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METHODOLOGY TO EVALUATE THE CONFIDENCE LEVEL FOR CALCULATION OF THE EXPANDED UNCERTAINTY OF MEASUREMENT OF IONS ACTIVITY

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In literary sources [1 - 3], only a partial consideration is given to ways of establishing a trust level for calculating the expanded uncertainty of measurement. The mathematical apparatus that would allow reasonable confidence level set in the concept of measurement uncertainty is not described. It is therefore advisable to propose and describe a methodology for estimating trust level based on the metrological risks of the manufacturer and the consumer, which will allow to establish the value of the coefficient of coverage k for the calculation of the expanded uncertainty of measurement on an example of the use of the developed means for measuring the activity of ions.

The confidence level for calculating the expanded uncertainty is proposed based on the metrological risks of the manufacturer and the consumer by the formula

$$D = 1 - P_n = 1 - (\alpha + \beta),$$
 (1)

where α - metrological risk of the manufacturer; β - metrological risk of the consumer; Pn - the total value of metrological risk.

On the basis of the measurement tool (MT) of an ions activity [4], we describe the procedure for determining the confidence level. The general law of error of measuring ion activity depends on many factors such as activity interfering ions limited properties of ion-selective electrodes, the presence of measurement error in temperature, zero drift, instability of power supply, etc., among which is difficult to identify dominant. This allows us to adopt the law of distribution of the centered value of the error of measuring the activity of ions in the normal, which we describe with the expression

$$p(\Delta pX) = \frac{1}{\sigma_{\Delta pX} \sqrt{2\pi}} \exp \left[-\frac{\Delta pX^2}{2\sigma_{\Delta pX}^2} \right], \quad (2)$$

where ΔpX is centered value of the error of measuring the activity of ions; $\sigma_{\Delta pX}$ is mean square deviation of the centered value of ion activity.

A compatible two-dimensional confidence level density when measuring the activity of ions taking into account the allowable deviation of measurement error ε , which is established by the consumer, is described by the expression [5]

$$p(\Delta pX, \varepsilon) = p(\Delta pX) \cdot p(\varepsilon) = \frac{1}{2\pi\sigma_{\Delta pX}\sigma_{\varepsilon}} \exp\left(-\frac{\Delta pX^{2}}{2\sigma_{\Delta pX}^{2}} - \frac{\varepsilon^{2}}{2\sigma_{\varepsilon}^{2}}\right).$$
(3)

Taking into account expression (3), the metrological risk of the manufacturer α is estimated by the formula

$$\alpha = \frac{1}{2\pi\sigma_{\Delta pX}}\int\limits_{-\Delta}^{\Delta} \int\limits_{-\Delta}^{-\Delta-\Delta pX} \exp\Biggl(-\frac{\Delta pX^2}{2\sigma_{\Delta pX}^2} - \frac{\epsilon^2}{2\sigma_{\epsilon}^2}\Biggr) d\epsilon + \int\limits_{\Delta-\Delta pX}^{\infty} \exp\Biggl(-\frac{\Delta pX^2}{2\sigma_{\Delta pX}^2} - \frac{\epsilon^2}{2\sigma_{\epsilon}^2}\Biggr) d\epsilon \Biggr] d\Delta pX \; , \tag{4}$$

the metrological risk of the consumer β is estimated by the formula

$$\beta = \frac{1}{2\pi\sigma_{\Delta pX}\sigma_{\epsilon}} \left[\int_{-\infty}^{\Delta} \int_{-\Delta - \Delta pX}^{\Delta - \Delta pX} exp \left(-\frac{\Delta pX^2}{2\sigma_{\Delta pX}^2} - \frac{\epsilon^2}{2\sigma_{\epsilon}^2} \right) d\Delta pX d\epsilon + \int_{\Delta}^{\infty} \int_{-\Delta - \Delta pX}^{\Delta - \Delta pX} exp \left(-\frac{\Delta pX^2}{2\sigma_{\Delta pX}^2} - \frac{\epsilon^2}{2\sigma_{\epsilon}^2} \right) d\Delta pX d\epsilon \right]. \tag{5}$$

The control increments of the tolerance fields Δ for the lower and upper limits are assumed to be equal to zero ($\Delta = 0$), and the admission field is the value of the measured value ΔpX , in this case, this is the activity of the pX ions of the constituent elements of humus, we define by the formula

$$\Delta pX = \frac{pX_{\text{max}}}{100} \delta_{\partial \text{max}},$$
 (6)

where pX_{max} is maximum activity of ions of constituent elements of humus (maximum value for nitrate nitrogen is 0,3 pX); $\delta_{\partial max}$ is the maximum permissible relative measurement error set by the consumer (normative documents on the measured value of physical value) - for the measuring channel of ion activity this value is 0.7% in the range of measurements from 6 to 0.3 pX.

Substituting calculated tolerances in the formula for estimating metrological risks manufacturer (4) and consumer (5) and solving them using the Maple 10.0 math package we obtain the following numerical values: $\alpha = 0.0375$, $\beta = 0.00195$. The total value of the metrological risk is $P_n = 0.0375 + 0.00195 = 0.00195$.

0.0395, and the confidence level for calculating the expanded uncertainty of the measurement, according to formula (1) will be equal to $D = 1-P_n = 1-0.0395 = 0.9605$.

Characteristics of the change of metrological risks of the manufacturer and the consumer depending on the parameter $\mu = \sigma_\epsilon/\sigma_{\Delta pX}$, which establishes the relationship between the mean square deviation of $\sigma\epsilon$, which is set by the manufacturer (developer) of mean of measurement of activity and mean square deviation of $\sigma\Delta pX$ of allowable error of measurement of ion activity, which is established by the consumer (normative documents, in which the admissible deviations of measurable values) for $\sigma_\epsilon < \sigma_{\Delta pX}$, $\sigma_\epsilon > \sigma_{\Delta pX}$ are shown in Fig. 1. Characteristics of the change of the general value of metrological risk due to the presence of metrological risks of the manufacturer and the consumer presented in fig. 2.

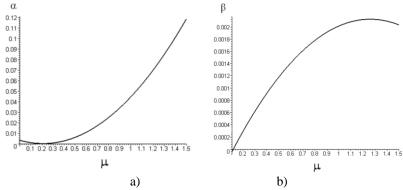


Fig.1 - Characteristics of the change of metrological risks: a) manufacturer; b) consumer

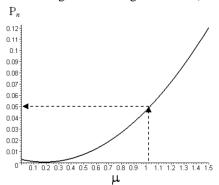


Fig. 2 - Characteristic of the change of metrological risk depending on the parameter μ

Thus, on the basis of the ratio of the value of mean square deviation that is experimentally installed by the manufacturer means of measuring to the value of mean square deviation permissible by the consumer deviation and the received characteristic of change of metrological risk we can be substantiated graphically determine the confidence level for calculating the expanded uncertainty of measurement (Fig. 2). That is, if the parameter μ , which is equal to the ratio of the above-mentioned mean square deviation is 0.93, then having conducted perpendicular to the line from the calculated value of the parameter $\mu=0.93$ to the intersection with the characteristic obtained Changes in metrological risk (dashed lines in Figure 2) can be used to determine the significance of the metrological risk of Pn on on the basis of which the confidence level D=1-Pn = 1-0.04 = 0.96 is calculated on the expression (1), within which it is probable the majority of the distribution of values that can be attributed to the measured value is located.

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