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Inverse correlation filters of objects features with optimized regularization for image processing

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ABSTRACT

The problem of extraction of the image objects features by means of using the inverse filters (IF) is considered. The IF are formed by the inversion of the matrix composed of correlation vectors of a set of objects templates examples. The inversion is made with the help of singular value decomposition. Three approaches to regularization and its impact on IF recognition properties are also considered. There was defined the functional that specifies minimal mutual relations between functions of the filters to obtain optimal separation of the features. A training process is used in order to obtain filters with high recognition performance.

Keywords: inverse filters, correlation filters, objects feature recognition, regularization

1. INTRODUCTION

Convolution networks (CN) is a widely used architecture in computer vision and machine learning. The convolution gives a correlation relation between two items. The correlation is used in two ways. The first one is the correlation of image fragments with known set of templates, such as Gabor and Haar functions, and forming multilevel mosaic of the correlation samples that relate to a desired object[1,2]. Another technique uses correlation of image fragments and desired objects features[1,3,4]. The objects features are created in such a way as to achieve the required quality of recognition. The response of the correlation of feature function and the recognizing object should have a known form – Heaviside or Gaussian manner[5,6]. Some restrictions to the form of the feature function are used in order to achieve the required quality of the object recognition while using correlation methods. The conditions of minimal average correlation energy, zero-aliasing, kernelized correlation, discriminative correlation, minimum output sum of squared error and other are using[1,3,7]. The correlation approach has some issues. The convolution has high resolution ability if the entire object is captured for correlation with the feature template and there are no fragments of external objects. Also, the recognition of many objects, some of which have a structure similarity in shape, is problematic[8,9,10]. Such problems exist in case of recognition of the license plates characters in real time when desired objects are presented by small data matrices; also, these are different characters, handwritten characters, aerial data of the land objects tracking. The problem of extracting the features of objects by learning and training and using them for further recognition by means of using the regularized inverse correlation filters is considered[11,12,13].

2. RELATED WORKS

The problem of feature filter implementation is formulated as follows. Consider a set of N image fragments $\mathbf{x}_{i=0\ldots N-1}$ associated with an object. The fragments $\mathbf{x}_i$ are vectors of the lexicographical presentation of image matrices of pixels. The scalar product of the filter characteristic vector and object fragment $\mathbf{x}_i^T\mathbf{h} = g_i$ gives desired response g_i, usually $g_i = 1$ (T is the transposition). The joined expression for all fragments is the following[14,15,16]:

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$$\mathbf{X}\cdot\mathbf{h}=[\mathbf{x}_0\mathbf{x}_1...\mathbf{x}_{N-1}]^T\mathbf{h}=[g_i]_{i=0...N-1} \tag{1}$$

The system of equation (1) is usually overdetermined and its solution with minimal square error can be defined as[8]

$$\mathbf{h}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{g} \tag{2}$$

The equation (2) includes averaged fragment vector $\overline{\mathbf{x}}=\mathbf{X}^T\mathbf{g}$ and the inverse correlation matrix $\mathbf{R}^{-1}=(\mathbf{X}^T\mathbf{X})^{-1}$. The correlation is average on value too. As soon as the operation of inversion is sensitive to small variations, the vector **h** (2) corresponds to the narrow class of vectors $\boldsymbol{x}_i$ which are close to $\overline{\mathbf{x}}$. As the result, the desired signal of the filter response will be obtained only for this narrow class of objects. The task is to expand filter action on entire set of object fragments with some allowable error of the object recognition. The problem solution is found by introduction to (1) constraints on the feature vector **h** in the manner of regularization[17,18,19].

$$J(\mathbf{h})=\underset{\mathbf{h}}{minarg}\left\{\ \left|\mathbf{X}\cdot\mathbf{h}-\mathbf{g}\right|^2+\lambda\cdot Reg(\mathbf{h})\ \right\} \tag{3}$$

where λ is regularization parameter – small positive value, $Reg(\mathbf{h})$ – some function, such as quadratic norm, total variance etc. The next approach to feature characteristic constraint is to define as a matrix **A** of additional conditions using non-object regions or spectral properties of the , for example, antialiasing[5]. This conditions may be presented as additional to (1) equations system $\mathbf{A}\cdot\mathbf{h}=0$.

Solutions of the optimization problem (3) in spatial and spectral domains are described[1,3-7]. The main mathematical action that affects the result is the inversion or pseudo-inversion of the correlation matrix. In the spatial domain, singular value decomposition (SVD) is commonly used because the correlation matrix can be singular[8]. In the spectral region, this is solved by inverting the values of the spectrum[1,5]. In both cases, the problem lies in the inversion of small components of the correlation matrix or spectrum values caused by fluctuations around the averaged values. As a result of the inversion, the influence of these small quantities becomes decisive.

There are some approaches to regularization of ill-posed matrices inversion using SVD[9,10]:

$$\mathbf{X}=\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^T=\left[x_{i,k}=\sum_{m=0}^{M-1}u_{i,m}v_{k,m}s_m\right]_{i=0...N-1,k=0...M} \tag{4}$$

where $\mathbf{X}=\left[x_{i,k}\right]_{i=0...N-1,k=0...M}$ is the matrix of a size $N\times M$, $\mathbf{U}=\left[u_{i,k}\right]_{i=0...N-1,k=0...N-1}$ and $\mathbf{V}=\left[v_{i,k}\right]_{i=0...M-1,k=0...M}$ the matrices of the unitary orthogonal vectors defined by indexes size, $\mathbf{S}=diag\left[s_0...s_{M-1}\right]$, $N\geq M$, s_m – singular values: $s_m > 0$. The inverse matrix

$$\mathbf{X}^{-1}=\left[\overline{x}_{i,k}=\sum_{m=0}^{M-1}v_{i,m}u_{k,m}/s_m\right]_{i=0...M-1,k=0...N-1} \tag{5}$$

The first approach uses truncated SVD which excludes the singular values, less than some threshold level, and their corresponding vectors in (4), (5)[9,10]. The next one is based on the regularization in (3) in the manner of quadratic norm[9], $\lambda\cdot Reg(\mathbf{h})=w^2\cdot\left|\mathbf{h}\right|^2$. It gives the filter of singular values

$$\phi_i=s_i^2/(s_i^2+w^2) \tag{6}$$

which is used as an additional multiplier in the sum of expression (5). The simplest way to eliminate the ill-conditioning of the correlation matrix is to change values of its diagonal elements.

$$\tilde{\mathbf{R}}=\mathbf{X}^T\mathbf{X}+w^2\mathbf{I}=\mathbf{V}\cdot\left(\mathbf{S}\cdot\mathbf{S}+w^2\mathbf{I}\right)\cdot\mathbf{V}^T \tag{7}$$

assuming (4), **I** is the identity diagonal matrix. The model (7) is a randomizing of correlation by including dispersion of Gaussian uncorrelated noise[8]. Known approaches to solving ill-posed problems are formulated by setting constraints on values of primary data and resulting model parameters, such as a correlation matrix or averaged samples of values, feature function that characterizes desired objects, although, the main target of the constraints is the process of inverting of matrix of equations system and the inverse matrix itself. The combined regularization of inverse and direct representations of data is investigated as the example of the problem of license plate characters' recognition. The SVD (4) allows to perform the combined regularization. The object of investigation are results of the application of mentioned above approaches to singular values regularization. Also, the objects of the study consider how regularization affects the expansion of the capture area of a set of objects of the same type by the inverse correlation filter, number of successful and unsuccessful recognitions of characters.

3. MULTICHANNEL INVERSE CORRELATION FILTER DESIGN

Let's consider the recognition of P = 35 characters of car license plate by P filters. Each character has some unique features that make it different from other ones. These features are represented by some fragments of full character image. The task is to find these fragments and use them for recognition.

The correlation matrix of the k-th frame of size $N \times N$ of i-th object is defined as $\mathbf{R}_N^{(i,k)} = \mathbf{X}_N^{(i,k)T} \mathbf{X}_N^{(i,k)}$ (i = 0,1,…, P–1). The general correlation matrices of the objects are the next:

$$\mathbf{R}_N^{(i)} = \frac{1}{N_i} \sum_{k=0}^{N_i - 1} \mathbf{R}_N^{(i,k)} \tag{8}$$

where N_i is the number of fragments of i-th object. The matrix $\mathbf{R}_{N^2 \times P}$ of pointed size can be compiled by the vectors $\mathbf{r}_{N^2}^{(i)} = \mathbf{lex}\left(\mathbf{R}_N^{(i)}\right)$ which are the lexicographical presentation of the corresponding correlation matrices. The pseudo-inversion of the matrix $\mathbf{R}_{N^2 \times P}$ by using the SVD like (4) gives the matrix $\mathbf{R}_{P \times N^2}^{-1}$. The rows of this matrix $\overline{\mathbf{r}}_{N^2}^{(i)}$ can be interpreted as feature filters of the objects set and can be used for object recognizing, because

$$\sum_{m=0}^{N^2 - 1} \overline{r}_m^{(i)} r_m^{(k)} = \delta_{i,k} \tag{9}$$

where δ_{ik} is δ-function. The filters in (9) are narrow, they can recognize only that characters which correlation differ from $\mathbf{r}^{(k)}$ by variation on the level of computation accuracy. The filter band (9) can be changed by randomizing of the correlation by (7) with setting of variations dispersion bound w^2. The problem is to determine the optimal boundary, so the filter (9) will have the best separation ability for a certain capture bandwidth of the objects features. The regularized filter characteristic in terms of the SVD is as follows.

$$\overline{\mathbf{r}}^{(i)} = \left[\overline{r}_k^{(i)}(\lambda) = \sum_{m=0}^{P-1} v_{k,m} u_{i,m} / \left(s_m + \lambda \right) \right]_{k=0 \ldots N^2 - 1} \tag{10}$$

where λ is a regularization positive variable, $v_{k,m}$ and $u_{i,m}$ are elements of the matrices $\mathbf{V}$ and $\mathbf{U}$ of the matrix $\mathbf{R}_{N^2 \times P}$ SVD. The regularization in (10) degrades the separation ability of the filter (9). The separation ability will be high if that mutual relation of the filters characteristics (10) is minimal as possible at optimal value of λ. This condition can be determined by the functional

$$\underset{\lambda}{argmin} \left\{ J(\lambda) = \sum_{i,k=0: i \neq k}^{P-1} \left| \sum_{m=0}^{N^2 - 1} \overline{r}_m^{(i)}(\lambda) \overline{r}_m^{(k)}(\lambda) \right|^{\gamma} \right\} \tag{11}$$

where γ = 1,2… . As it follows from (10), the functional (11) is not limited: $J(\lambda \to \infty) \to 0$. The energy of the regularized correlation matrix increases indefinitely under this condition. Therefore, it should be supplemented with a functional that limits the energy of the regularized correlation matrix:

$$I(\lambda) = \sum_{i,k=0}^{P-1} \left| \sum_{m=0}^{N^2 - 1} r_m^{(i)}(\lambda) r_m^{(k)}(\lambda) \right|^{\gamma} \tag{12}$$

The functionals (11) and (12) are mutually inversed on values, related as x and $1/x$, and are not compatible. Therefore, their further normalization is needed. They can be joined on the base of condition of balanced relative variations, such as

$$Y(\lambda) = \underset{\lambda}{argmin}\left\{\frac{J(\lambda)}{J(0)} + \frac{I(\lambda)}{I(0)}\right\}. \tag{13}$$

The condition $\partial Y(\lambda)/\partial\lambda = 0$ yields the following equation in terms of matrix $\mathbf{R}_{N^2\times P}$ SVD and $\gamma = 2$.

$$-\frac{1}{J(0)}\sum_{i,k=0:i\neq k}^{P-1}\sum_{m=0}^{P-1}\frac{v_{i,m}v_{k,m}}{\left(s_m+\lambda\right)^3} + \frac{1}{I(0)}\sum_{i,k=0}^{P-1}\sum_{m=0}^{P-1}v_{i,m}v_{k,m}\left(s_m+\lambda\right) = 0 \tag{14}$$

The closest to zero value of the expression (14) and corresponding to it λ_{opt} can be found numerically. The desired objects features are defined by using a training set of characters. At each step of a training process P spectral values for the correlation matrix are found as $\mathbf{r}_{N^2}^{(i,k)} = \mathbf{lex}\left(\mathbf{R}_N^{(i,k)}\right)$ of each k-th frame of i-th object.

$$\sigma_j = \sum_{m=0}^{N^2-1}\overline{r}_m^{(j)}(\lambda_{opt})\cdot r_m^{(i,k)} \tag{15}$$

If $\max(\sigma_j): j = i$ then correlation matrix of the frame is accumulated to general correlation matrix $\mathbf{R}_{true,N}^{(i)}$ as a true part of (8). The matrices $\mathbf{R}_{true,N}^{(i)}$ are used to create the filters for the next step of the training. The process stops when number of feature frames reaches some value and does not change.

4. EXPERIMENTAL ANALYSIS OF THE FILTERS. METHODS OF ERRORS ELIMINATING

Analysis of the inverse correlation filters effectivity with different approaches to regularization was made by means of using 2612 images of 35 license plate characters of a size from 9×9 to 41×41, cropped from car images. In total, there were used 373622 fragments of a size $N\times N$ = 9×9, 267290 of a size 11×11 and 178872 of a size 13×13.

Initially, the recognition by the spectrum (15) is not effective, the number of errors is slightly less then number of successful recognitions even after completed training by selection of the feature frames. Therefore, the following methods were used to reduce the level of object recognition errors.

4.1 Extension of the Spectrum

Along with the correlation matrix $\mathbf{R}_{true,N}^{(i)}$, the matrix $\mathbf{R}_{false,N}^{(i)}$ of frames which gives a false result, can be accumulated too. The inversion of the matrix $\left[\mathbf{R}_{true,N^2\times P}\mathbf{R}_{false,N^2\times P}\right]$ compiled by these two parts gives 2P filters. The extended spectrum allows separate non-feature objects fragments.

4.2 Combination of Inverse Correlation Spectra

Let $P = T\cdot Q$. Recognized objects can be separated on T groups of size Q. The filters (10) can be defined for each of T groups using the pseudoinversion of the matrices $\mathbf{R}_{true,N^2\times Q}$ of vectors which corresponds to frames of the objects group. Also, can be defined filters for Q groups by T objects. Then, if maximal spectral element σ_J in (15) was found then should be maximal element $\sigma_{mod_Q J}^{[J/T]}$ – $mod_Q J$ -th element of the $[J/T]$-th spectral group, $[J/T]$ is an integer part. Also, there should be a maximal element 2P. If these conditions are met then the maximum σ_J is true with more higher probability. The basis of *2P* regularized filters and group filters have different recognizing ability because they are defined on different sets of correlations. Their properties can be joined.

4.3 Spectrum Selection

Let the i-th object was recognized as j-th object in the training process. Such mistake was caused by the form of the spectrum (15). The spectrum can be accumulated by averaging as the matrix $\left[\varsigma_{i,j,m}\right]_{i,j,m=0...P-1}$. The first dimension relates to the current object, second dimention with recognized object and third – to P spectral samples (15). This matrix fixes the cases when different objects can include similar frames. Such frames should be eliminated. The true spectrum can be selected by finding the minimal spectral distance. If spectrum (15) points on the i-th object and then object is recognized as true, otherwise as false.

4.4 Spectra Superposition

As it follows from the paragraphs 3, 4.1 and 4.2, the spectral analysis (15) are made by using $2P + Q + T$ filters. The fragments of minimal size can be used for analysis of image rectangles, obtained by standard methods, candidates on plate, to minimize computational complexity of the recognizing. The image fragments of recognized objects features are extended for analysis by filters of the next size. The combining of recognized objects allows to reduce error level too.

4.5 Branching of the Recognizing

The purpose of the training process is to obtain true recognition with minimal error level. Not all objects templates will be recognized by created filters basis. The objects which are recognized create the branch of the recognizing. Remained templates can be used for obtaining the next branches while all templates have been recognized. Each true object recognition can be in a single branch. This is a criterion to eliminating errors as well.

4.6 Experimental Data

The recognizing while using regularized filters of fragments of a pointed above size was investigated and results are presented in the tables 1-4. The results of the optimized filters (10) are presented in Table 1.

Table 1. Dynamic of training process with optimized regularization.

Training step	Feature fragments	True fragments	False fragments	True rating
0	2183	753	1430	0.3449
	1558	947	611	0.6078
	1163	868	295	0.7463
20	2148	947	1201	0.4408
	1073	822	251	0.7660
	943	845	98	0.8960
40	2082	981	1101	0.4711
	1051	839	212	0.7982
	864	805	59	0.9317
60	1822	887	935	0.4868
	961	774	187	0.8054
	839	791	48	0.9427
80	1810	897	913	0.4955
	925	769	156	0.8313
	806	768	38	0.9528
100	1820	897	923	0.4928
	916	765	151	0.8351
	794	760	34	0.9571

For each of the training steps three numbers corresponding to the fragments size: N = 9, 11, 13 are presented. The obtained by optimization regularization parameter $\lambda_{opt} \sim 0.01 \cdot s_0$, where s_0 is a maximal singular value. As can be seen from the table, the true rating shows, the size of the fragment affects the quality of recognition. For comparison, Table 2 shows the results of using filter (10) with a non-optimal regularization parameter, the fragment capture range has sharply decreased, and only some objects are reliably recognized. Data are given only for $N = 13$. Table 3 shows the results of the filter with using cutoff of singular values on the level of λ_{opt} in the inverse SVD. Table 4 shows usage of the singular value filter (6) for $w^2 = \lambda_{opt}$.

Table 2. Dynamic of training process with regularization by $\lambda << \lambda$.

Training step	Feature fragments	True fragments	False fragments	True rating
0	272	154	118	0.5661
10	73	72	1	0.9853
20	66	66	0	1.0
40	54	54	0	1.0

Table 3. Dynamic of training process with truncated SVD on level λ_{opt}.

Training step	Feature fragments	True fragments	False fragments	True rating
0	2245	1138	1107	0.5069
10	2546	1289	1257	0.5062
20	2608	1286	1342	0.4854

Table 4. Dynamic of training process with filter of SVD with λ_{opt}.

Training step	Feature fragments	True fragments	False fragments	True rating
0	2599	1503	1096	0.5782
10	2532	1557	975	0.6149
20	2501	1485	1016	0.5937
40	2616	1561	1055	0.5967

5. CONCLUSION

As can be seen from the presented data, only filters based on the randomized model (7) make it possible to obtain a stable learning process with a gradually improving result. The parameter λ in (11) can be used to control the range of fragments capture, when it is increased, the range expands, but the number of errors also increases.

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