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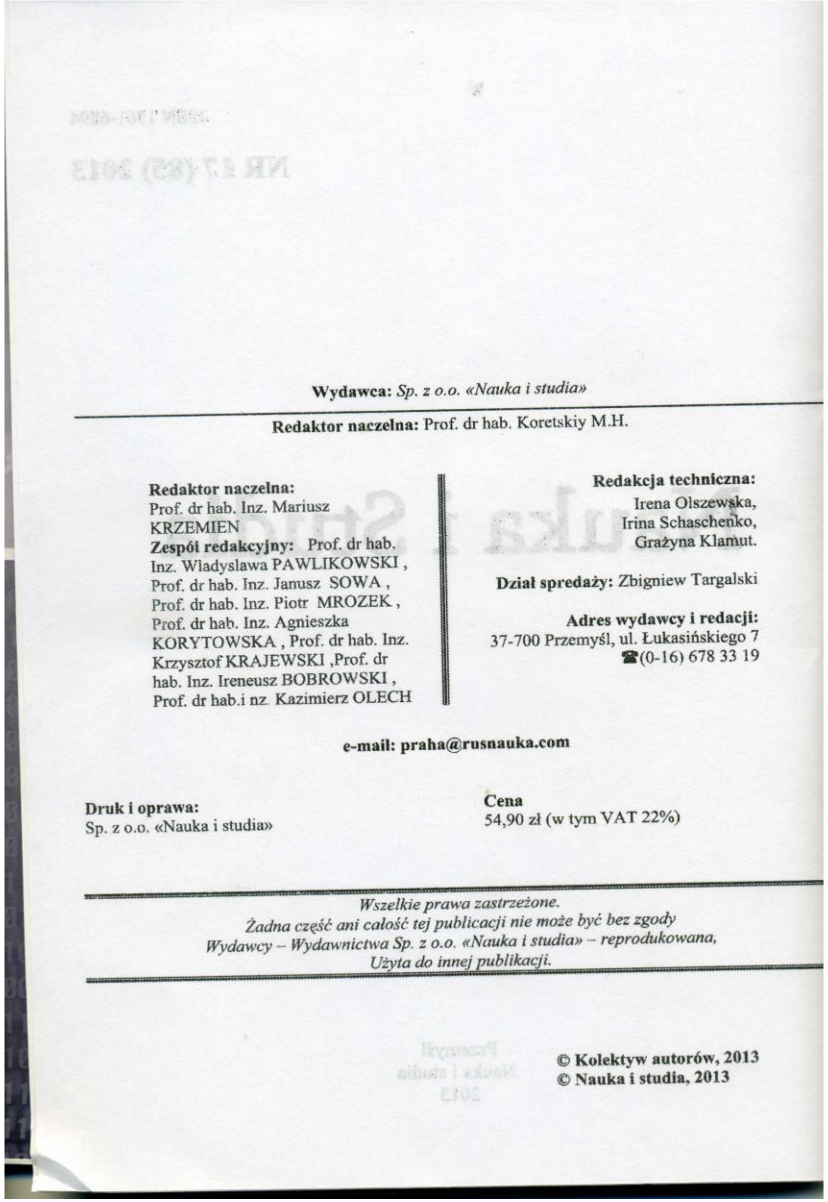

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TREŚĆ

TECHNICZNE NAUKI

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UNCERTAIN GRAPH AS THE MODEL OF BRANCHING TECHNOLOGICAL PROCESS

Annotation

This paper discusses methods of modeling of branching technological processes (BTP) under uncertainty based on uncertain graphs. Simulation of BTP involves the formation of the graph and its modification in case of restructuring of BTP. This formalization takes the form of operations on uncertain graph. To use undefined graph as the model of BTP in decision support system (DSS) the concept of transition uncertainty function between states and transition risk uncertainty functions is introduced. Model of BTP as uncertain graph in combination with certain operations on graphs and operator method of uncertainties conversion is described as convenient basis for creating DSS in management systems of BTP.

Key words: *branching technological process, uncertain graph, decision-making, decision support system, risk uncertainty function.*

I. Introduction

The important problem of industry development as of today is relevant quality control improvement of complex technological processes and objects. The complexity of modern manufacturing systems, which include branching technological processes, in many cases leads to uncertainty in management decisions. Therefore it is necessary to develop methodologies, models, information technology, and decision support systems (DSS) providing opportunity to take the necessary decisions in complex technological processes management [3].

Recently researchersfocus on complex technological processes with many ramifications, including irregular and complex processes and which structure is being changed during the process [4]. For such processes it is necessary to generate stochastic graphs with a vast number of vertices. In general, the model of branching technological process is a random graph where the rule of mutual allocation of edges and vertices is set by probability distribution.

Today four basic approaches to the modeling of complex technological processes are formulated: Poisson random graphs and generalized random graphs; Markov random graphs and walk model for «Count Count» with probabilities that are proportional to the desired properties; a model of «small-world» and its generalizations; evolutionary model of the grid of Barabash and Albert, Price's model [5]. The first three approaches involve the generation of random graphs with a known number of vertices and the set of probabilistic properties.

However, the needs of automation of branching technological processes in a possible equipment and technology upgrade leading to process restructuring, and determination of the end points and sequence of operations based on the results of both hardware control with random errors, and peer review of fuzzy type, require improvement approaches to modeling of BTP. Such approaches should be targeted for use in automated human-machine control systems of BTP.

II. Purpose

The **purpose** of the article is to improve the modeling method of branching technological processes under uncertainty based on uncertain graphs.

III. Results

Branching technological process consists of individual technological operations and preferably has a discrete character. It is represented as directed graph, an example is shown in Fig. 1.

The vertices of the graph represent certain state of BTP, transitions – the implementation process of decision to BTP implementation, subgraph that corresponds to the «past» – the actual previous implementation of BTP, subgraph that corresponds to the «future» – performance forecast of BTP.

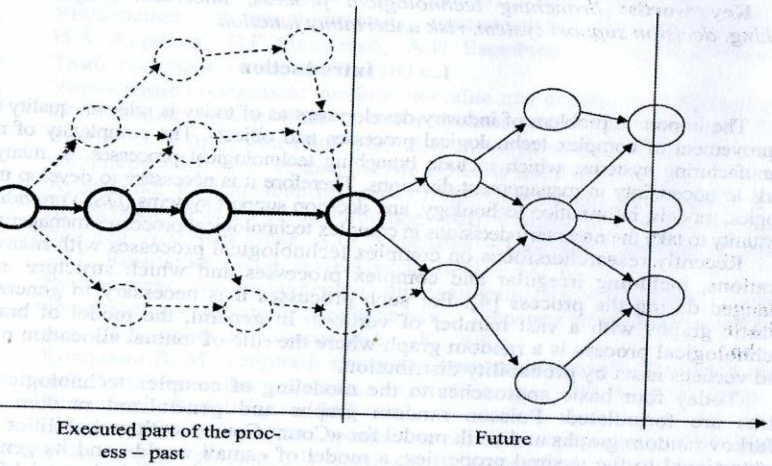

Fig.1. General view of BTP graph :
——— The actual implementation of the process;
- - - - Options for the future implementation of the process;
. . . . . The process that maytake place in other decisions.

Let name the graph $G[S,A]$, where S – the set of vertices (states of BTP), A – the set of connections between them. Let's choose to study a description of the set A with the adjacency matrix.

Each transition between states is related to the cost c_{ij} and time of its implementation t_{ij}, where i – number of top original condition; j – number of vertices of the next state. Each state is characterized by a vector of parameters X_i, and the corresponding effect / losses q_i.

The transition between operations is carried out by deciding so at the end of the operation. Since decisions are made under incomplete information about the parameters of BTP and conditions of its implementation, then decisions themselves and BTP sequence are not fully defined.

Let's characterize definitions and terms of BTP by uncertainty function β, that generalizes the probability distribution density of stochastic data and function of fuzzy data [6]. Similarly certain multi-step BTP management strategy is characterized by a general average effect / losses (risk):

$$R_i = \sum_{j=1}^{i}\left[\int_0^{\infty} c_{j-1,j}\beta(c_{j-1,j})dc_{j-1,j} - \int_{-\infty}^{+\infty} q_j\beta(q_j)dq_j\right]. \quad (1)$$

Additive form (1) allows access to the recursive definition of BTP as the addition new vertices and edges to the graph (2):

$$\begin{gathered} t = t_0; \\ G(S,D,t) = \{S_0; \emptyset\}; \\ G(S,D,t+\Delta t) = G(S,D,t) \cup \{s : D : \forall(S,s)\}, \\ t = t + \Delta t,\ t \le t_n, \end{gathered} \quad (2)$$

where S – the set of possible states at time t (at time t_0 this set contains one element S_0 – real state); D – set of processes of solutions implementing for BTP managing.

BTP modeling involves the graph formation and its modification in case of BTP restructuring. The formalization of this takes the form of operations on uncertain graph:

1. Finding the initial state *root* $[G] = S_0 : \{\xi \forall (\xi, S_0) = \emptyset\}$
2. Finding the final state *terminal* $[G] = S_0 : \{\xi \forall (S_0, \xi) = \emptyset\}$
3. Operation of alternatives removal $G_1 = (S, A \setminus \{a_i\})$
4. Operation of state removal $G_1(V_1, E_1) = G(V,E) \setminus G_0\left(\{v_0\}, \forall_i \{e_{i0}, e_{0i}\}\right)$.
5. Operation of subprocesses union $G_1 = G_1 \cup G_2 = (S_1 \cup S_2, A_1 \cup A_2)$
6. Operation of alternative input $G_1 = (V, E \cup \{e\})$, де $a = (S_u, S_v)$.
7. Operation of intermediate state input $G_1 = (S \cup s_k, A / [u,v] \cup [u,k] \cup [k,v])$

8. Operation of subprocesses supplement $G \setminus G_1 = (S \cup S_1, A \cup A_1)$.

9. States aggregation S_u, S_v

$G_1 = G \setminus S_v \setminus S_u \cup S_a \cup ([S_\xi, S_a], A\xi : \exists[S_\xi, S_u]) \cup ([S_a, S_\theta], \forall\theta : \exists[S_\theta, S_a])$

10. Operation of state bifurcation (splitting)

$G_1 = S \setminus S_0 \cup S_0^I \cup S_0^{II}, A \setminus \xi[\xi, S_a] \setminus \forall[S_0, \xi] \cup \forall[\xi, S_0^I] \cup \forall[S_0^{II}, \xi] \cup [S_0^I, S_0^{II}])$

11. Operation of subprocesses connection

$G = G_1 \overline{\cup} G_2 = G(V_1 \cup V_2, E_1 \cup E_2 \cup \{E(v_1, v_2) : v_1 \in V_1, v_2 \in V_2\})$.

In the process of BTP graph formation and finding optimal way of its passing the risk (1) is assessed in recursive form

$$R_i = R_{i-1} + \left[\int_0^{\infty} c_{i-1,i}\beta(c_{i-1,i})dc_{i-1,i} - \int_{-\infty}^{+\infty} q_i \beta(q_i)dq_i \right] \tag{3}$$

The uncertainty of the graph $G[S,A]$ is based on two components:

– the uncertainty of the conversion options, which assumes the risk (1);

– the uncertainty as to the choice of making transitions between operations, resulting in uncertainty values at risk.

We estimate the probability of the decision-making to move to the next operation.

In operator form uncertainty function of risk:

$$\beta(R_i) = \Phi^{(3)}\beta(R_{i-1})\beta(\overline{c}_{i-1,i})\beta(\overline{q}_i), \tag{4}$$

where Φ – conversion operator of uncertainty function, which follows from the definition of the expression

$$\beta(Y) = \overline{\Phi^{(n)}}[\beta(X)] \tag{5}$$

or in expanded form

$$\beta(y_i) = \Phi_i^{(n)}[\beta(x_1, x_2, \ldots x_m)], \; i = 1 \ldots k,$$

where Y – the vector of states (outputs) of system with dimension k; X – the vector of input actions with dimension m; $\overline{\Phi^{(n)}}$ – the vector of transform operators of generalized uncertainty functions (GF) with dimension k; n – order of operators.

Operators of model (5) are implemented using integral transforms

$$\beta(y_i) = \int_{\Omega_1} \overset{(n)}{\cdots} \int_{\Omega_m} \phi\left[y_i, X, \|R_{x_i x_j}\|\right] \beta(x_1, x_2, \ldots x_m)(dx_1)^{n_1} \ldots (dx_m)^{n_m},$$

where $n = \sum_{j=1}^{m} n_j$ – order of operator (multiplicity of integration); $\phi[\bullet]$ – core of conversion; $\|R_{x_i x_j}\|$ – correlation matrix.

Operations with umbrella function are defined – unary, binary operations, comparison operations of uncertain data and aggravation.

<u>The result of *unary operations* o^1 on uncertain data $x_l \in B_l$ is uncertain data for which</u>

$$\int_{B_2} \beta(x_2)dx_2 = \int_{B_l} \beta(x_l)dx_l, \tag{6}$$

moreover $B_1 \subset B$, $B_2 \subset B$, $B_1 : \forall x_1 \to x_2 = o^1(x_1)$.

$$\beta_y(y) = \Phi^{(1)}[\beta_x(x), N(x, y)] = \int_{-\infty}^{+\infty} \beta_x(x)\varphi(x, y)dx, \tag{7}$$

where is core $\varphi(x, y) = \delta[y - N(x)]$ – Dirac delta function.

Integral-differential operation

$$\beta_Y(y) = \Phi^{(n)}\beta_X(x) = \int_{-\infty}^{+\infty} \beta_X(x_n - m_X^{(1)}(t))\varphi^{(n)}(x_n, y, \omega)dx_n, \tag{8}$$

$$\varphi^{(n)}(x, y, \omega) = \int_{-\infty}^{+\infty} \overset{n-1}{\cdots} \int_{-\infty}^{+\infty} \prod_{i=1}^{n-1} \beta_X\left(x_1 - m_X^{(1)}\right) \times$$

$$\times \delta\left[y - (1 - a)\, m_Y^{(1)} - a \sum_{i=1}^{n-1} x_{n-1}(t - i\tau)\, g(i\tau) \right] dx_1 \cdots dx_{n-1};$$

where $n = ent[t/\Delta\tau]$; $a = \sqrt{\dfrac{D_Y^{(2)}}{\tau D_X^{(2)} \sum_{i=1}^{n-1} g_0^2(i\tau)}}$; $g(t)$ – pulse transient response;

$g_0(i\Delta\tau) = \dfrac{1}{\Delta\tau} \int_{i\Delta\tau}^{(i+1)\Delta\tau} g(\tau)d\tau$; $\Delta\tau$ – sampling interval.

<u>The result of *binary operations* o^2 on uncertain data $x_1 \in B_1$ i $x_2 \in B_2$ is uncertain data $x_3 \in B_3$, for which</u>

$$\int_{B_3} \beta(x_3)dx_3 = \int_{B_1} \int_{B_2} \beta(x_1, x_2)dx_1 dx_2, \tag{9}$$

moreover $B_1 \subset B, \quad B_2 \subset B, \quad B_3 \subset B$, $B_3: \forall x_1, x_2 \to x_3 = x_1 o^2 x_2$.

$$\beta_Y(y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \beta_X(\overline{x_1})\beta_X(\overline{x_2})\varphi^{(2)}(x_1, x_2, y)\,d x_1 d x_2, \qquad (10)$$

where

$$\varphi^{(2)}(x_1,x_2,y)= \int_{-\infty}^{+\infty} \delta[y - N(x_1,\zeta)]\delta\left[\xi - r_{X1,X2}\sqrt{\frac{D^{(2)}_{X2}}{D^{(2)}_{X1}}}\left(x_1 - m^{(1)}_{X1}\right) - \sqrt{1 - r^2_{X1,X2}}\left(x_2 - m^{(1)}_{X2}\right) - m^{(1)}_{X2}\right] d\zeta;$$

m_{x1} – first initial moment X_1; m_{x2} – first initial moment X_2; D_{x1} – second central moment X_1; D_{x2} – second central moment X_2; $r_{X1,X2}$ – second mixed normalized central moment X_1 and X_2.

The ratio of equation: uncertain data x, y are considered equal if $\beta_X = \beta_Y$.

The ratio of inequality: for uncertain data $X > Y$ if $Z = X - Y$ and

$$\int_0^{+\infty} \beta_Z dz > \int_{-\infty}^{0} \beta_Z dz. \qquad (11)$$

The uncertain data x' is exacerbation of uncertain data x if:

1. $X_x = X_{x'}$, where X – domain of definition of GF;
2. $M_x = M_{x'}$, where M – mode of GF;
3. $\exists\,[a,b]$: $M_{X'} \in [a,b]$; $\forall x \in (a,b) \to \beta_{X'}(x)\beta_X(x)$; $(x=a)\vee(x=b) \to \beta_{X'}(x)=\beta_X(x)$.

More complex transformations are defined by decomposition *to consider three types of transformations.*

The vector of BTP parameters X can be determined by iterative procedure:

$$\text{while } X\text{-}X_0 \geq \varepsilon \text{ do } \{\forall i : X := X_0 * A_i * W^T;\ X_0 := X\}, \qquad (12)$$

where $\boldsymbol{A_i}$ – row vector of the matrix $\boldsymbol{A}$; $\boldsymbol{W}$ – operator of acceptance and implementation of solutions.

The elements of the set $\boldsymbol{A}$ with already defined structure take values $\{a_{ij} = (0/1)\}$. In an uncertain graph let's describe the relationship between subsystems using GF in the interval [0,1]. Let's name

$\beta(A_i) = \{\beta(a_{ij})\}$

$\beta(X) = \{\beta(x_j)\}$

Then in the operator form (12) can be written

while $\beta(X\text{-}X_0) > \varepsilon$ do

$$\{\forall i : \beta(X) := \{\Phi^{(n)}(W)\}^T\,[\Phi^{(2)}[\beta(X_0), \beta(A_i)]];\ \beta(X_0) := \beta(X)\} \qquad (13)$$

To use an undefined graph $G[\boldsymbol{S},A]$ as a model of BTP in the decision support system let's add its matrix $M_A|\beta_A(a_{i,j})|, M_R|\beta_R(r_{i,j})|$, where β_A - function of the transition uncertainty between states, β_R – function of the uncertainty of risk transfer. For the elements of the matrix M the following condition is true: $\forall i : \Sigma\beta(a_{ij}) = 1$.

Then, in case of independent decisions in individual states and additivity of loss function the uncertainty of BTP implementation is:

$$B = M^n_a.$$

BTP Model as uncertain graph is implemented in «TP Modeling» software of information technology for decision making of BTP management.

IV. Conclusions

BTP Model as uncertain graph in combination with certain operations on graphs and operator method of uncertainties conversion is a convenient basis for creating DSS in BTP management systems.

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