_i} = -\frac{l_i}{E I_i} M_i , \quad \frac{\partial M_i}{\partial \xi_i} = \frac{l_i \kappa_i G A_i}{\kappa_i G A_i - P_i} Q_i +$$

$$+ \frac{l_i \kappa_i G A_i P_i}{\kappa_i G A_i - P_i} \varphi_i - l_i \rho_i I_i \frac{\partial^2 \varphi_i}{\partial t^2} ,$$

$$\frac{\partial Q_i}{\partial \xi_i} = l_i \rho_i A_i \frac{\partial^2 w_i}{\partial t^2} \quad (i = 1, 2, ..., n-1), \quad (1)$$

where E, G are moduli of elasticity of the material of the first and second kind; φ_i is the angle of the tangent to the curved axis of the beam from the bending moment, Mi, Qi are the bending moment and shear force arising in a cross section perpendicular to the undeformed axis ; t is time.

Upshot of equations (1) corresponding to harmonic oscillation are expressed with the following:

$$w_i = W_i(\xi_i)\sin \omega t, \qquad \varphi_i = \Phi_i(\xi_i)\sin \omega t,$$

$$M_i = M_{0i}(\xi_i)\sin \omega t, \quad Q_i = Q_{0i}(\xi_i)\sin \omega t,$$

$$(i = 1, 2, ..., n-1), \quad (2)$$

where $W_i(\xi_i)$, $\Phi_i(\xi_i)$ are the peak functions of transverse and rotational displacements of the beam cross-section; $M_i(\xi_i)$, $Q_i(\xi_i)$ are function amplitudes of bending moments and transverse forces; ω is the cyclic frequency.

Taking into account (2), we simplify equation (1) to the form below

$$\frac{dF_{Yi}(\xi_i)}{d\xi_i} = D_{Yi}(\xi_i) F_{Yi}(\xi_i) \ (i = 1, 2, ..., n-1), \quad (3)$$

where

$$F_{Yi} = \mathrm{col}\big(W_i(\xi_i), \Phi_i(\xi_i), M_{0i}(\xi_i), Q_{0i}(\xi_i)\big),$$

$$D_{Yi}(\xi_i) = \begin{pmatrix} 0 & \frac{l_i \beta_i}{\beta_i - P_i} & 0 & \frac{l_i}{\beta_i - P_i} \\ 0 & 0 & -\frac{l_i}{\alpha_i} & 0 \\ 0 & \frac{l_i P_i \beta_i}{\beta_i - P_i} + l_i \delta_i & 0 & \frac{l_i \beta_i}{\beta_i - P_i} \\ -l_i \gamma_i & 0 & 0 & 0 \end{pmatrix},$$

with indicators

$$\alpha_i = EI_i; \quad \beta_i = \kappa_i G A_i; \quad \gamma_i = \rho_i A_i \omega^2;$$

$$\delta_i = \rho_i I_i \omega^2; \quad \chi_i = EA_i .$$

Integrating the system of differential equations with variable coefficients (3) we determine the amplitude functions of geometric and force parameters in randomized cross-section of the bearing structure for the values of these parameters at the starting point of the section. This procedure is substituted for operator $L_{Fi}(\xi_i)$ of the initial conditions F_i (0). Then the connection of matrix columns of geometric and force parameters on the ends of the section is determined by the equation below:

$$F_i(1) = L_{Fi}(1) \cdot F_i(0). \quad (4)$$

Taking into account the D'Alembert's principle, we note the boundary conditions for the left end of the metal:

$$m_1 \frac{\partial^2 w_1(0,t)}{\partial t^2} - Q_1(0,t) + c_{y1} w_1(0,t) = 0;$$

$$M_1(0,t) = 0. \quad (5)$$

Given the specifics of the interaction between the system of connecting elements, we formulate the boundary conditions for joints of adjacent sections

$$m_i \frac{\partial^2 w_i(0,t)}{\partial t^2} - Q_i(0,t) +$$

$$+ Q_{i-1}(1,t) + c_{yi} w_i(0,t) = 0;$$

$$M_{i-1}(1,t) = 0; M_i(0,t) = 0;$$

$$w_i(0,t) = w_{i-1}(1,t)$$

$$(i = 2,3,...,n-1). \qquad (6)$$

Similarly, we note the boundary conditions for the right end of the structure

$$m_n \frac{\partial^2 w_{n-1}(1,t)}{\partial t^2} + Q_{n-1}(1,t) + c_{yn} w_{n-1}(1,t) = 0;$$

$$M_{n-1}(1,t) = 0. \qquad (7)$$

Substituting (2) into the boundary conditions (5) - (7) and excluding the time function, we form the matrix equality

$$F_1(0) = H_{F1} F_0,$$

$$F_i(0) = H_{Fi} F_{i-1}(1) \quad (i = 2,3,...,n-1),$$

$$F_n = H_{Fn} F_{n-1}(1), \qquad (8)$$

Where

$$F_0 = \mathrm{col}(W_1(0), \Phi_1(0), 0, 0),$$

$$F_n = \mathrm{col}(W_{n-1}(1), \Phi_{n-1}(1), 0, 0),$$

$$H_{F1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c_{y1} - m_1\omega^2 & 0 & 0 & 1 \end{pmatrix};$$

$$H_{Fi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c_{yi} - m_i\omega^2 & 0 & 0 & 1 \end{pmatrix};$$

$$H_{Fn} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c_{yn} - m_n\omega^2 & 0 & 0 & 1 \end{pmatrix}.$$

In the process of conducting transitions through connecting nodes using the second matrix equation (8), vibration amplitude rotation angles and the initial cross-sections were assigned zero values. The actual values of these amplitudes are given by the following algorithm. The system of ordinary differential equations (3) for each of the sections is integrated in determining the unknown functions $0 \le \xi_i \le 1$ twice: first using initial conditions obtained in a matrix column-Fi (0) as a result of crossing connecting node; upon re-integrating, we accept $F_i(0)$=col(0, 1, 0, 0). The resulting matrix with columns of geometric and force parameters will be denoted by one and two asterisks, respectively.

Whereas the second condition (6), we can express with the equality:

$$M_i^*(1) + \Phi_i(0) M_i^{**}(1) = 0, \qquad (9)$$

since the elements of the matrix column $F_i^{**}(1)$ are the reaction of geometric and strength parameters of the rightmost section for the single angle $\Phi_i(0)$.

Determining the value of (9) $\Phi_i(0)$, we finally determine:

$$F_i(1) = F_i^*(1) + \Phi_i(0) F_i^{**}(1). \qquad (10)$$

Solving for the matrix columns F_i (0) in equations (4) and (10), we substitute for operator $R_i(1)$, i.e.

$$F_i(1) = R_{Fi}(1) F_i(0). \qquad (11)$$

Using (8), (11), we record value matrix

$$F_n = H_{Fn} \left\{ \prod_{j=n-1}^{1} \left[R_{Fj}(1) H_{Fj} \right] \right\} F_0. \qquad (12)$$

When performing calculations using equation (12), zero-value of the third element column matrix F_n is reached. The condition of finding the natural frequency of the mechanical system is a zero-value of the fourth element of the specified matrix. The value W_i (0) in the process of solving is selected at random, equal to one.

We calculate the modes of vibrations of the mechanical system by numerical integration of differential equations (3) with initial conditions corresponding to the specified natural frequencies.

3 Results of modal analysis Arm

Consider the results of a calculation of frequencies and forms of free vibrations of the boom structure, whose parameters are given in Table. 1.

Table 1. Parameters of the Boom Structure

Parameter	Unit	Value
l_1	m	12,00
l_2	m	9,00
l_5	m	3,00
$E_1, ..., E_3$	N/m^2	$2,10\odot10^{11}$
$G_1 ..., G_3$	N/m^2	$8,10\odot10^{10}$
$\rho_1, ..., \rho_3$	kg/m^3	$1,25\odot10^{4}$
$A_1, ..., A_3$	m^2	$3,34\odot10^{-3}$
$I_1, ..., I_3$	m^4	$3,26\odot10^{-4}$
$\kappa_1, ..., \kappa_3$	–	0,15
P_1	N	$3,13\odot10^{4}$
P_2	N	$2,75\odot10^{4}$
P_3	N	$2,49\odot10^{4}$
m_1	kg	200,0
m_2, m_3	kg	10,0
m_4	kg	35,0
J_1	kg$\odot$m^2	100,0
J_2, J_3	kg$\odot$m^2	1,0
J_4	kg$\odot$m^2	2,3
c_{y1}	N/m	$1\odot10^{9}$
c_{y2}, c_{y3}	N/m	$4,4\odot10^{7}$
c_{y4}	N/m	0

The values of natural frequencies of the mechanical system are given in Table. 2. As seen from the results, flexibility of the connection between the first and second sections has the biggest impact on the second frequency. Fig. 2 and 3 below provide data for three natural vibration modes of boom structure under absolute rigidity of the connection sections (Fig. 2), and flexible connection in the first and second sections (Fig. 3). Bold curves below show first forms of free vibrations, curves in medium bold show second forms, and regular curves show natural forms. As seen from the graphs, connections in the vicinity of boom structures with cables gain minimal amplitude functions.

Table 2. The dependence of the natural frequency of the boom structure on the flexibility of the connection between the first and second sections.

Flexibility of the connection/(N·m)	Frequency Value, Hz				
	1	2	3	4	5
0	17,88	21,46	32,15	49,69	65,92
$0,1\cdot10^{-6}$	17,76	18,35	30,98	48,89	65,72
$0,1\cdot10^{-5}$	17,64	18,01	30,50	48,55	65,64
$0,1\cdot10^{-4}$	17,44	17,96	30,43	48,50	65,62
$0,1\cdot10^{-3}$	17,42	17,95	30,42	48,49	65,62
$0,1\cdot10^{-2}$	17,42	17,95	30,42	48,49	65,62

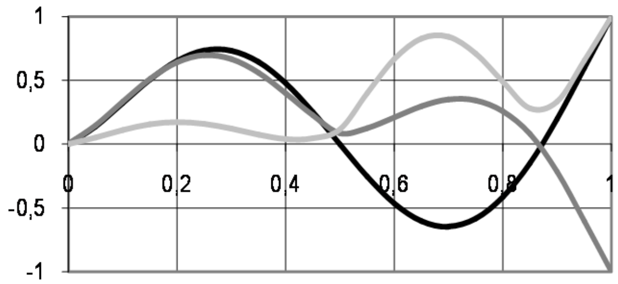

Figure 2: – Forms of free oscillations boom with rigid connections sections.

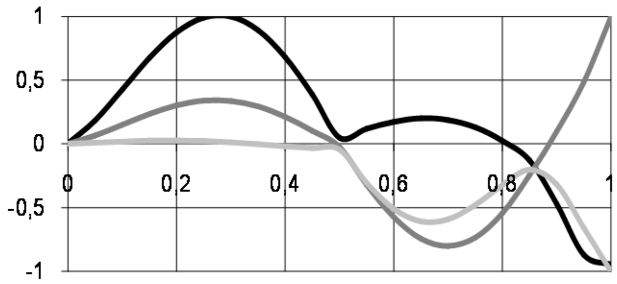

Figure 3: – Forms of free oscillations arrows connections with sections malleability $0,1\cdot10^{-3}$ rad/(N·m).

The flexibility of the connection between boom structure sections significantly impacts all custom forms. However, this effect is most evident in the first natural form. The amplitude function that corresponds to the first form of fluctuations in flexible connection of the sections is expressed with a broken line.

4. Conclusions

A mathematical model of dynamic phenomena in the jib designs including bending deformation and shear elastic compliances supports and connections enables detailed modal analysis of technical objects with arbitrary number of sections.

The demonstrated mathematical model allows to analyze free vibrations of pivotally connected Timoshenko beams with removable elastic-inertial characteristics.

The algorithm of the calculation is sufficiently convenient in numerical implementation and can be used in computer-aided design of bearing steel constructions to ensure rational parameters of frequency spectra, as well as forms of deformation vibrations.

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