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MATHEMATICAL AND COMPUTER MODELING OF POLITICAL ELECTIONS

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Abstract

In this paper, we have modeled the dynamics of an election process in the case of the transformation of bipartisan (the ruling party and one opposition party) election into three-party (ruling and two opposition parties) election during the inter-election period. Special case is considered.

In the particular case, with the zero demographic factor of all three political parties, and also with some relation between the variable coefficients of the model in the time segment [0, T_1] and the constancy of the model coefficients in the time interval (T_1, T), exact analytic solutions are found.

For computer modeling of the process of transformation of bipartisan elections into three-party, in general, the model coefficients are taken as variable time functions. Also, the process of the appearance of a new party in the period between two elections (in many countries this period is T = 4 years = 1460 days) is considered after 400, 800 and 1200 days from the date of previous elections. To analyze the results obtained, the case T_1 = 400 days is taken. It should be specially noted that with some selection of the variable coefficients of the model (for example, when the coefficient of attracting supporters for the emerging second opposition party significantly exceeds the coefficients of attracting the other two parties), in the time interval (T_1, T) there is an effect of an increase and a subsequent decrease in the number of supporters of the first opposition and the ruling party, i.e. the functions have a stationary points.

Аннотация

В этой статье мы моделировали динамику избирательного процесса в случае трансформации двухпартийной (правящей партии и одной оппозиционной партии) выборов в трехсторонние (правящие и две оппозиционные партии) выборы в межвыборный период.

В частном случае с нулевым демографическим фактором всех трех политических партий, а также с некоторой зависимостью между переменными коэффициентами модели в сегменте времени [0, T_1] и постоянством коэффициентов модели во временном интервале (T_1 , T) найдены точные аналитические решения.

Для компьютерного моделирования процесса трансформации двухпартийных выборов в трехпартийные, в общем, коэффициенты модели принимаются как переменные функции времени. Кроме того, процесс появления новой партии в период между двумя выборами (во многих странах этот период составляет T = 4 года = 1460 дней) рассматривается после 400, 800 и 1200 дней с даты предыдущих выборов. Для анализа полученных результатов был взят случай T_1 = 400 дней. Следует особо отметить, что при некотором выборе переменных коэффициентов модели (например, когда коэффициент притяжения сторонников для новой второй оппозиционной партии значительно превышает коэффициенты притяжения двух других сторон), в интервале времени (T_1, T) наблюдается эффект увеличения и последующего уменьшения числа сторонников первой оппозиции и правящей партии, т. е. функции имеют стационарные точки.

Introduction

Mathematical and computer modeling has been widely recognized in such disciplines as sociology, economics, history, political science and others [1,2]. Of great interest is mathematical modeling of political elections. Many scientists deal with this topic. In most cases, stochastic models of political elections are proposed, which involve an analysis of the statistics of elections already held [3,4]. Extremely important is the creation of a mathematical model, which would give an opportunity to define the dynamics of change in the number of supporters of different political subjects during the election period and a possible forecast of the election results. In [5] is proposed the nonlinear mathematical model with variable coefficients in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the progovernment and two opposition parties from election to election. In the particular case obtained exact analytical solutions.

1. General mathematical model of transformation of two-party elections to three-party elections

We will consider the general mathematical model of transformation of two-party elections to three-party elections, from the accounting of demographic factor of elections which we have an appearance:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t)-\alpha_2(t))N_1(t)N_2(t) + (\alpha_1(t)-\alpha_3(t))N_1(t)N_3(t) - F_1(N_1(t),t) + \gamma_1(t)N_1(t) \\ \\ \dfrac{dN_2(t)}{dt} = (\alpha_2(t)-\alpha_1(t))N_1(t)N_2(t) + (\alpha_2(t)-\alpha_3(t))N_2(t)N_3(t) - F_2(N_2(t),t) + \gamma_2(t)N_2(t) \\ \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t)-\alpha_1(t))N_1(t)N_3(t) + (\alpha_3(t)-\alpha_2(t))N_2(t)N_3(t) + F_1(N_1,t) + F_2(N_2,t) + \gamma_3(t)N_3(t) \end{cases} \quad (1.1)$$

$t \in (0,T]$,

$$N_1(0) = N_{10},\ N_2(0) = 0,\ N_3(0) = N_{30},\ N_{30} > N_{10} \quad (1.2)$$

$N_2(t) \equiv 0,\ F_2(N_2(t),t) \equiv 0,\ t \in [0,T_1),\ T_1 < T \quad N_2(T_1) = N_{20} > 0$

In this nonlinear mathematical model(1.1), (1.2) all coefficients are variables and demographic factors are taken into consideration. Equations (1.1) is defined in the interval $t \in (0,T]$, and initial conditions (of Cauchy) $t = 0$ moment of time. We look for the solution of the Cauchy problemon the segment $t \in [0,T]$ in the class of continuous differentiable functions $N_1(t), N_2(t), N_3[t] \in C^1[0,T]$. In a nonlinear system of differential equations (1.1): $N_1(t), N_2(t), N_3(t)$ – are the numbers of supports of two opposition and one ruling party $N_3(t)$ at time *t*; $t = 0$ - is the time of previous elections, when one of the parties $N_3(t)$ won the elections and became the ruling party; $t = T$ – is the time of the following elections (in many cases *T*= 4 years or 1460 days); $t = T_1$($0 < T_1 < T$) - time-point, when to political arena there is the second opposition party ; $\alpha_1(t), \alpha_2(t), \alpha_3(t)$ - are the coefficients of attracting votes bythe first and second opposition party and the ruling party at time *t*. They largely depend on the action programs, as well as financial, technological and informational capacities of the political parties; $F_1(N_1(t),t), F_2(N_2(t),t)$ - are the continuous positive functions, that define the scale of used administrative resources; $\gamma_1(t), \gamma_2(t), \gamma_3(t)$ - are the coefficients that describe demographic changes of the parties.

2.A special case with zero demographic factor and variable coefficient of elections

We will consider a special case when the functions characterizing use of administrative resources are linear concerning the first variables, demographic factors of elections of all three parties are equal to zero

$$F_1(N_1(t),t) = \beta_1(t)N_1(t),\ F_2(N_1(t),t) = \beta_2(t)N_2(t), \quad (2.1)$$

Taking into account (2.1), (1.1) and (1.2) will correspond in the following look

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_3(t))N_1(t)N_3(t) - \beta_1(t)N_1(t) + \gamma_1(t)N_1(t) \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t) - \alpha_1(t))N_1(t)N_3(t) + \beta_1(t)N_1(t) + \gamma_3(t)N_3(t) \end{cases} \quad (2.2)$$

$t \in (0, T_1]$ $N_1(0) = N_{10}$, $N_3(0) = N_{30}$, $N_{30} > N_{10}$

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_{11}(t) - \alpha_{31}(t))N_1(t)N_3(t) + (\alpha_{11}(t) - \alpha_2(t))N_1(t)N_2(t) - \beta_{11}(t)N_1(t) + \gamma_1(t)N_1(t) \\ \dfrac{dN_2(t)}{dt} = (-\alpha_{11}(t) + \alpha_2(t))N_1(t)N_2(t) + (\alpha_2(t) - \alpha_{31}(t))N_3(t)N_2(t) - \beta_2(t)N_2(t) + \gamma_2(t)N_2(t) \\ \dfrac{dN_3(t)}{dt} = (-\alpha_{11}(t) + \alpha_{31}(t))N_1(t)N_3(t) + (-\alpha_2(t) + \alpha_{31}(t))N_3(t)N_2(t) + \beta_{11}(t)N_1(t) + \beta_2(t)N_2(t) + \gamma_3(t)N_3(t) \end{cases}$$

$$t \in (T_1, T] \quad N_1(T_1) = N_{11}, \quad N_2(T_1) = N_{20} > 0, \quad N_3(T_1) = N_{31} \quad (2.3)$$

Thus, we get the two Cauchy problem (2.2), (2.3).
Consider the case, when

$$\gamma_1(t) = \gamma_2(t) = \gamma_3(t) \equiv 0 \qquad t \in [0_1, T] \quad (2.4)$$

Then the exact solution of the Cauchy problem (2.3) has the form

$$\begin{cases} N_1(t) = \dfrac{\dfrac{(\alpha_{11} - \alpha_{31})c - \beta_{11}}{\alpha_{11} - \alpha_{31} + p(\alpha_2 - \alpha_{31})} N_{11} \exp\{[(\alpha_{11} - \alpha_{31})c - \beta_{11}](t - T_1)\}}{\dfrac{(\alpha_{11} - \alpha_{31})c - \beta_{11}}{\alpha_{11} - \alpha_{31} + p(\alpha_2 - \alpha_{31})} + N_{11} \exp\{[(\alpha_{11} - \alpha_{31})c - \beta_{11}](t - T_1)\} - N_{11}} \\ N_2(t) = pN_1(t) \\ N_3(t) = c - (p+1)N_1(t) \end{cases} \quad (2.5)$$

$t \in [T_1, T]$

3. Computer modeling

For the computer simulation of the process of transforming bipartisan elections into three-party elections in the general case (1.1), the model coefficients are taken as variable time functions. The process of entering the new opposition party's political arena in the period between the two elections for definiteness was considered after 400, 800 and 1200 days from the date of the previous elections. For a specific numerical count, the coefficients of the model are taken as exponentially increasing functions, although in the sequel can be considered other cases. For clarity, with a numerical calculation, the number of voters was chosen by the example of two countries - Georgia and Australia. It should be specially noted that with some selection of the variable coefficients of the model (for example, when the coefficient of attracting supporters for the emerging second opposition party significantly exceeds the coefficients of attracting the other two parties), in the time interval (T_1, T) there is an effect of an increase and a subsequent decrease in the number of supporters of the first opposition and the ruling party, i.e. the functions N_1 (t) and N_3 (t), $t\in (T_1, T)$, have a stationary point $t^* \in (T_1, T)$, where $N_1^{'}(t^*)=0$ and stationary point $t^{**} \in (T_1, T)$, where $N_3^{'}(t^{**})=0$ (Fig. 1).

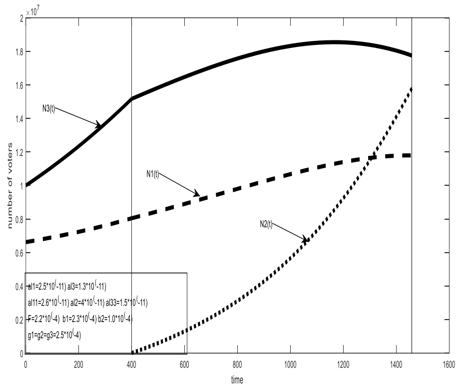

Fig 1.

Conclusions

The paper considers a nonlinear mathematical model of the transition of bipartisan elections to three-party elections, when in the period between elections to the political arena the second opposition party stands. In the case, with the zero demographic factor of all three batches, and also with some relation between the variable coefficients of the model in the segment $[0, T_1]$ and the constancy of the model coefficients in the interval (T_1, T), exact solutions are found. In the case of variability of all coefficients of the model, numerous computer calculations are performed, in which, depending on the choice of variable coefficients and initial data, different election results are obtained. The results of the numerical account can be used by both the ruling and opposition parties to achieve their goals.

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