NANOSTRUCTURE WITH PERIODIC POSITION DEPENDENT ELECTRON EFFECTIVE MASS

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For a long time yet semiconductors superlattices turned out the main subjects of a lot of nanoelectronics devices such as pillar lasers structures or light emmiting diodes et.cet. Technological progress based on molecular beam epitaxy methods or STM and AFM technologies allows to create nanosystems with demanded spectral properties. As a rule, electron energetic spectrum consisted of narrow minibands, is formed due to the periodic modulation of the basic properties of supperlattice components such as the conduct band bottom and the valency band top.

In this communication will be considered electron energy eugenvalues of an alternative layered system composed of layers with position dependent electron effective mass. Dynamics of a particle with effective mass dependent of coordinates reveals in detail discussed[1] peculiarities. One of them relates to search of models which admits existence of exact solutions. General approach to finding exact results has been proposed in reference [2] for polynomial and exponential dependence mass on coordinates. However, neither eigenvalues or eigenfuntions have been presented. Therefore some exactly soluble model is trated here.

Inside of a layer the electron effective mass depends on coordinate x directed along the supperlattice axis accordingly to the expression proposed by von Ross[3]:

$$m(x) = m_0 f(x) \tag{1}$$

where $f(x)$ is periodic function given as follows:

$$f(x) = \frac{\kappa(x - nL)}{L} + 1 \; ; x \in [nL,(n+1)L] \; ; n \in Z \tag{2}$$

Here L is the period of the layered structure and κ is the coefficient of the linear function slop.

Wave equation after separation of variables has being made assumed its usual form, namely

$$-\frac{\hbar^2}{2m_0}\partial_x \frac{1}{f(x)}\partial_x \Psi = E\Psi \tag{3}$$

In the Schrödinger equation (3) Ψ is dependent only on x component of wave function responsible for a movement along supperlattice axis and E is the energy taken from the bottom of 2dimensional band corresponding to the free translations of electron along the layer.

Due to the coefficients of Schrödinger equation (2) are periodical functions with the period L its solutions have to satisfy to certain boundary conditions. Having recourse to Fluquet theorem one can write the first boundary condition as follows:

$$\Psi(x)\big|_{x=0} = e^{iqL}\Psi(x)\big|_{x=L} \tag{4}$$

Concerning the second one it comes up of probabilitie's flux density continuity, that is:

$$\frac{1}{m_0 f(x)}\partial_x \Psi(x)\Big|_{x=0} = e^{iqL}\frac{1}{m_0 f(x)}\partial_x \Psi(x)\Big|_{x=L} \tag{5}$$

Here q is the wave vector which runs quase continuous values into the first Brilluene zone $-\frac{\pi}{L} < q \le \frac{\pi}{L}$

One can verify that equation (3) has exact solution expressed as linear combination of Bessel and Neumann functions with fractional indices, that is:

$$\Psi(x) = A\left(\frac{\kappa x}{L}+1\right)J_{2/3}\left[\frac{2}{3}Q\left(\frac{\kappa x}{L}+1\right)^{3/2}\right] + B\left(\frac{\kappa x}{L}+1\right)N_{2/3}\left[\frac{2}{3}Q\left(\frac{\kappa x}{L}+1\right)^{3/2}\right] \tag{6}$$

with no dimensional wave number Q determined by formula $Q = \sqrt{\frac{2m_0 E}{\hbar^2}\left(\frac{L}{\kappa}\right)^2}$.

Substitution (6) into the boundary conditions (4) and (5) leads to the system of linear homogeneous equations refered to the coefficients A and B. By equaling to zero the determinant of this system one can find the dispersion law given as follows:

$$E_n(q) = \frac{\hbar^2}{2m_0}\left(\frac{3\kappa}{2L(y_L^{3/2}-1)}\right)^2\left\{\pi n + arcCos\left[\frac{2CosqL}{y_L^{1/4}+y_L^{-1/4}}\right]\right\}^2 \tag{7}$$

where $y_L = \kappa + 1$ and $n \in Z$.

Accordingly to the obtained result (7) the electron spectrum shows features correspondent to the supperlattice structure that is it consists of subbands separated by forbidden zons with energetic gaps

$$E_{gn} = \frac{\pi^2\hbar^2}{m_0}\left(\frac{3\kappa}{2L(y_L^{3/2}-1)}\right)^2\left(n + \frac{1}{2} + \frac{1}{\pi}\arccos\frac{2}{y_L^{1/4}+y_L^{-1/4}}\right) \tag{8}$$

Consequently periodic modulation of position dependent effective mass as well as the modulation of crystal potential allows to create supperlattice whose demanded spectral properties can be achieved by fitting of parameters L and κ.

References

1. Levy-Leblond J.M.,Phys.Rev. A 52 (1995) 1845
2. Koç R.,Saym S.,J.Phys.A: Math.Theor.43(2010) 455203
3. Von Roos O., Mavromatis H., Phys. Rev. B31 (1985), 2294.