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EVALUATION OF DYNAMIC MEASUREMENT UNCERTAINTY OF VIBRATION ACCELERATION

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In accordance with [1-5], in a linear approximation, the mechanical oscillating system can be represented by one or a combination of the links of the first and second order, i.e. oscillatory link. In this case, the vibrations that are registered at the point of positioning of the accelerometer on the node of the electrospindle represent its response to the impact of the vector generating process. The differential equation describing the dynamic relationship of the input and output values of the vibration acceleration measuring transducer has the form [4, 5]

$$d^2 X_s(t)/\left(dt^2\right) + 2h\,dX_s(t)/(dt) + h_k^2 X_s(t) = F_0/(m)\sin(\omega_0 t), \tag{1}$$

where $F(t) = F_0 sin(\omega_0 t)$ is the harmonic forced power of the oscillation of the surface of the object (input value); F_0 is the force amplitude; ω_0 is the angular frequency of forced power; $X_s(t)$ represents the the mechanical vibrations of the inertial mass; m is the mass of the accelerometer; c is the damping variable; k is the equivalent rigidity of the piezoelements, $h = c/2m$ is the damping coefficient; $h_k = \sqrt{k/m}$ is the critical value damping coefficient [4, 5].

The transfer function of the measuring device will take the form of

$$H(s) = K_{MM} / \left(s^2 + 2hs + h_k^2\right), \tag{2}$$

where K_{MM} is the coefficient of proportionality of the measuring channel of vibration acceleration.

Turning to the domains of frequency and separating the real and imaginary parts, we obtain an expression for the module of the frequency characteristics of the measuring device for vibration acceleration

$$\left|\mathrm{K_C}(j\omega)\right| = \left|\frac{K_{MM}}{(j\omega)^2 + 2h(j\omega) + h_k^2}\right| = \left[\frac{K_{MM}}{\omega^4 - 2\omega^2 \mathrm{h}_k^2 + 4\omega^2 h^2 + h_k^4}\right]^{1/2}. \tag{3}$$

The input signal $F_0 m^{-1} sin(\omega_0 t)$ of vibration acceleration has the form of

$$X(j\omega) = j\omega_0 F_0 \left(\omega_0^2 + (j\omega)^2\right)^{-1} m^{-1}, \tag{4}$$

where ω_0 is the frequency input vibration acceleration, which ranges from 6 to 10 kHz that is, with a minimum value of 18,849.5 and the maximum value is 31,415.9 radians/second [4].

The module image of the input vibration acceleration is written as

$$\left|X(j\omega)\right| = \omega_0 F_0 \left(\omega_0^2 - \omega^2\right)^{-1} m^{-1}. \tag{5}$$

From source literature [6 – 10], it is known that the amplitude of forced harmonic power F_0 is $3 \cdot 10^{-4}$ m. The mass of the accelerometer is $m = 4 \cdot 10^{-2}$ kg**.** The damping variable for the piezoelectric accelerometers is equal to 0.5, equivalent rigidity of the piezoelements is k =2, and the minimum observation time T = 300 s. The proportionality factor or gain K_{MM} of the measuring channel of the vibration acceleration is 10^5 [6, 7, 10].

Substituting the resulting values of the module of the frequency characteristics (3) and the image of the input signal (5) in equation

$$\sigma_{\mathrm{Y}} = \pi^{-1/2}\left(\mathrm{T}^{-1}\int_0^\infty \left|K_C(j\omega)\right|^2 \left|X(j\omega)\right|^2 d\omega\right)^{1/2}, \tag{6}$$

where $\left|\mathrm{K_C}(\mathrm{j}\omega)\right|$ is the frequency response module of the measuring device, used for dynamic measurements [1, 4, 5], we obtain an expression for the evaluation of the uncertainty of dynamic measurement of vibration acceleration in the spectral area

$$\sigma_Y = \pi^{-1/2}\left(\mathrm{T}^{-1}\int_0^{\infty}\frac{K_{MM}\,\omega_0^2 F_0^{\,2}\left(\omega_0^2-\omega^2\right)^{-2} m^{-2}}{\omega^4-2\omega^2\mathrm{h}_k^2+4\omega^2 h^2+h_k^4}\,d\omega\right)^{1/2}. \qquad (7)$$

To represent the characteristicsof the changes in the uncertainty in the dynamic measurement vibration acceleration in the time domain, which is caused by the inertial properties of the measuring transducer in its dynamic mode we must express a Fourier expression for inverse transformation in the form of

$$\mathrm{u_D}(\mathrm{t}) = \pi^{-1/2}\int_0^{\infty}\sigma_{\mathrm{Y}} e^{j\omega t} d\omega = \pi^{-1/2}\left[\int_0^{\infty}\sigma_{\mathrm{Y}}\cos(\omega \mathrm{t})d\omega + j\int_0^{\infty}\sigma_{\mathrm{Y}}\sin(\omega \mathrm{t})d\omega\right]. \qquad (8)$$

Since expression (8) consists of real and imaginary parts, and in assessing the uncertainty we are interested in the amplitude value of dynamic uncertainty, expression (8) may now be written as

$$\left|\mathrm{u_D}(\mathrm{t})\right| = \left[\pi^{-1}\int_0^{\infty}\left(\sigma_Y\cos(\omega \mathrm{t})\right)^2 d\omega + \pi^{-1}\int_0^{\infty}\left(\sigma_Y\sin(\omega \mathrm{t})\right)^2 d\omega\right]^{1/2}. \qquad (9)$$

For the solution of equation (9) in the light of equation (7) we used the Maple 12 mathematical package. At the minimum frequency of the input signal of the vibration acceleration of 6 kHz, and with an observation time of 300 s, the value of dynamic uncertainty is 0.14 m/s^2 (Tab. 1). If the observation period increased to 600 s at a frequency of input signal of the vibration acceleration of 6 kHz, the value of dynamic uncertainty is reduced to 0.1 m/s^2 (Tab. 1). The evaluation value of the signal for vibration acceleration of the bearings of the electrospindle of the motor is 2.34 m/s^2 (Tab. 1).

Table 1. Uncertainty budget of the constituent elements of the measuring channel of vibration acceleration

Quantity	**Mean value, m/s^2**	**Frequency of the study, kHz**	**Observation time, s**	**Value of dynamic uncertainty, m/s^2**	**The expanded dynamicuncertainty** (coverage factor 1.96 at confidence level 95%), **m/s^2**	**Value of relative dynamic uncertainty,%**
The vibration acceleration	2.34	6	300	0.14	0.27	5.98
		6	600	0.1	0.2	4.27
		10	300	0.11	0.22	4.7
		10	600	0.08	0.18	3.29

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