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THE BIT ERROR PROBABILITY COMPUTATION IN AN M -CHANNEL DATA TRANSFER SYSTEM WITH IMPLEMENTED FIRST M ORTHOGONAL WALSH FUNCTIONS, WHERE THE INTEGER M VARIES BETWEEN 17 AND 31

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Анотація. Проведені обчислення імовірності бітової помилки в M -канальній системі передачі даних з використанням перших M ортогональних функцій Уолша, де M змінюється від 17 до 31. Встановлено, що оптимальним варіантом є використання 19-елементної системи функцій Уолша.

Abstract. There have been carried out the computations of the bit error probability in an M -channel data transfer system with applying the first M orthogonal Walsh functions, where M varies between 17 and 31. Ascertained that the optimal variant is applying the 19-element system of Walsh functions.

Аннотация. Проведены вычисления вероятности битовой ошибки в M -канальной системе передачи данных с использованием первых M ортогональных функций Уолша, где M изменяется от 17 до 31. Установлено, что оптимальным вариантом является использование 19-элементной системы функций Уолша.

FOREWORD

It is well known, that the task of the efficiency evaluation of the CDMA principle in the multichannel data transmission network is very urgent. The trials [1 — 7] to compute the theoretic probability $p_e = p_e\left(Q,\, D(\tilde{n}),\, N,\, u\right)$ of the single bit detection error for such systems have driven to the next lower and upper restrictions in the inequalities: for the power SNR $Q \in (3;\, 16]$ and variance $D(\tilde{n}) \in [0.1;\, 1000]$.

$$p_e\left(Q,\, D(\tilde{n}),\, 8,\, \{1,\, 7\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 8,\, \{6,\, 9\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 8,\, 3\right), \tag{1}$$

for $Q \in (3;\, 16]$ and $D(\tilde{n}) < 20$.

$$p_e\left(Q,\, D(\tilde{n}),\, 9,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 9,\, \{3,\, 4\}\right) \tag{2}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 10,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 10,\, \{3,\, 4\}\right), \tag{3}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 11,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 11,\, \{3,\, 4,\, 7\}\right), \tag{4}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 12,\, \{1,\, 5,\, 8\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 12,\, \{3,\, 7\}\right), \tag{5}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 13,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 13,\, \{3,\, 7\}\right), \tag{6}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 14,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 14,\, \{3,\, 7,\, 8\}\right), \tag{7}$$

$$p_e\left(Q,\, D(\tilde{n}),\, 15,\, \{1,\, 5\}\right) < p_e\left(Q,\, D(\tilde{n}),\, 15,\, \{2,\, 3,\, 7\}\right); \tag{8}$$

for $Q \in (3; 16]$ and $D(\tilde{n}) > 20$.

$$p_e(Q, D(\tilde{n}), 9, 5) < p_e(Q, D(\tilde{n}), 9, 3), \tag{9}$$

$$p_e(Q, D(\tilde{n}), 10, 5) < p_e(Q, D(\tilde{n}), 10, 3), \tag{10}$$

$$p_e(Q, D(\tilde{n}), 11, 5) < p_e(Q, D(\tilde{n}), 11, 3), \tag{11}$$

$$p_e(Q, D(\tilde{n}), 12, 5) < p_e(Q, D(\tilde{n}), 12, 3), \tag{12}$$

$$p_e(Q, D(\tilde{n}), 13, 5) < p_e(Q, D(\tilde{n}), 13, 3), \tag{13}$$

$$p_e(Q, D(\tilde{n}), 14, \{4, 5\}) < p_e(Q, D(\tilde{n}), 14, 3), \tag{14}$$

$$p_e(Q, D(\tilde{n}), 15, \{4, 5\}) < p_e(Q, D(\tilde{n}), 15, 3), \tag{15}$$

for $Q \in [0.25; 16]$ and $D(\tilde{n}) \in (0.1; 1)$

$$p_e(Q, D(\tilde{n}), 15, 9) < p_e(Q, D(\tilde{n}), 14, 9) < p_e(Q, D(\tilde{n}), 13, 9) < p_e(Q, D(\tilde{n}), 12, 9) <$$

$$< p_e(Q, D(\tilde{n}), 11, 9) < p_e(Q, D(\tilde{n}), 10, 9) < p_e(Q, D(\tilde{n}), 9, 9) \tag{16}$$

for $Q \in [0.25; 16]$ and $D(\tilde{n}) \in (60; 1000)$

$$p_e(Q, D(\tilde{n}), 12, 9) < p_e(Q, D(\tilde{n}), 15, 9) < p_e(Q, D(\tilde{n}), 14, 9) < p_e(Q, D(\tilde{n}), 9, 9) \tag{17}$$

for $Q \in (3; 16]$ and $D(\tilde{n}) \in [0.1; 1000]$

$$p_e(Q, D(\tilde{n}), 16, \{1, 5\}) < p_e(Q, D(\tilde{n}), 16, 2) < p_e(Q, D(\tilde{n}), 16, 9), \tag{18}$$

for $Q \in (3; 16]$ and $D(\tilde{n}) \in [0.1; 1000]$

$$p_e(Q, D(\tilde{n}), 32, 7) < p_e(Q, D(\tilde{n}), 32, 3) < p_e(Q, D(\tilde{n}), 32, 9), \tag{19}$$

for $Q \in (3; 16]$ and $D(\tilde{n}) > 100$

$$p_e(Q, D(\tilde{n}), 64, 7) < p_e(Q, D(\tilde{n}), 64, 5) <$$

$$< p_e(Q, D(\tilde{n}), 64, 3) < p_e(Q, D(\tilde{n}), 64, 9); \tag{20}$$

for $Q \in (3; 16]$ and $D(\tilde{n}) < 1$

$$p_e(Q, D(\tilde{n}), 64, \{3, 6\}) < p_e(Q, D(\tilde{n}), 64, 1) <$$

$$< p_e(Q, D(\tilde{n}), 64, 5) < p_e(Q, D(\tilde{n}), 64, 9). \tag{21}$$

Here $u = \overline{1, 8}$ if there is used [6 — 8] system $\left\{ \mathrm{rom}_u \left(r, \frac{t}{T_0} \right) \right\}_{r=0}^{N-1}$, and $u = 9$, if used $\left\{ \mathrm{wal} \left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{N-1}$, $t \in [0; T_0)$, $N \in \left\{ \overline{8, 16}, 32, 64 \right\}$. At the present work here stands the objective problem to rearrange into inequalities with 15 functions the relationship between the following dependences:

$$p_e = p_e \left(Q, D(\tilde{n}), M, 9 \right), \ M = \overline{17, 31}. \tag{22}$$

Otherwise, is this problem is unsolvable, — to determine the optimal integer $M^* \in \left\{ \overline{17, 31} \right\}$, by which

$$p_e \left(Q, D(\tilde{n}), M^*, 9 \right) = \min_{M = \overline{17, 31}} p_e \left(Q, D(\tilde{n}), M, 9 \right). \tag{23}$$

COMPUTATION AND THE RESULT VISUALIZATION

Will be computing

$$p_e = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{Q}} e^{-z^2/2} dz + \left[\frac{\alpha_b}{6} \sigma_b \left(Q^{1.5} - \sqrt{Q} \right) + \frac{\gamma_b}{24} \sigma_b^2 \left(Q^{2.5} - 3Q^{1.5} \right) + \right.$$

$$\left. + \frac{\alpha_b}{72} \sigma_b^2 \left(Q^{3.5} - 10Q^{2.5} + 15Q^{1.5} \right) \right] \frac{e^{-Q/2}}{\sqrt{2\pi}}, \tag{24}$$

where [1 — 6]

$$\alpha_b = \frac{\mu_3^{(b)}}{\left(\mu_2^{(b)} \right)^{\frac{3}{2}}} = \frac{\sum_{n=-31}^{31} \left(b(n) - \sum_{m=-31}^{31} b(m) f[b(m)] \right)^3 f[b(n)]}{\sqrt{\left(\sum_{n=-31}^{31} \left(b(n) - \sum_{m=-31}^{31} b(m) f[b(m)] \right)^2 f[b(n)] \right)^3}}, \tag{25}$$

$$\sigma_b = \sqrt{\mu_2^{(b)}} = \sqrt{\sum_{n=-31}^{31} \left(b(n) - \sum_{m=-31}^{31} b(m) f[b(m)] \right)^2 f[b(n)]}, \tag{26}$$

$$\gamma_b = \frac{\mu_4^{(b)}}{\left(\mu_2^{(b)} \right)^2} - 3 = \frac{\sum_{n=-31}^{31} \left(b(n) - \sum_{m=-31}^{31} b(m) f[b(m)] \right)^4 f[b(n)]}{\left(\sum_{n=-31}^{31} \left(b(n) - \sum_{m=-31}^{31} b(m) f[b(m)] \right)^2 f[b(n)] \right)^2} - 3, \tag{27}$$

$$f\left[b(n)\right]=\frac{1}{\sqrt{2\pi D(\tilde{n})}}\exp\left(-\frac{n^2}{2D(\tilde{n})}\right),\ n=\overline{-31,\,31}, \tag{28}$$

$$b(n)=\frac{2}{M(M-1)}\sum_{r=1}^{M-1}\sum_{q=r+1}^{M}\left(\frac{1}{32}\sum_{l=1}^{32}\mathrm{wal}\left(r,\,\frac{l-1}{32}\right)\mathrm{wal}\left(q,\,\frac{l-1+n}{32}\right)\right),\ n=\overline{-31,\,31}, \tag{29}$$

$$\mathrm{wal}\left(q,\,\frac{t}{T_0}\right)\equiv 0,\ \forall\ \frac{t}{T_0}\notin[0;\,1), \tag{30}$$

for $M=\overline{17,\,31}$, $Q\in\left[\frac{1}{4};\,16\right]$, $D(\tilde{n})\in[0.1;\,1000]$. The step of the SNR varying is 0.25, so all 64 values of the SNR are

$$\{0.25,\,0.5,\,0.75,\,1,\,1.25,\,\ldots,\,15.25,\,15.5,\,15.75,\,16\}. \tag{31}$$

And all calculable values of the variance $D(\tilde{n})$ are in the 25-element set

$$\{0.1,\,0.2,\,0.3,\,0.4,\,0.5,\,0.6,\,0.7,\,0.8,\,0.9,\,1,\,2,\,4,\,6,\,8,\,10\}\cup$$

$$\cup\{20,\,40,\,60,\,80,\,100,\,200,\,400,\,600,\,800,\,1000\}. \tag{32}$$

Performing this, there are the 15 surfaces (22), plotted for $Q\in\left[\frac{1}{4};\,16\right]$ and $D(\tilde{n})\in[0.1;\,1000]$. One of those surfaces is shown in the figure 1, and the rest 14 surfaces practically don't differ from that surface. For sorting these surfaces in order of the theoretic probability p_e decreasing may use the distance between the points of surface $p_e=p_e\left(Q,\,D(\tilde{n}),\,M,\,9\right)$ and the points of surface $p_e=0$, $M=\overline{17,\,31}$. The less this distance the preferable the corresponding orthogonal Walsh functions $\left\{\mathrm{wal}\left(w,\,\frac{t}{T_0}\right)\right\}_{w=0}^{M-1}$ system to be used. If denoting the k-th element of the set (31) as Q_k, $k=\overline{1,\,64}$, and the j-th element of the set (32) as D_j, $j=\overline{1,\,25}$, then this distance may be defined as

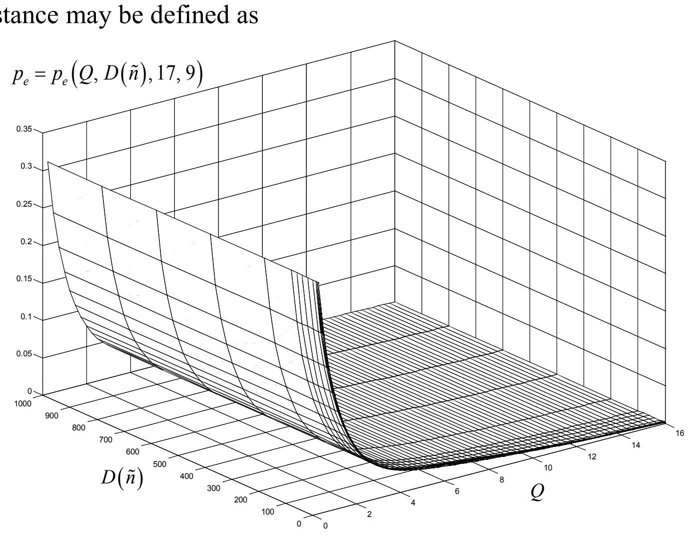

Figure 1. Surface $p_e\left(Q,\,D(\tilde{n}),\,17,\,9\right)$

$$d_1(M) = \sum_{j=1}^{25} \sum_{k=1}^{64} p_e\left(Q_k, D_j, M, 9\right). \tag{33}$$

The dependence $d_1(M)$ is shown in the figure 2, from which

$$\arg \max_{M=\overline{17, 31}} d_1(M) = 19, \tag{34}$$

$$\arg \min_{M=\overline{17, 31}} d_1(M) = 31, \tag{35}$$

that is by this distance the most preferable multichannel data transfer system should use $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{30}$ functions, and the worst case is 19-channel data transfer system with implemented $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{18}$ functions.

Otherwise the distance may be defined by raising each value $p_e\left(Q_k, D_j, M, 9\right)$, $k = \overline{1, 64}$, $j = \overline{1, 25}$, to some power β, that may belong to $[1; 2]$:

$$d_\beta(M) = \sum_{j=1}^{25} \sum_{k=1}^{64} \left(p_e\left(Q_k, D_j, M, 9\right) \right)^\beta \tag{36}$$

The normed dependences (36)

$$\tilde{d}_\beta(M) = \frac{d_\beta(M)}{\max\limits_{M=\overline{17, 31}} d_\beta(M)} \tag{37}$$

by $\beta \in [1; 2]$ with spacing 0.05 are plotted in the figure 3, from which in a majority the most preferable case is using $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{30}$ orthogonal system or $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{23}$ orthogonal system, and the worst case remains the same — 19-channel data transfer system with implemented $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{18}$ functions.

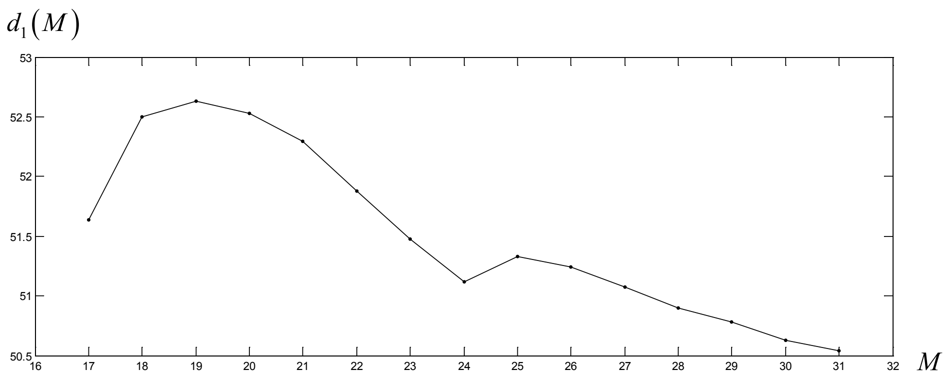

Figure 2. Dependence $d_1(M)$

Furthermore, there may be introduced a distribution function of the SNR, since the values (31) have different significances. So, each element Q_k , $k=\overline{1,\,64}$, of the set (31) may have its weight y_k , $k=\overline{1,\,64}$. It is conjectured that the SNR distribution function is normal with the math expectance $Q_a=1$ and the variance $\sigma^2_{\text{SNR}}=4$ (figure 4):

$$y(Q)=\frac{1}{\sigma_{\text{SNR}}\sqrt{2\pi}}\exp\left(-\frac{(Q-Q_a)^2}{2\sigma^2_{\text{SNR}}}\right)=\frac{1}{2\sqrt{2\pi}}\exp\left(-\frac{(Q-1)^2}{8}\right). \quad (38)$$

Then the distance between the surface $p_e=p_e\left(Q,\,D(\tilde{n}),\,M,\,9\right)$ and surface $p_e=0$ with (38)

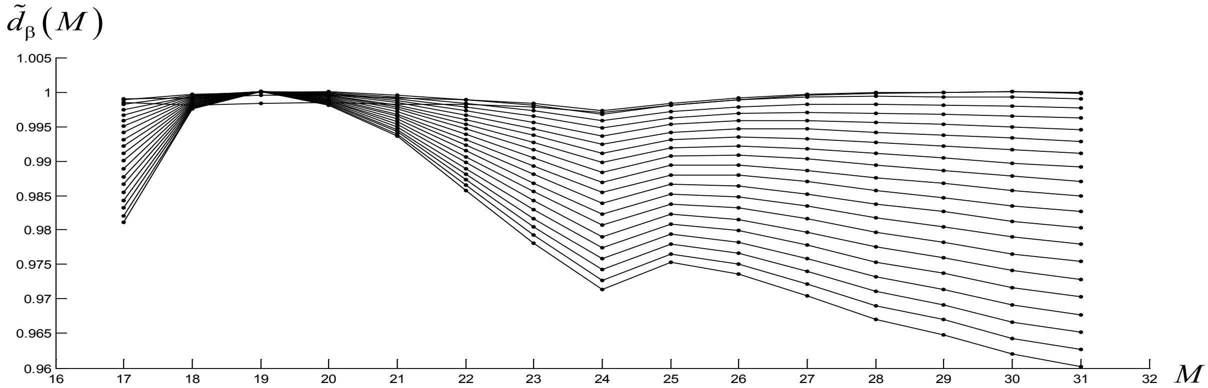

Figure 3. Normed dependences $\tilde{d}_\beta(M)$

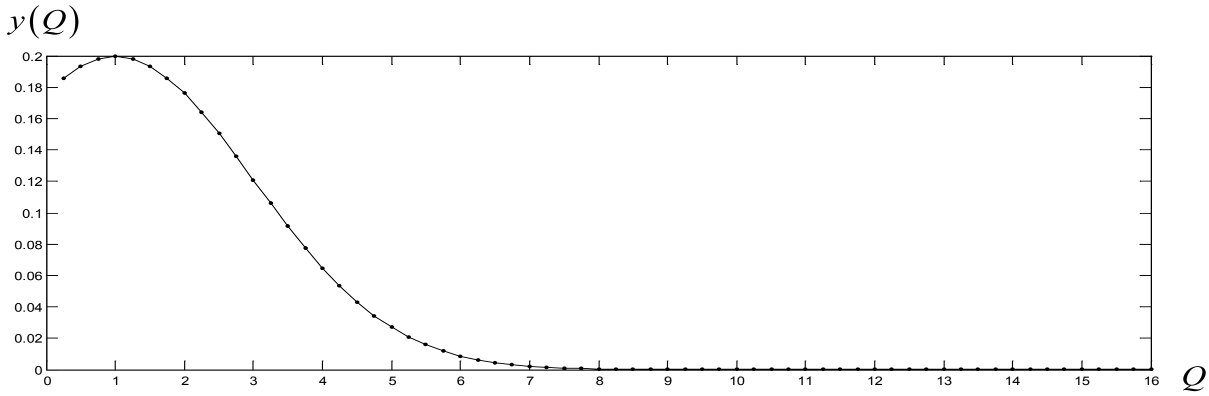

Figure 4. SNR distribution function $y(Q)$

Then the distance between the surface $p_e=p_e\left(Q,\,D(\tilde{n}),\,M,\,9\right)$ and surface $p_e=0$ with (38) $y_k=y(Q_k)$, $k=\overline{1,\,64}$, should be defined as

$$d^{\langle y\rangle}(M)=\sum_{j=1}^{25}\sum_{k=1}^{64}p_e\left(Q_k,\,D_j,\,M,\,9\right)y_k=$$

$$=\sum_{j=1}^{25}\sum_{k=1}^{64}p_e\left(Q_k,\,D_j,\,M,\,9\right)\frac{1}{2\sqrt{2\pi}}\exp\left(-\frac{(Q_k-1)^2}{8}\right). \quad (39)$$

The dependence $d^{\langle y\rangle}(M)$ is shown in the figure 5, from which

$$\arg\max_{M=\overline{17,\,31}} d^{\langle y\rangle}(M)=30\,, \quad (40)$$

$$\arg \min_{M=\overline{17,\, 31}} d^{\langle y \rangle}(M) = 18 \,, \tag{41}$$

that is by the distance (39) the most preferable multichannel data transfer system should use.

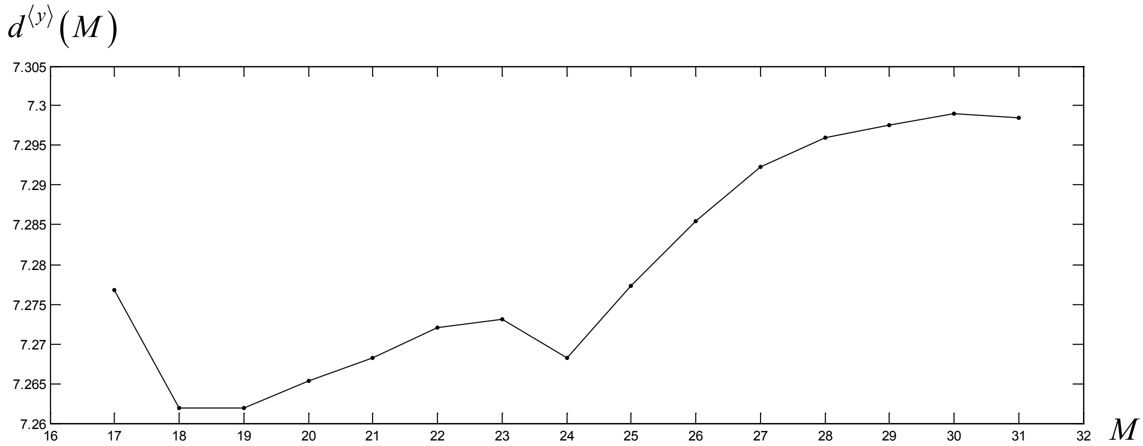

Figure 5. Dependence $d^{\langle y \rangle}(M)$

that is by the distance (39) the most preferable multichannel data transfer system should use $\left\{ \text{wal}\left(w,\, \frac{t}{T_0} \right) \right\}_{w=0}^{17}$ functions, and the worst case is 30-channel data transfer system with implemented $\left\{ \text{wal}\left(w,\, \frac{t}{T_0} \right) \right\}_{w=0}^{29}$ functions.

The fourth variant of the distance may be defined by raising each value $p_e\left(Q_k,\, D_j,\, M,\, 9\right)$, $k = \overline{1,\, 64}$, $j = \overline{1,\, 25}$, in the (39) to some power β, that may belong to the same segment $[1;\, 2]$:

$$d_{\beta}^{\langle y \rangle}(M) = \sum_{j=1}^{25} \sum_{k=1}^{64} \left(p_e\left(Q_k,\, D_j,\, M,\, 9\right) \right)^{\beta} y_k =$$

$$= \sum_{j=1}^{25} \sum_{k=1}^{64} \left(p_e\left(Q_k,\, D_j,\, M,\, 9\right) \right)^{\beta} \frac{1}{2\sqrt{2\pi}} \exp\left(-\frac{(Q_k - 1)^2}{8} \right). \tag{42}$$

The normed dependences (42)

$$\tilde{d}_{\beta}^{\langle y \rangle}(M) = \frac{d_{\beta}^{\langle y \rangle}(M)}{\max\limits_{M=\overline{17,\, 31}} d_{\beta}^{\langle y \rangle}(M)} \tag{43}$$

by $\beta \in [1;\, 2]$ with spacing 0.05 are plotted in the figure 6, from which in a majority the most preferable case is using $\left\{ \text{wal}\left(w,\, \frac{t}{T_0} \right) \right\}_{w=0}^{18}$ orthogonal system, and the worst cases are 30-channel and 31-channel data transfer systems with implemented $\left\{ \text{wal}\left(w,\, \frac{t}{T_0} \right) \right\}_{w=0}^{29}$ and $\left\{ \text{wal}\left(w,\, \frac{t}{T_0} \right) \right\}_{w=0}^{30}$ functions respectively. These consequences are antithetic to the consequences on using $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{M-1}$ preference, where the distances (33) and (36) are applied. Nevertheless, the distance $d_\beta^{\langle y \rangle}(M)$, defined for the distribution function (38) of the SNR, looks more efficiently.

And now it is possible to sort the surfaces (22) by their proximity to the zero level surface $p_e = 0$. This is less severe to make, as it is well seen from the figures 3 and 6, with using the distance (42). In such a way, resorting to the decision-making theory symbolics, have the following:

$$p_e(Q, D(\tilde{n}), 31, 9) \succ p_e(Q, D(\tilde{n}), 30, 9) \succ p_e(Q, D(\tilde{n}), 29, 9) \succ p_e(Q, D(\tilde{n}), 28, 9) \succ$$

$$\succ p_e(Q, D(\tilde{n}), 27, 9) \succ p_e(Q, D(\tilde{n}), 26, 9) \succ p_e(Q, D(\tilde{n}), 25, 9) \succ$$

$$\succ \left(p_e(Q, D(\tilde{n}), 17, 9) \sim p_e(Q, D(\tilde{n}), 23, 9) \sim p_e(Q, D(\tilde{n}), 24, 9) \right) \succ$$

$$\succ p_e(Q, D(\tilde{n}), 22, 9) \succ p_e(Q, D(\tilde{n}), 21, 9) \succ p_e(Q, D(\tilde{n}), 20, 9) \succ$$

$$\succ p_e(Q, D(\tilde{n}), 18, 9) \succ p_e(Q, D(\tilde{n}), 19, 9), \quad (44)$$

where the symbol of succeeding means the farther distancing to the zero level surface $p_e = 0$.

After the planned at the start rearranging of the dependences (22) into relations (44), the optimization problem (23) is solvable with the help of decision-making theory techniques:

$$\min_{M=\overline{17,\, 31}} p_e(Q, D(\tilde{n}), M, 9) \approx p_e(Q, D(\tilde{n}), 19, 9), \quad (45)$$

but at the same time

$$\max_{M=\overline{17,\, 31}} p_e(Q, D(\tilde{n}), M, 9) \approx p_e(Q, D(\tilde{n}), 31, 9). \quad (46)$$

CONCLUSIONS

After having analyzed the rate of the single bit detection error probability with the distances $d_1(M)$, $d_\beta(M)$, $d^{\langle y \rangle}(M)$, $d_\beta^{\langle y \rangle}(M)$, there appeared a conclusion about the theoretically optimal system of orthogonal binary functions to be implemented into an M-channel data transfer system, where $M = \overline{17,\, 31}$. The optimal system among the Walsh orthogonal binary functions systems $\left\{ \left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{M-1} \right\}_{M=17}^{31}$ is the 19-element system $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{18}$, because this orthogonal system insures by the distance $d_\beta^{\langle y \rangle}(19)$ the minimized probability $p_e(Q, D(\tilde{n}), 19, 9)$. Another defined and considered distance $d^{\langle y \rangle}(M)$ is a special case of the distance $d_\beta^{\langle y \rangle}(M)$, and the other distances $d_\beta(M)$ with $\beta \in [1;\, 2]$ drive to inconsistent results. Therefore they cannot be applied for the further explorations. Eventually, applying the distance $d_\beta^{\langle y \rangle}(M)$, there have been detected the most erroneous data transfer system that is the 31-channel system, where the orthogonal system $\left\{ \mathrm{wal}\left(w, \frac{t}{T_0} \right) \right\}_{w=0}^{30}$ is implemented.

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