MATHEMATICAL MODEL OF MUSCLE TISSUE REDUCTION ACTIVITY

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Abstract. The article considers improved mathematical models of the physiological process of muscle contraction based on the known hypotheses of the process of the human body musculoskeletal system functioning. A mathematical model of changing the force load of muscle tissue for the modes of isometric tetanus and contraction (elongation) of the muscle at a constant rate was developed according to the first phenomenological hypothesis of A. Hill. Based on A. Huxley's hypothesis, a

mathematical model of muscle tissue force loading was developed, which depends on the distribution function of the number of transverse bridges.

Introduction. Currently, the specialized application directions of the mathematical modeling functioning of living organisms constituent systems is developing rapidly [1]. One of such components is the system of muscle tissue, which provides the mechanical function of all internal organs [2]. One of the urgent problems is to establish adequate patterns of the functioning of muscle tissue mechanism at different modes of its loading (isometric, isotonic) [2, 3].

Providing the accuracy of muscle physiological process contraction identification will allow optimizing the methods of treatment, rehabilitation, and sports training [4].

The aim is to increase the accuracy of identification of the physiological process of muscle contraction by developing effective methods of mathematical modeling based on differential equations of these organs functioning, which will increase the accuracy of predicting kinematic and force parameters of the musculoskeletal system.

Problem solving. An adequate mathematical model of real muscle tissue should provide the ability to describe the anisotropy of properties [5], the nonlinearity of its deformation, changes in mechanical characteristics upon activation of contractile function, and the influence of these factors on the activation process.

To determine the function of changes in the force load on the muscle in the modes of isometric tetanus and muscle relaxation at a constant rate, we use the hypothesis of A. Hill, which uses the assumption that any material can be represented as a rheological model - a combination of viscous and elastic elements [5, 6]. Assuming that the force generated by the elastic element is a function of its length $p=P(x)$ and using the rule of differentiation of a complex function, we obtain a mathematical model of the change in force load on the muscle in the form of a differential equation [7]:

$$\frac{dp}{dt}=\frac{dP}{dx}\frac{dx}{dt}=\frac{dP}{dx}\left(\frac{dL}{dt}-\frac{dl}{dt}\right)=\frac{dP}{dx}\left(\frac{dL}{dt}+\upsilon\right)=\frac{dP}{dx}\left(\frac{dL}{dt}+\frac{b(p_0-p)}{p+a}\right). \quad (1)$$

where:

l – the length of the muscle fiber contractile element;

x – deformation (reduction) of the elastic element, muscle fiber;

$L=l+x$ – total muscle length.

If the skeletal muscle is brought to a state of tetanus by periodic stimulation, then after a certain period the muscle will develop isometric tension. Under isometric conditions, during muscle contraction, its length will not change, so $dL/dt = 0$ and the solution of differential equation (1) will take the form [8]:

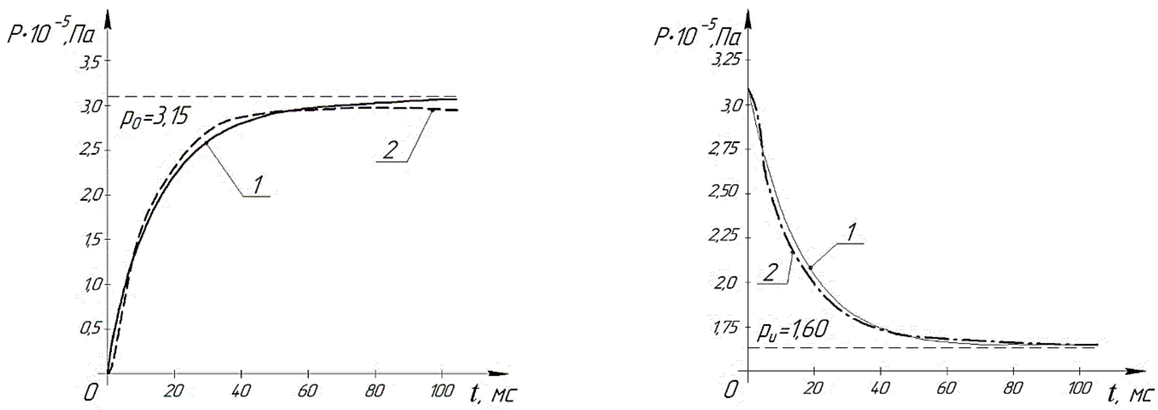
$$-p-(p_0+a)\ln\left(\frac{p_0-p}{p_0}\right)=\alpha bt\,. \qquad (2)$$

Function (2) implicitly describes the change in the force load of muscle tissue as a function of time (Fig. 1 a) at the values of the parameters a = 0,14 Pa, b = 1,03 cm/sec and the maximum load $p_0 = 0{,}031\cdot10^{-5}$ Pa ($t\to\infty,\ p\to p_0$). The result of comparing the obtained functional dependence (see fig. 3) by equation (2) with experimental data [9] allowed obtaining the adequacy index of the mathematical model in the form of the average approximation error, which was 5,6%.

For the mode of releasing food at a constant speed, it should be added that the consumer, which is first supported in the isometric reinforcement released, and then reduced at a constant speed and. Then the solution of the differential level (1) is obtained [10]:

$$p_0-p+(p_u+a)\ln\frac{p_0-p_u}{p-p_u}=\alpha(b+u)t\,. \qquad (3)$$

Function (3) implicitly describes the change in the force load of muscle tissue as a function of time (fig. 1) at the same values of parameters a and b as in equation (2), and the maximum load p_u=$1{,}6\cdot10^{-5}$ Pa ($t\to\infty$, $p\to p_u$).



a) б)

1 – theoretical dependence, 2 – experimental dependence

Fig. 1. The diagram of change of force loading of muscular fabric in the modes:

a) isometric tetanus; б) muscle relaxation at a constant rate

The result of comparing the obtained functional dependence (fig. 1 b) according to equation (3) with experimental data [11, 12] allowed obtaining the adequacy of the mathematical model in the form of the average approximation error, which amounted to 4,8%.

The parameter $n(x, t)$ indicates the fraction of bridges with an offset x that is in the bound state. Simplifying the mechanism of the cycling reaction (attachment-detachment) of the bridge and introducing the assumption that the bridge can be in only two states – unbound or strongly bound, i.e. in a state where it generates force, then the transitions between states are described by the following scheme [4, 7]:

$$U \underset{g(x)}{\overset{f(x)}{\rightleftarrows}} B, \tag{4}$$

where:

U, B are the rate constants of the line f (x) and g (x) of the inverse reactions are the displacement functions x.

Using the kinetic equation for the fraction of attached bridges, we obtain the equation of the dynamics of transverse bridges according to the hypothesis of A. Huxley [8, 12]:

$$\frac{\partial n}{\partial t} - \upsilon \frac{\partial n}{\partial x} = f(x)(1-n) - g(x)n, \tag{5}$$

where:

n(x, t) is the fraction of bridges with an offset x that is in the bound state.

Assuming that the bridge is a linear elastic element, i.e. the force of elasticity that it develops, and then the force that develops the muscle is determined by the formula:

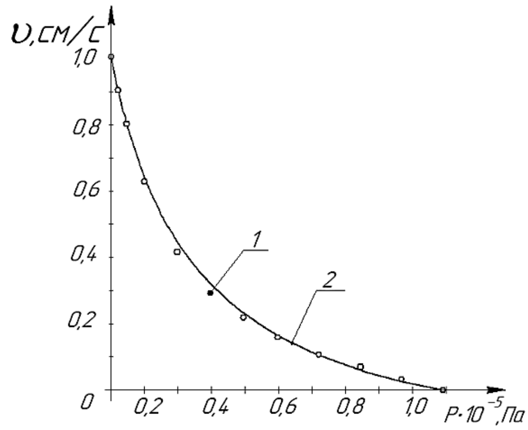
$$p = \rho k \int_{-\infty}^{+\infty} x \cdot n(x) dx, \tag{6}$$

where:

$$n(x) = \begin{cases} Ae^{\frac{g_2 x}{\upsilon}}, & x \le 0; \\ \dfrac{f_1}{g_1 + f_1} + B\exp\left(\dfrac{x^2(g_1+f_1)}{2\upsilon h}\right), & 0 < x \le h; \\ 0, & x > h. \end{cases}$$ – *the solution of the differential level in parts of the output (5);*

$$A = \frac{f_1}{g_1 + f_1} - \frac{f_1}{g_1 + f_1}\exp\left(-\frac{h(g_1 + f_1)}{2\upsilon}\right),$$

$$B = -\frac{f_1}{g_1 + f_1}\exp\left(-\frac{h(g_1 + f_1)}{2\upsilon}\right).$$



1 – the Hill model ; 2 – Huxley model

Fig. 2. **Comparative diagram of the dependence of the force load on the muscle depending on the rate of contraction**

Figure 2 shows comparative diagrams of the power load dependence on the number depending on the rate of reduction. The diagram based on the hypothesis of A. Huxley is constructed for the following parameters: the maximum distance between cities h=10^{-6} cm; the average speed of direct and reverse reactions of attachment of bridges, respectively – f_1=50 sec^{-1}; g_1=230 sec^{-1}; maximum speed of muscle mixing speed - υ_{max}=1,0 cm/sec.

A comparison of the results of the theoretical study of the force load on the mixture for Huxley's hypothesis (see fig. 2) shows a coincidence with the theoretical data for the Hill model with an accuracy of 4,2%. The obtained results of theoretical and experimental [2, 6] study of the physiological process of muscle contraction provide rational use of theories of sliding threads and the cycle of transition cities as the basis of silo generation and movement reduction.

Conclusions. During the work were:

- developed mathematical models of the physiological process of muscle contraction on the basis of known hypotheses of functioning of these types of organs, which allows to determine with high accuracy the physiological characteristics of muscle tissue at different modes of its loading.

- in order to determine the reliability of the developed mathematical models, were studied a comparative analysis of the theoretical physiological process results of the muscle contraction based on the developed mathematical models with the results of experimental studies of these type organs functioning where the adequacy index averaged 4,5%.

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