

# NEW METHOD OF MEASUREMENT OF A MOMENT OF INERTIA OF AN ELECTRICAL MACHINES

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*Abstract:* In the given article the method of a moment of inertia measurement of Electrical Machines is considered. Various modes of Electrical Machines operations are investigated. Probably also determination of a moment of mechanical losses in function of rotation angular velocity. Questions of a Computer-Measuring System construction and experimental results of research are indicated. The functional opportunities of the Computer-Measuring System need to be expanded for determination moment of inertia  $J$  and moment of mechanical losses of various gears rotated parts.

*Keywords:* moment of inertia, electrical machines, Computer-Measuring System

## 1 INTRODUCTION

The moment of inertia of a rotor of electrical machines (EM) is one of the most important parameters, which determines its dynamic properties. However in directories it is not always determined. At designing various electrical drives, electrical machines and systems of automatics it is frequently necessary to know exact significances of moment of inertia of the EM rotor (and sometimes and in tax with running gears), as they determine velocity and thermal mode of operations of the EM. To determine a moment of inertia of a rotor or whole unit, that rotates, it is possible calculating and experimental ways [1, 2].

## 2 THE EXISTING METHODS FOR MOMENT OF INERTIA OF THE EM OVERVIEW

The accounts of a moment of inertia of the EM are labour-consuming of non-uniform materials and complex geometrical form of a rotor of the EM. They do not differ by high accuracy and are used seldom.

Experimentally the moment of inertia is determined by several methods. State standard of Ukraine provides three methods of determination of a moment of inertia: *method of an auxiliary pendulum; method of self-braking; method of turning fluctuations.*

Defects of a *method of an auxiliary pendulum* is the fact, that it does not differ by high accuracy, because it is heavy to determine distance of a centre of gravity of a pendulum from axis of rotation, as thus owes be discounted weight of the lever. A rotor owes it be good balanced, as him non-balanced deforms results of measurements.

By use of a *method of turning fluctuations* a rotor suspend on wire and result in turning oscillatory movement. Period of small fluctuations is thus determined, which is then compared to period of fluctuations of a reference body with known moment of inertia. The standard claims, that before experience the rotor was balanced and the point of hang up was precisely on axis of rotation. If the point of hang up will be displaced from axis of rotation, an error of determination of a moment of inertia is increased. Main defects of this method such: necessity of analysis of the EM and significant labour input of realization of experience.

*The method of self-braking* claims preliminary determination of mechanical losses (for example, from experience of a single course). Defect of this method is the fact, that the mechanical losses are determined as a constant, when actually they are function from angular

velocity of rotation. It causes low determination accuracy of a moment of inertia of the EM by a self-braking method.

### 3 RESEARCH OF THE ELECTRICAL MACHINES OPERATIONS MODES

One of the most important research problems of measurement objects is the problem of determination of influence of measurement object internal parameters in various modes of operations on its target characteristics. The decision of this problem permits to answer the question, which of operations modes of measurement object the most effective (informative) for measurement or identification of same internal parameter of measurement object. Such problem concerning parameters  $J$  and  $M_o$  was not considered in the known literature, and its decision is very important for the construction of measurement effective algorithms.

The EM can work in various modes. Considering them, main modes can be allocated [3]: *start-up* - transient mode with change of angular velocity from 0 up to  $\omega_{max}$ ; *statica* - process of the EM work with constant velocity  $\omega_{max}$ ; *reverse* - the EM rotor rotation direction change; *a shock load* - unexpected change of a resistance moment on a shaft for a short period; *a repeatedly-dynamic mode* - the EM mode switch on - switch off or regulation law change of velocity during its rotation; *self-braking* - braking of a switched off the EM, stimulated by the action of the resistance moment on its shaft.

For research of the EM internal parameters influence on its movement characteristics the sensitivity theory is used, according to which sensitivity function are determined as in [3-5]:

$$\frac{d\mathbf{U}(t)}{dt} = \frac{d\mathbf{F}}{d\mathbf{I}} \cdot \mathbf{U}(t) + \frac{d\mathbf{F}}{d\mathbf{A}}, \quad (1)$$

where

$$\mathbf{I} = [i_{s\alpha}; i_{s\beta}; i_{r\alpha}; i_{r\beta}]^T; \mathbf{A} = [J; M_o]^T. \quad (2)$$

The object of research represents the EM as asynchronous motor, the mathematical model of which is recorded in a kind of a system of nonlinear differential equations as in [3, 4]. The results of a sensitivity functions calculate is considered on Figures 3-4.

Having decided with the help of numerical methods the equations system, we shall find sensitivity functions, which were modelled on a personal computer. For the comparison of received results analysis we shall put the Table 1.

In the Table following conditional designations are used:  $I$  - large significances and regularity of sensitivity functions;  $\theta$  - the sensitivity is not present or small significances of sensitivity functions;  $X$  - variable (oscillatory) character of sensitivity functions. The first symbol designates sensitivity function on  $i_{s\alpha}(t)$ , second - on  $i_{s\beta}(t)$ , and third - on  $\omega(t)$ .

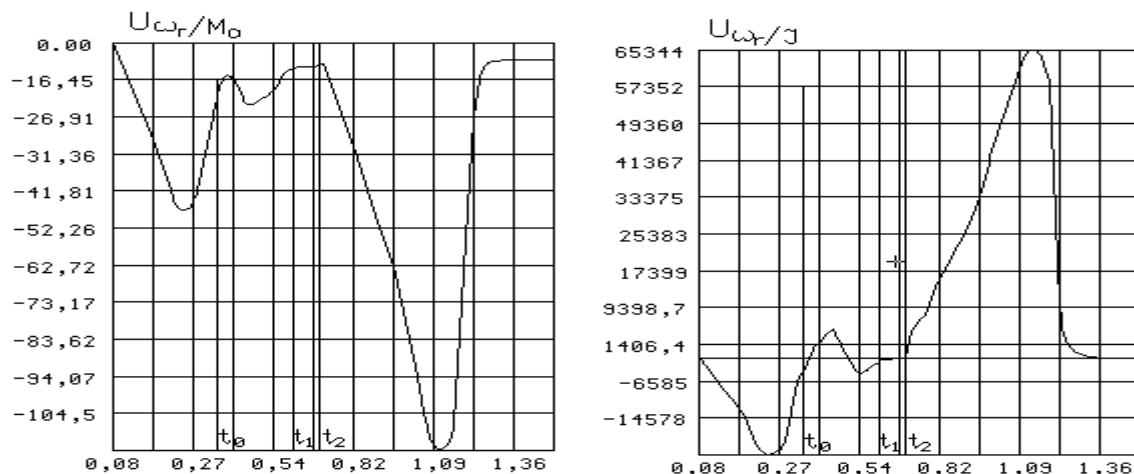


Figure 1. Modes: Start-up - Repeatedly-Dynamic - Statica - Self-Braking.

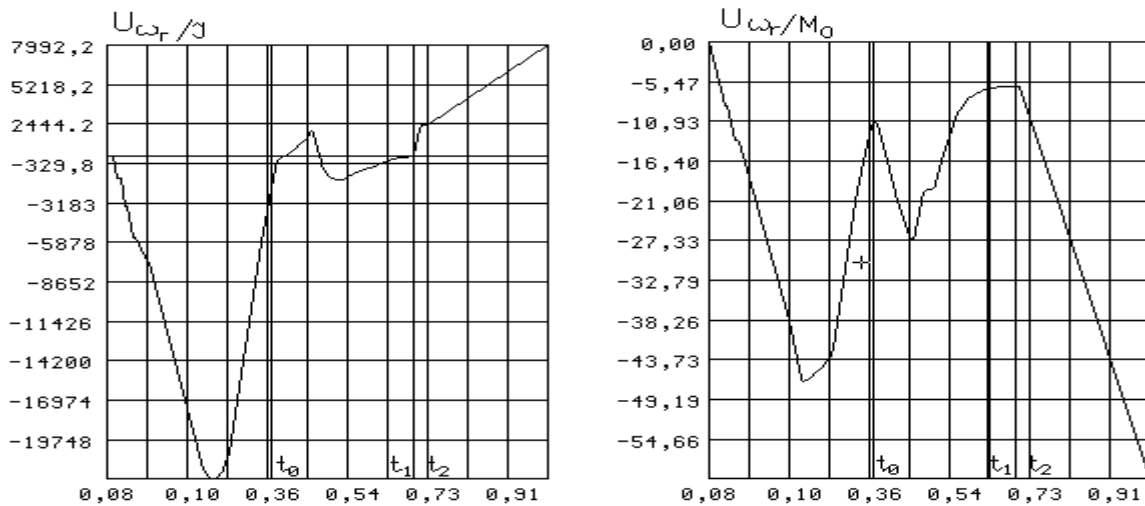


Figure 2. Modes: Start-up - Shock Load - Statica - Reverse.

Table 1. The sensitivity in different operations modes of the EM.

N	Modes	Parameters	
		$J$	$M_0$
1.	Start-up	XXI	XXI
2.	Self-braking	00I	XXI
3.	Statica	000	XXI
4.	Reverse	XXI	XXI
5.	Shock load	XXI	XXI
6.	Repeatedly-dynamic	XXI	XXI

The analysis of the table permits to make following main conclusions:

- parameter  $M_0$  needs to be determined in all modes of the EM operations;
- static mode not effective for measurement of the inertia moment  $J$ ;
- in modes of start-up, reverse, shock load and repeatedly-dynamic mode (i.e. in dynamic modes) it is probable to determine  $M_0$  and  $J$ ;
- the sensitivity functions on currents  $i_{s\varphi}(t)$  and  $i_{s\sigma}(t)$  have exclusively oscillatory character;
- the sensitivity functions on angular velocity  $\diamond(t)$  have large significances and have not oscillatory character, that specifies a opportunity of determination  $J$  and  $M_0$  on process  $\diamond(t)$ .

#### 4 A NEW MEASUREMENT METHOD OF A MOMENT OF INERTIA DESCRIPTION

Being based on conducted analysis, the authors have offered a method of determination of a moment of inertia and moment of mechanical losses in function of angular velocity of rotation, which is displayed below.

Necessary condition of realization of a method is availability of a reference moment of inertia, body executed in a kind of rotation, for example, disk or cillinder. The reference moment of inertia is calculated at geometrical and weight parameters of a body of rotation.

On output part of the shaft of the EM is established a reference moment of inertia and switch-on of the EM. After ending of transient the EM disconnect and during its self-braking register angular velocity  $\diamond_r$ . Registration of angular velocity  $\diamond_r$  in mode of self-braking carry out also at disconnected reference moment of inertia.

The equations of movement of the EM accordingly without reference moment and with reference moment in mode of self-braking have a kind:

$$\begin{cases} 0 = M_0(\omega_r) + M_{MS}(\omega_r) + (J + J_{MS}(\omega_r)) \cdot \varepsilon_{b1}(\omega_r); \\ 0 = M_0(\omega_r) + M_{MS}(\omega_r) + (J + J_{MS}(\omega_r) + J_M) \cdot \varepsilon_{b2}(\omega_r). \end{cases} \quad (3)$$

Thus we shall note, that all listed sizes are submitted in function of angular velocity of rotation  $\omega_r$ .

The realization of two experiments of self-braking is a methodical basis of an offered method and permits with use of system (3) to combine and to decide system from two equations with two unknown -  $M_0$  and  $J$ .

We shall record system (3), where we shall transfer unknown to left-hand parts of equations and having omitted for convenience angular velocity of rotation  $\omega_r$ :

$$\begin{cases} -M_0 - J \cdot \varepsilon_{b1} = J_{MS} \cdot \varepsilon_{b1} + M_{MS}; \\ -M_0 - J \cdot \varepsilon_{b2} = (J_{MS} + J_M) \cdot \varepsilon_{b2} + M_{MS}. \end{cases} \quad (4)$$

As  $J_{MS} \ll J$  and  $M_{MS} \ll M_0$ , the significances  $J_{MS}$ ,  $M_{MS}$  can be neglected. Then the system (4) will be recorded as:

$$\begin{cases} -M_0 - J \cdot \varepsilon_{b1} = 0; \\ -M_0 - J \cdot \varepsilon_{b2} = J_M \cdot \varepsilon_{b2}. \end{cases} \quad (5)$$

The decision of system (5) has a kind:

$$M_0 = \frac{\varepsilon_{b1} \varepsilon_{b2}}{\varepsilon_{b2} - \varepsilon_{b1}} \cdot J_M; \quad J = \frac{\varepsilon_{b2}}{\varepsilon_{b2} - \varepsilon_{b1}} \cdot J_M. \quad (6)$$

The moment of inertia  $J$  of the EM is calculated as average significance on whole interval of change of rotation angular velocity in self-braking mode.

## 5 THE COMPUTER-MEASURING SYSTEM FOR REALIZING OF PROPOCED METHOD

For realization of a method the test bed is developed, simplified design drawing which is represented on Figure 3. The reference moment of inertia 4 incorporates to shaft of the EM 1 through muff 5. At rotation of the EM 1 in one party of the muff enters into coupling with reference moment 4 and is loaded by this moment. At rotation of the EM in other party of the muff is not linked to reference moment and then the EM rotates in single mode.

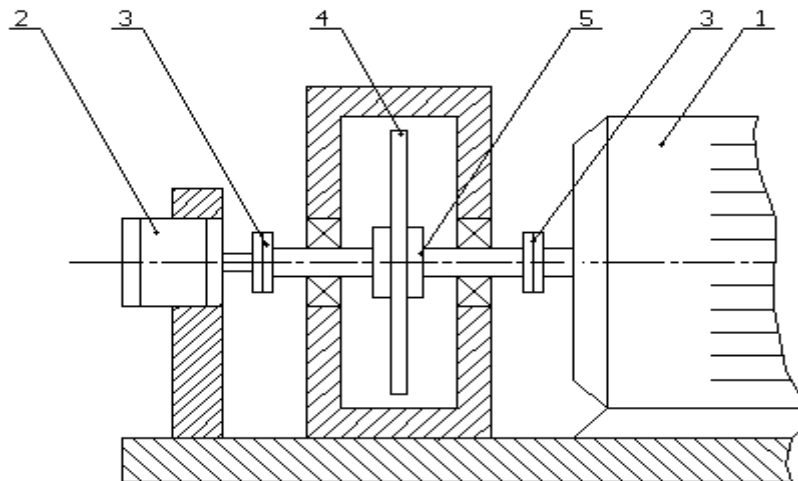


Figure 3. The design drawing of the test bed (1 - the EM; 2 - the sensor of angular velocity; 3 - the interface muff; 4 - the reference moment of inertia; 5 - the muff).

For realization of a method the measuring system should contain only channel of measurement of angular velocity of rotation (Figure 4).

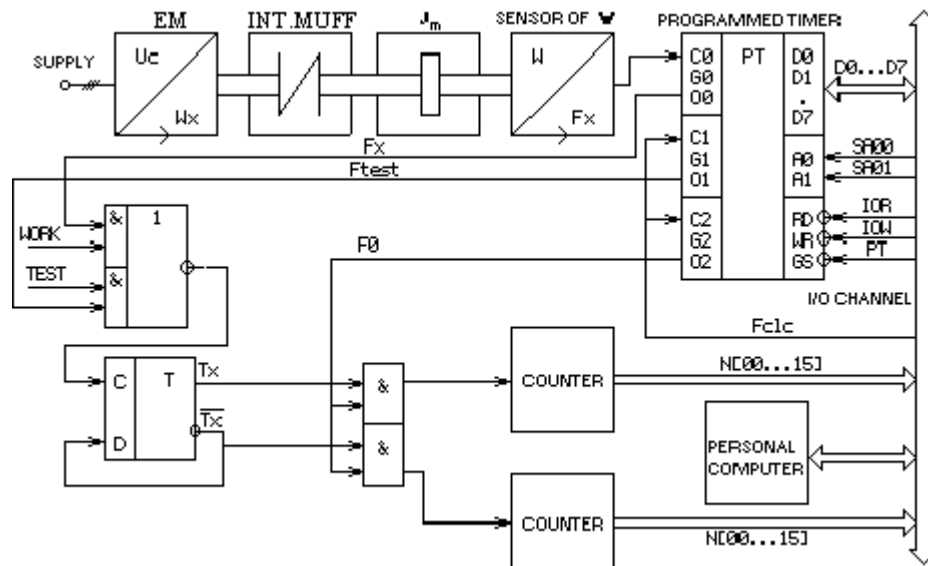


Figure 4. The block diagram of the Computer-Measuring System.

Basic elements of the channel of measurement of rotation angular velocity of the Measuring System, realized with use of the personal computer IBM PC, is: object of measurement - EM, interface muff, reference moment of inertia, sensor of rotation angular velocity and measuring circuit of period of signals.

The functional software of Measuring System contains such basic elements: the driver of connection with measuring channel, filtering procedure of measurements results, procedure of differentiation and procedure of graphic display of measurements results.

## 6 EXPERIMENTAL RESULTS

On Figures 5, 6 some experimental results, received for a asynchronous EM (type 4A71A4CY1) and submitted which evidently wide functional opportunities of a offered Computer-Measuring System represent.

The character of dependence of a moment of inertia from rotation angular velocity can be explained by that the moment of inertia of a rotor depend on its angular position and it influces measurement result.

## 7 CONCLUSIONS

1. Various modes of Electrical Machines operations are investigated by sensitivity functions: *start-up, statica, reverse, shock load, repeatedly - dynamic mode, self-braking*. Conclusions about expediency of a moment of inertia and moment of mechanical losses determination in these modes are made.

2. The new method of the moment of inertia and moment of mechanical losses in function of rotation angular velocity of Electrical Machines determination is developed.

3. The basis of an offered method is measurement of rotation angular velocity in *dynamic modes (self-braking mode)* of the Electrical Machines.

4. The block diagram of a Computer-Measuring System of the moment of inertia is offered.

5. Examples of experimental researches are shown.

6. The functional opportunities of the Computer-Measuring System need to be expanded for determination moment of inertia  $J$  and moment of mechanical losses of various gears rotated parts.

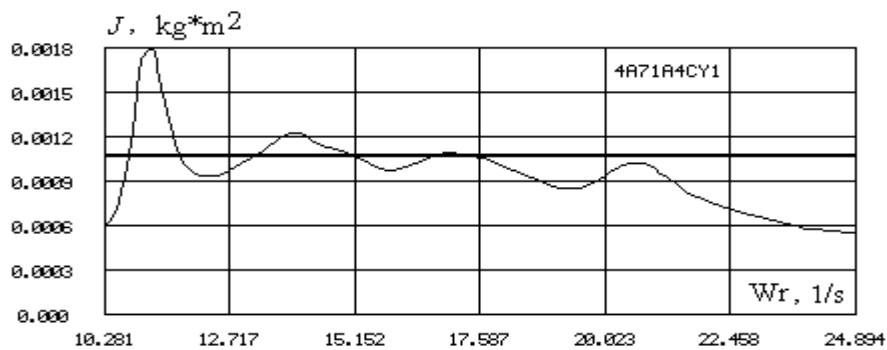


Figure 5. The moment of inertia of the EM (the greasy line designates average significance of a moment of inertia).

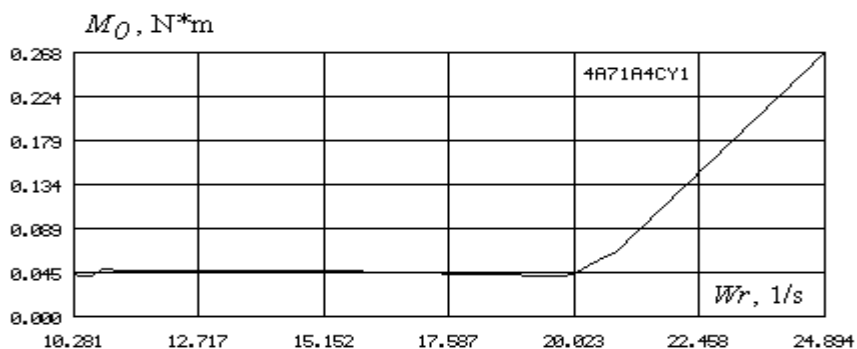


Figure 6. The moment of mechanical losses of the EM.

## NOMENCLATURES

**I** - vector of a researched system condition; **F** - vector of a researched system right part; **A** - vector of researched parameters; **U** - matrix of sensitivity functions;  $i_{s\varnothing}$  - current in stator winds in axis  $\varnothing$  (coordinate system  $\varnothing, \varrho, 0$ );  $i_{s\varrho}$  - current in stator winds in axis  $\varrho$ ;  $i_{r\varnothing}$  - current in rotor winds in axis  $\varnothing$ ;  $i_{r\varrho}$  - current in rotor winds in axis  $\varrho$ ;  $M_o$  - moment of mechanical losses;  $M_{MS}$  - moment of mechanical losses of input shaft of Measuring System;  $J$  - rotor inertia moment;  $J_M$  - reference inertia moment;  $J_{MS}$  - inertia moment of input shaft of Measuring System;  $\omega_r$  - rotor angular velocity;  $\omega_{b1}$  - rotor angular velocity in self-braking mode without a reference inertia moment;  $\omega_{b2}$  - rotor angular velocity in self-braking mode with a reference inertia moment;  $\ddot{m}_{b1}$  - rotor angular acceleration in self-braking mode without a reference inertia moment;  $\ddot{m}_{b2}$  - rotor angular acceleration in self-braking mode with a reference inertia moment;  $t$  - time.

## REFERENCES

- [1] G.K. Gerve, The Industrial Tests of the Electrical Machines, Energoatomizdat, St.-Peterburg, 1984, 408 p.
- [2] M.M.Gernet, V.F. Ratobylsky, The moment of inertia determination, Mashinostroenie, Moskow, 1969, 248 p.
- [3] V.Podzharenko, V.Kucheruk, Computer-Measuring System of the Mechanical Characteristics of Electrical Drives, in: V.Fedak, J.Fedor (ed.), *Proceeding of the EDPE'96 Int. Conf. on Electrical Drives and Power Electronics* (KOSICE 1-3. October 1996), Technical University of Kosice, Slovakia, 1996, volume 2, p.633-637.
- [4] V. Kucheruk, V.Podzharenko, Identification of the internal parameters of Electrical Motors, *Visnyk VPI* 1 (2) (1994) 10-13.
- [5] E.N. Rozenvasser, R.M. Yusupov, The Sensitivity of the Control Systems, Science, Moskow, 1981, 464 p.

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