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A VITERBI ALGORITHM AS A KEY TO DECODING A TURBO-CODE

Annotation. *Questions, related to modern turbo-codes, are examined. The turbo-coding system and schemes of encoder and decoder is rotined for work with this code. The Viterbi algorithm and his modification SOVA, which is one of the methods of work with turbo-code, perspective for information transmission, is considered. The algorithm of work for SOVA decoder is resulted for the «hard» and «soft» finding of decisions. A simulation is also conducted for the SOVA algorithm in a channel with additive white Gaussian noise.*

Key of words: *turbo-code, probability, Viterbi algorithm, SOVA, bit error rate.*

I. Introduction

Turbo-code – the method of error correcting coding, which has an important advantage, that allows to provide reliable performance very close to the Claude Shannon theoretical limit. This code consists of cascade of the parallell connected systematic codes. These constituents are named as component codes, and for them can be used codes of Reed-Solomon, Bose-Chaudhuri-Hocquenghem or convolutional codes [1]. Turbo-codes – a class of methods, that determine the iterative (probabilistic) coding with errors correction, and improved in the direction of increasing speed and reliability of the data transmission, making them the most promising for using in digital television, space and mobile satellite communications. The considered code is used in modern standards for transfer digital information, such as DVB-RCS, W-CDMA, CDMA2000, IEEE P1901, ViaSat, HDTV. For turbo-code many developments is conducted, that allows to improve efficiency of this code in distributed computer systems of the different functional purposes. For example, the improving class of turbo-codes was patented by companies France Telecom and Telediffusion de France for using in the modern digital systems for transmission data, and Advanced Hardware Architectures (AHA) has achieved most successes in the construction of microcircuits for decoding of this code. Using of turbo-coding allows to increase the error correcting reception of signals on 1-3 dB and more depending on type of signals, speed, type of coding, quality of communication channel [2].

II. Target of setting

Reception and forming of the turbo-code at planning of the modern digital systems of transmission data are the actual theoretical and applied problems [1 – 4]. One

of basic ideas of decoding the turbo-codes is a Viterbi algorithm, the basis for developing modifications, which minimize hardware expenses and reduce the computational complexity of decoding. The purpose of the article is research of probabilistic-energy characteristics of information transmission with the usage decoding method for turbo-code which called SOVA.

III. Research results

An example of turbo-code system is shown in fig. 1, where $\overline{U_i}$ – submitted sequence of information bits; P_{1i}, P_{2i} – dibit from encoder 1 and 2; $\overline{V_i}$ – information symbolic data; p_{1i}, p_{2i} – symbols of parity; σ^2 – value of dispersion from the Gaussian normal distribution; $\overline{r_i}$ – noisy information symbolic data; p'_{1k}, p'_{2k} – noisy parity symbols; $LLR(\overline{U_i})$ – «soft» decision from decoder; $\hat{U}_i$ – «hard» decision from decoder [1].

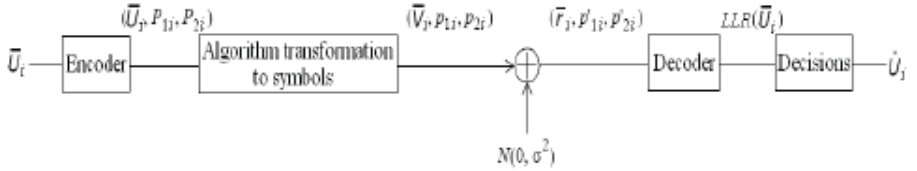

Figure 1 – The system of turbo-code

Scheme of turbo-encoder with two convolutional identical encoders and interleaver P is resulted on fig. 2. The structure of turbo-decoder (fig. 3) consists of pair of decoders, which work jointly, improving the estimation of initial informative bits. Decoders work on the basis decoding algorithm on the maximum of a posteriori probability and produce information of «soft» decision. Initially decoder № 1 begins to work without initialising of information (a priori estimations equal a zero). In next iterations information of «soft» decisions is used to activate the other decoder. Information from decoder rotating the loop to that moment, while «soft» decisions do not converge on the stable set of values. The last «soft» decisions are used to recover the original binary sequence [1].

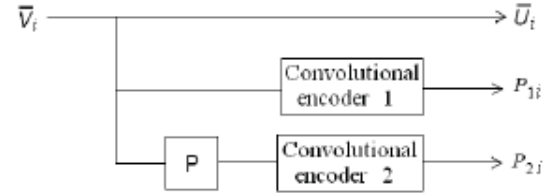

Figure 2 – Scheme of turbo-encoder

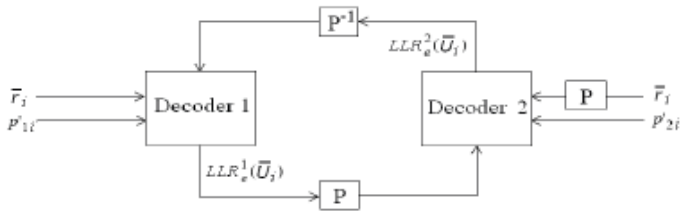

Figure 3 – Scheme of turbo-decoder

The Viterbi decoder [3] in the case of error-free reception of all the sequence dibits will have information about this sequence, and also about the diagram of encoder states. Having all of this information, encoder must restore the sequence of bits, which was at first. For this purpose use appropriate table and calculate metric of errors. Consequently, is built a trellis, in which lay the path, that has the shortest metric accumulated errors for all encoder tacts. The method of decoding by the Viterbi algorithm shows by itself decoding on the maximum likelihood. The idea of algorithm consists, that the decoder reproduce all possible ways sequential changes states signal by comparing the received code symbols with an approved analogues on a communication channel. On the basis of analysis of errors between the accepted and necessary symbols determine an optimum way (an optimum is consider, that sequence, for which Hamming distance from the accepted sequence is minimum), that conduct search of the most advantageous maximally likelihood way [3].

Decoding of block code is possible with the usage of «hard» (hard decision decoding – HDD) or «soft» (soft decision decoding – SDD) decision-making, and on the output from decoder are «hard» or «soft» data. A decoder with a «soft» decision-making accepts besides a binary size 1 or 0 also trust value, which is related with given bit. A decoder with a «soft» input and «soft» output (soft-input soft-output – SISO) accepts information with a «soft» decision and produces information with a «soft» decision. Conception of SISO decoders is applied in turbo-codes. Turbo-code gives data, which demodulating, with a «soft» decision in the SISO decoder. The output data of this decoder are given in the same (or other) SISO decoder. Then an operation repeats again. This iterative process lasts until will not be accepted a certainty decision. Conception submission initial data back onto the input is analogical to the turbocharger of engine [1, 3].

For the decoding of turbo-codes it is possible to use a several methods: a «soft» output Viterbi algorithm (SOVA); a decoding on the maximum of a posteriori probability (MAP) and his modifications for decreasing the calculable complication (log-MAP and max-log-MAP algorithms) [4].

Will consider the category of «soft» algorithms, which includes maximal likelihood, that minimizes probability of error on the bit of information due to the forecast of a posteriori probability of every separate bit. It is a «soft» output Viterbi algorithm. The usage of SOVA complicates the scheme of Viterbi decoder on 70-80% [2]. Given modification of Viterbi algorithm is developed in 1989 [3], allows to calculate

reliability of information symbols as a logarithm of likelihood ratio (LLR), that it is possible to write down in a formula

$$LLR(\overline{U}_i) = \log\left(\frac{p(\overline{U}_i = 1 | \overline{r})}{p(\overline{U}_i = 0 | \overline{r})}\right). \quad (1)$$

The work of SOVA decoder is divided into two parts [4]. The first part of decoding procedure is carried out like the ordinary Viterbi algorithm, herewith select the most probable code sequence, which answers a way, to laid on a trellis «forward». But there is a condition for saving all metrics for every step and for every state of the decoder. In the second part of SOVA the Viterbi algorithm is executed «back», saving metrics for every state of trellis, herewith form two metrics for the knot of trellis. On the stage of forming «soft» output of SOVA algorithm for i-section determine the most likelihood value of information symbol $\overline{U}_i = a \in \{0; 1\}$ and an appropriate maximal metric, which is found on direct way the Viterbi algorithm, is set equal $Y_i(\overline{U}_i)$.

If to mark $Y_i(1) \approx Y_{\max}$ and $Y_i(0)$ name the maximal metric of the way, associated with the addition of information symbol $\overline{U}_i$, it is possible to point a formula for the calculation of «soft» output.

$$LLR(\overline{U}_i) = Y_i(1) - Y_i(0). \quad (2)$$

A metric of best way for addition $Y_i(\overline{U}_i \oplus 1)$ can be found by the formula [4]

$$Y_i(\omega_i \oplus 1) = \min_{k_1, k_2} \{Y_f(S_{i-1}^{(k_1)}) + BY_i^{(k_1)}(\overline{U}_i \oplus 1) + Y_b(S_i^{(k_2)})\}, \quad (3)$$

where ω – the most probable sequence found by the Viterbi algorithm, $Y_f(S_{i-1}^{(k_1)})$ – metric of way, that «survived» on the direct way for the $(i-1)$-section and state $S^{(k_1)}$, $BY_i^{(k_1)}(\overline{U}_i \oplus 1)$ – metric for the inverted information symbol associated with the transition from a state $S^{(k_1)}$ in $S^{(k_2)}$, $Y_b(S_i^{(k_2)})$ – metric of the way, that «survived» on the way back for the i-moment and state $S^{(k_2)}$, $\{k_1, k_2\} \in \{0, 1, 2, \ldots, 2^n - 1\}$.

Logarithm of likelihood ratio in channel with additive white Gaussian noise (AWGN) at $\sigma^2 = 0{,}5 \cdot N_0$, where N_0 is a spectral density of noise, can be represented in the formula

$$LLR(\overline{U}_i) = 4 \cdot \frac{\overline{r}_i}{N_0}. \quad (4)$$

The result of work with the usage of SOVA decoding algorithm (fig. 4) in a channel with AWGN shows the bit error rate (BER), which follows to the minimum,

which determine by the theoretical possible limit for given code, when there is error-free transmission and decoding of the information. For turbo-codes these values are maximally close to this limit.

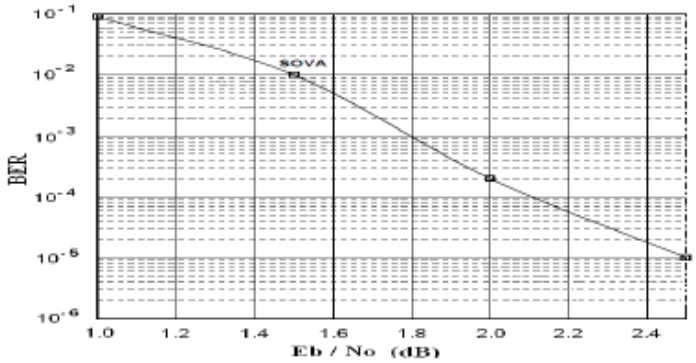

Figure 4 – Simulation for SOVA decoder (E_b – signal energy)

IV. Summary

Consequently, turbo-codes use to encode volume information messages for high-speed data transfer with large error correcting. A main principle of turbo-coding is the use of a several parallell working elementary encoders. In the cascade scheme, such as turbo-code for reliable work decoding algorithm should not be limited by the submission on decoders «hard» scheme decisions. Therefore, for better usage of information, obtained from every decoder apply the «soft» decoding scheme, for example SOVA, which complicates the Viterbi algorithm by 70-80%. The widespread use of SOVA decoder with 3-bit output. Conducted a computer probabilistic simulation showed BER near to 10^{-5}. As marked before, turbo-codes promising method for working with information, for this reason, researches of the systems with application of these codes are actual. Consequently, turbo-codes become a standard for coding the technique of this millennium and of the future [2].

Literature

1. Кулик А.Я. Турбо-код: властивості та застосування для завадостійкого кодування / А.Я. Кулик, С.Г. Кривогубченко, Ю.Ю. Іванов // Оптико-електронні інформаційно-енергетичні технології. – 2011. – № 2(22). – С. 25-33.

2. Sripimanwat K. Turbo Code Applications: Journey From Paper to Realization / K. Sripimanwat. – New York: Springer, 2005. – 386 p. – ISBN 1-402-03686-8.
3. Витерби А.Д. Принципы цифровой связи и кодирования / А.Д. Витерби, Дж.К. Омура. – М.: Радио и связь, 1982. – 536 с.
4. Moon T.K. Error Correction Coding: Mathematical Methods and Algorithms / T.K. Moon. – John Wiley & Sons, Ltd, 2005. – 750 p. – ISBN 0-471-73914-6.