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22–25 September 2015
Lublin-Nałęczów, Poland

Editors
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The use polynomials as a possible variant analytical processing of logic-time functions

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ABSTRACT

The paper discusses the possibility of time-logic functions (LTF) using polynomial, extending the formal mathematical apparatus to practical problems of pattern recognition. Reviewed operations on LTF have been presented in case of submitting them in the form of polynomials as well as hardware implementation of these operations have been discussed.

Keywords: time-logic functions, pattern recognition

1. INTRODUCTION

Modern state of hardware development shows search for architectures of the new generation computers, which theoretical basis frequently is based on the theory of artificial intelligence and their practical implementation now uses optical elements and gadgets.

Since information processing is a sequence of certain operations in time domain, all the signals related to the processing can be treated as logical functions of time. Such an approach is especially important, as stated in paper[2], for describing optical elements[1]. The notion of logic-time function (LTF)[2] was introduced by V. Kozhemiako. Later, to ensure analytical processing of LTF, mathematical apparatus was created, which enables improvement of the formal modelling process[4].

2. MATHEMATICAL APPARATUS FOR ANALYTICAL PROCESSING OF LTF

The mathematical apparatus is based on Δ-partitioning of LTF with regard to time. A certain time interval $[t_k, t_{k+1}]$ can be splitted into Δ-intervals. It should be underlined, that while constructing arbitrary Δ- partition, it is necessary to follow the following rules:

Rule 1. Boundaries of a Δ-interval are regarded to be beginnings of the corresponding Δ- partition, i.e. the mentioned partitioning happens on both sides of the Δ- interval.

Rule 2. If fractional part of $\frac{t_{k+1} - t_k}{\Delta_i}$ is not less than 0.5, then the number of the obtained intervals is increased by one.

At an arbitrary Δ–interval, LTF partition can change its value. In such cases it is reasonable to correct value of the corresponding function. That correction is identical to quantization, but for LTF such correction is performed on the same coordinate as the mentioned discretization. In this context it is better to use the term "filtration".

Due to the mentioned Δ-partitioning, any LTF of the $f(t,t_1,T_1)$ form can be presented in form of $f(t,t_1,\Delta_i,t_1+\Delta_i,\Delta_i,t_1+2\Delta_i,\Delta_i,...,t_1+(n-1)\Delta_i,\Delta_i)$, if $T_1 = n\Delta_i$.

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1 – 21
X – 5

The properties and operations on LTF were studied[4,6,7], including specific features of finding ordinary and parametrical derivatives of the first and higher orders for both binary and multi-valued functions, antiderivation, etc.

It should be noted that the mentioned mathematical description of LTF enables studying of logic-time functions only using heuristic approach. Thus, the problem of further formalization of LTF presentation suitable for further automatic processing turns to be actual.

3. POSSIBLE VARIANT OF LTF PRESENTATION IN FORM OF POLYNOMIALS

In this case it should be underlined that any logic-time function at an arbitrary interval can be presented as a code combination where each digit corresponds to certain Δ-interval4. It gives grounds to present any LTF in polynomial form. The goal of such approach is an attempt to present wide spectrum of operations on LTF, first of all – the shift operation.

Let time interval includes Δ-intervals, then the corresponding to this function polynomial of the $(n-1)$-th power is:

$$P_{n-1}(t) = p_{n-1}t^{n-1} + p_{n-2}t^{n-2} + ... + p_1 t + p_0, \qquad (1)$$

where $p_i = \{0,1\}$, moreover $p_i = 0$ correspond to zero amplitude of LTF, $p_i = 1$ – non-zero.

For example:

$P_4(t) = t^4 + t^3 + t^2$ polynomial corresponds to $f_1(t, t_1, T_1)$ function (Fig. 1),

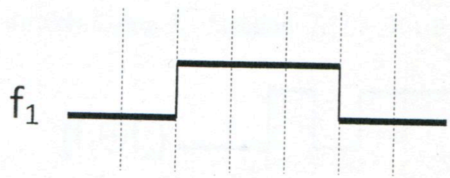


Figure 1. Possible variant of f_1 LTF, which between two zeroes takes constant value.

$P_5(t) = t^5 + t^4$ polynomial corresponds to $f_1(t, t_2, T_2)$ function (Fig. 2),

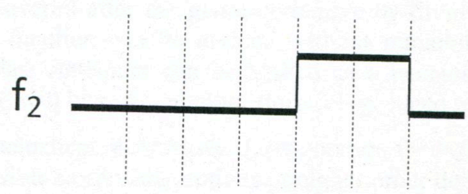


Figure 2. Possible variant of f_2 LTF, which between two zeroes takes constant value

4. OPERATIONS ON LTF IN CASE OF PRESENTING THEM IN POLYNOMIAL FORM

Operations of addition and subtraction in modulus 2 can be performed on polynomials. The operations are commutative and associative. Thus, to functions $f_1(t, t_1, T_1) \oplus f_2(t, t_2, T_2)$ (Fig. 3)

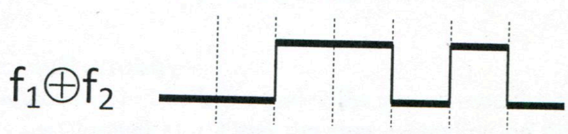


Figure 3. Result of adding functions f_1 and f_2 in modulus two.

corresponds polynomial $P_5(t) = t^5 + t^3 + t^2$.

It should be noted that operation of division is normal division of polynomials, the only difference is, instead of subtraction addition in modulus 2 is used. For example,

$$
\begin{array}{l|l}
t^6+t^5+t^4+t^2 & t+1 \\ \hline
t^6+t^5 & t^5+t^3+t^2 \\ \hline
\quad t^4+t^2 & \\
\quad t^4+t^3 & \\ \hline
\qquad t^3+t^2 & \\
\qquad t^3+t^2 & \\ \hline
\qquad\quad 0 &
\end{array}
\tag{2}
$$

One more operation necessary to process information in logic-time environment is operation of differentiation, which is the result of adding in modulus 2 any logic-time function and this very function with Δ-interval delay. Since one Δ-interval delay for polynomial is its multiplication by t to the first power, then differentiation for polynomial is reduced to its multiplication by polynomial $(t+1)$.

For example, polynomial $P_5(t) = t^5+t^3+t^2$ corresponds to function $f_1(t,t_1,T_1)\oplus f_2(t,t_2,T_2)$. Then, to derivative $(f_1(t,t_1,T_1)\oplus f_2(t,t_2,T_2))'$ corresponds polynomial:

$$P(t)=(t^5+t^3+t^2)(t+1)=t^6+t^5+t^4+t^3+t^3+t^2=t^6+t^5+t^4+t^2. \tag{3}$$

The same result can be obtained by differentiating the function $f_1(t,t_1,T_1)\oplus f_2(t,t_2,T_2)$ in traditional way (Fig. 4).

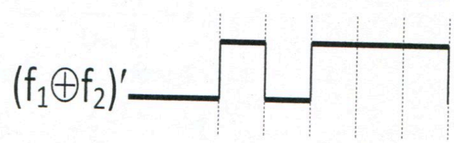


Figure 4. Result of differentiating the sum in modulus 2 of two functions f_1 and f_2.

It's clear that function can be recovered after the given derivative by dividing it by $(t+1)$ (see example of division of polynomials). In other words, if function can be divided without remainder by $(t+1)$, then it means that the given function can be integrated. The last statement can be treated as a necessary condition of the function integrability. Moreover, integration itself of LTF will be rather simple, since it is reduced to the division of polynomials.

Note, that for convenience of mathematical records, we'll use notation $f = P_n(t)$.

Like in classical calculus, it is possible to consider notions of higher order derivatives, in particular:

$$f'' = (f')' = f\cdot(t+1)(t+1) = f\cdot(t+1)^2, \text{ where } (t+1)^2 = t^2+1; \tag{4}$$

$$f''' = (f'')' = f\cdot(t+1)^3, \text{ where } (t+1)^3 = t^3+t^2+t+1; \tag{5}$$

$$\vdots$$

$$f^{(n)} = f\cdot(t+1)^n \tag{6}$$

It is possible to show that properties of binary LTF derivative are valid in the case of polynomial addition as well[4]. Among the basic properties are:

1) LTF and its inverse have equal derivatives.
2) Derivative of sum in modulus two of LTF's is equal to the sum in modulus two of the derivatives of LTF's.
3) Higher order derivatives transformed into initial function depending on the input signal duration expressed in Δ-intervals:
   a) when input signal duration is 2 to 4 Δ - intervals, the fourth derivative returns to the initial function,
   b) 5 to 8 Δ - intervals – the eighth derivative returns to the initial function,
   c) 9 to 16 – sixteenth derivative returns to the given function and so on.

As an example, let us prove the second property for two functions case. Indeed, let we have two logic-time functions $f_1 = P_n(t)$ and $f_2 = P_m(t)$. Since $f_1 \oplus f_2 = P_n(t) + P_m(t)$, then:

$$(f_1 \oplus f_2)' = (P_n(t) + P_m(t))(t+1) = P_n(t)(t+1) + P_m(t)(t+1) = f_1' \oplus f_2' . \tag{7}$$

Also there are differential operators controlled by parameter. These operators are used for processing matrices of binary data and are the base for compiling complex controlled procedures, in particular, compression, expansion and multiplication. In many cases there is possibility of restoring the initial structure of binary data by the results of transformation. This property is used when constructing calculation processes under limited memory.

Parametric Boolean derivative is defined as:

$$\frac{\partial f(X)}{\partial(\tau X)} = f(X) \oplus f(X+\tau), \tag{8}$$

where τ - parameter that specifies the value of the change of X. It determines the fact of the change of the $f(X)$ function value under change of the argument X by parameter τ.

Boolean derivatives with parameters are used in analysis and synthesis of binary automata, when detecting dynamical errors in automata when processing and synthesizing binary images.

In LTF case, parameter τ is replaced with $p\Delta$, where p=1,2,…, τ. Then finding of parametric derivative after parameter $\tau = p\Delta$ is reduced to multiplying the initial function by the $(t^p + 1)$ polynomial:

$$f^{\tau} = f^{(p\Delta)} = f \cdot (t^p + 1) \cdot \tag{9}$$

The antiderivative of LTF, when presented by polynomial, is found by the formula:

$$\int P(t)dt = \frac{P(t)}{(t+1)} + T, \tag{10}$$

where T is constant. Similarly, parametrical antiderivative is found as:

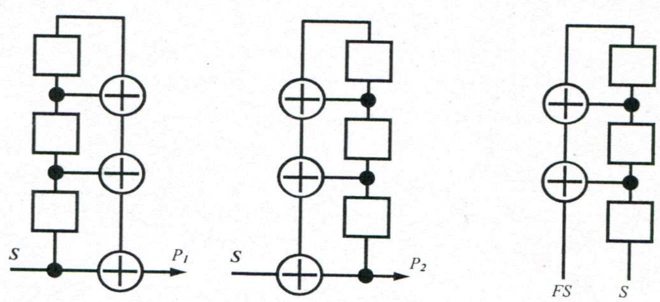
$$\int P(t)dt = \frac{P(t)}{(t^p+1)} + T. \tag{11}$$

It is clear that not all LTF's are subjected to integration. Besides, n-fold integration is possible as inverse operation to finding n-th derivative.

5. HARDWARE REALIZATION OF OPERATIONS OVER LTF

Cannot be ignored the fact that finding of derivatives, especially of higher orders and parametric, resembles cyclic encoding in which input message is multiplied by code-forming polynomial. As we can see, physical sense of such encoding may lie in differentiating of input code; respectively, decoding – is finding of antiderivative (integration). All this provides new reciprocal possibilities of studying both LTF and polynomial, cyclic encoding.

Hardware realization of operations over LTF may conveniently be performed on the base of shift registers with direct and inverse links (fig. 5). To analyze their work, we'll use three-digit registers which process input sequence ***S***, and output sequence denoted as ***P*** [4].



a) direct links b) inverse links c) operator *F*

Figure 5. Differentiation and integration schemes

Similarly, an t operator was introduced, which stands for one-bit delay of the input sequence. Respectively, St^2 stands for the sequence S with two-bit delay. Then the direct links register, in our case, is described with expression:

$$P_1 = S(1+t+t^2+t^3) = (1+F)S, \quad F = t+t^2+t^3, \tag{12}$$

where addition is performed in modulus 2.

Operator F is introduced in order to describe register part regardless of the use of direct or inverse links. So, if P_2 output of a register with inverse link, then FP_2 returns after going through the inverse link loop and $S+FP_2$ goes out of the adder creating P_2. Respectively,

$$P_2 = S + FP_2 \tag{13}$$

and therefore:

$$P_2 = \frac{1}{1+F}S = \frac{S}{1+t+t^2+t^3}\ . \tag{14}$$

Certainly, registers with direct and inverse links, due to their operators $1+F$ and $\frac{1}{1+F}$, are obviously reciprocal.

6. CONCLUSIONS

For the first time presentation of logic-time functions with the help of polynomials has been proposed; that extended formal apparatus for LTF analysis for practical problems. Considered properties of operations over LTF in polynomial form extend the knowledge base of LTF theory. Hardware realization of operations over LTF based on shift registers with direct and inverse links has been proposed.

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