UDC 621.316
DOI: 10.15587/1729-4061.2022.260080

DEVELOPMENT OF A METHOD FOR OPTIMIZING INDUSTRIAL ENERGY STORAGE UNITS PLACEMENT IN ELECTRIC DISTRIBUTION NETWORKS ON THE BASIS OF IDEAL CURRENT DISTRIBUTION

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The object of this study is the process of integrating industrial energy storage units (IESU) into distribution electrical networks. Their connection helps reduce peak loads on the elements of networks and improve the quality of voltage. However, determining the optimal IESU capacities and their places of connection to networks is associated with objective difficulties. It is necessary to apply comprehensive optimality criteria and take into consideration active restrictions. In addition, the trends in the development of distribution networks and pricing in the energy market are partially undefined. The current study proposes the formalization of the problem of optimizing the placement of IESU in distribution networks and reports a new method to solve it. Its application contributes to a reasonable definition of the volume of investments in the development of IESU, taking into consideration technical restrictions on the part of distribution networks. To solve the problem of multifactorial optimization of the energy storage system, the decomposition and method of ideal current distribution (for electricity losses) were applied. It is shown that this problem can be reduced to an iterative calculation of current distribution in the substitution circuit of power grids with active resistances. And, to take into consideration economic factors, a technique to determine and adjust fictitious resistances was devised. An optimization algorithm has been proposed that ensures a decrease in the number of computing operations and an increase in the reliability of obtaining an optimal solution. Its application makes it possible to take into consideration the dynamics of pricing, consumption, and electricity generation processes over long periods. This contributes to the formation of sound design decisions on the connection of IESU to distribution networks

Keywords: energy storage system, distribution electrical network, optimization, losses, quality of electricity

Received date 03.05.2022
Accepted date 20.06.2022
Published date 30.06.2022

How to Cite: *Kulyk, V., Teptya, V., Vishnevskyi, S., Hrytsiuk, Y., Hrytsiuk, I., Zatkhei, M. (2022). Development of a method for optimizing industrial energy storage units placement in electric distribution networks on the basis of ideal current distribution. Eastern-European Journal of Enterprise Technologies, 3 (8 (117)), 6–16. doi: https://doi.org/10.15587/1729-4061.2022.260080*

1. Introduction

Previously, distribution electrical networks (DEN) were designed for centralized power supply. Therefore, the increase in the installed capacity of power plants using solar, wind, and hydropower, the introduction of new types of electricity consumers, in particular electric transport, heat pumps, etc., has led to significant changes in the functioning and operation of DEN [1]. For these reasons, distribution system operators (DSO) faced new technical problems, especially due to the volatility of generating solar and wind power, as well as the consumption of charging stations for electric vehicles [2]. It turned out that in order to solve new problems, it is necessary to carry out large-scale measures for the reconstruction of power grids, in particular with the transfer to the upper voltage classes [3]. However, this approach does not eliminate the causes of the problem. With a sharply variable load of power lines and transformers, the efficiency of their use is low. And, therefore, the payback of such decisions is unsatisfactory. Further growth of the installed capacity of renewable energy sources and the number of consumers-suppliers of electricity will increase the volatility of energy flows. And this, in the future, can adversely affect the work of transmission system operators (TSO) and the energy market as a whole [4].

The integration of energy storage systems contributes to the solution of the problem of planning DEN modes with a sharply variable load and dispersed generation [5]. The use of industrial energy storage units (IESU) contributes to improving operational safety and stability of distribution and main networks by reducing peak load. As a result, this reduces the price of electricity in the intraday market and the cost of electricity transmission and distribution. In addition, this approach postpones the need to implement measures to strengthen DEN until the technical re-equipment is due to the growth of electricity distribution, that is, it will become economically justified.

Despite these advantages, the development of IESU in distribution networks is associated with a number of technical problems that were not solved in scientific research. Therefore, studies on improving methods and means of optimizing the integration of energy storage systems (ESS) into distribution networks are relevant.

2. Literature review and problem statement

World experience in the operation of modern distribution grids shows [6] that an alternative to their structural reinforcement is the introduction of ESS. Their advantages and economic efficiency are actively discussed by the scientific community. In particular, paper [7] provides an analysis of the main system services that can be provided by the owners of IESU (operators of accumulation systems), provides options for the implementation of services and analyzes financial revenues. In addition, it is shown that studies related to the placement and management of ESS require the use of specific modeling methods. Studies [8, 9] report an analysis of economic factors and market mechanisms that may impede the implementation of ESS. In [10, 11] and others, it is shown that due to the flexibility and expected reduction in specific value, electrochemical ESS are promising for industrial use.

Methods for determining the optimal connection points, rated capacities, and capacities of battery IESU are analyzed in [12–15]. Paper [12] provides an overview of the results of scientific research in order to analyze the main trends and technical solutions for the introduction of electricity storage systems. Among the unresolved tasks, the formation of market prices for specific types of services for operators of energy storage systems (SSO) is mentioned. In addition, it is important to form the theoretical basis for the comprehensive optimization of capacity and placement of IESU, taking into consideration the possibility of providing various services to several stakeholders. The reason for the lack of solution to this problem is the complexity of developing an effect assessment model for several energy market entities. Among the unresolved problems of integration of IESU into distribution networks, there is a lack of effective incentive mechanisms. The reason is the lack of a theoretical basis and regulatory framework for the evaluation and monetization of additional advantages, which ensures the integration of IESU. In particular, improving the reliability of electricity supply, reducing electricity losses and improving its quality.

Paper [13] provides an overview of the results of scientific research tackling the analysis of economic factors and technical limitations on the path of development of electricity storage systems. According to the results of the analysis, the limits of application of individual optimality criteria, as well as individual groups of optimization methods, were established. It is shown that to optimize the development of small-capacity ESS, it is advisable to use purely economic criteria. However, for optimal integration of IESU into electricity distribution systems, the criteria should take into consideration technical factors. The most used optimization methods are heuristic simulation algorithms, which is explained by their popularity and the use of simplified formulations of the optimization problem. Among the unresolved tasks, it is noted the need to improve models to assess the effectiveness of technical solutions for integrating IESU into electrical networks. In particular, environmental factors are considered relevant in complex efficiency criteria, especially for distribution networks with significant generation from renewable sources.

Study [14] provides an overview of the results of scientific research addressing the improvement of mathematical methods and algorithms for optimizing the introduction of ESS in electrical grids. A detailed analysis of mathematical models used to calculate the mode parameters of distribution networks during optimization of IESU placement is given. As a promising area of research, it noted increasing the accuracy of modeling energy flows in DEN, in particular by moving from single-line to three-phase replacement circuits. The authors include meta-resource optimization algorithms as in the previous review. Among the directions of improvement of methods for solving the problem of optimization of energy storage systems in DEN is the need to improve the quality of forecasting of influential factors. And to do this, algorithms must ensure the processing of significant amounts of retrospective data on load changes, generation of dispersed energy sources, etc.

In [15], it is noted that the design statement of the problem of placing energy storage systems in DEN requires taking into consideration a wide range of factors affecting the efficiency of consumers, the quality of electricity, as well as the economic efficiency of the solution.

For IESU, a number of parameters can be distinguished that are subject to optimization when they are connected to DEN. First of all, this is the connection point (supply unit), rated capacity (MW) and energy capacity (MWh). Further, depending on the required charge speed and frequency of responses, they determine a control and communication technique.

Taking into consideration the complexity of integrating IESU, well-known approaches [12–16] provide optimization mainly by rated power, energy capacity, and location in the network. The rated capacity and energy intensity are considered as independent technical indicators of IESU, which are determined taking into consideration investment opportunities. These parameters are discrete but they are usually defined as continuous variables using appropriate optimization methods.

Simultaneous optimization of placement and power (energy intensity) of IESU is a difficult task. Paper [16] analyzes a number of methods to solve it, which can be divided into four groups according to the methodology used: analytical methods, methods of mathematical programming, methods of exhaustive search (optimized selection of variants), and heuristic methods. According to the results of the analysis, it was established that for large-dimensional power grids, the problem of integrated IESU optimization was not effectively solved.

An important limitation that must be taken into consideration when modeling and optimizing the technical characteristics of IESU is the efficiency of the charge/discharge cycle, or the limitation of the energy intensity of the charge and discharge of batteries. These values are always different, and the difference depends on the technology of energy storage in IESU. The complexity of the optimization task is further increased if we take into consideration the economic factors necessary to achieve proper investment efficiency and planning.

Our review of the results of scientific research has established that the problem of introducing and operating electricity storage systems in DEN has a number of unresolved issues. To formulate the optimization problem, complex optimality criteria should be applied and formalized. It is necessary to improve the procedures for finding optimal solutions to take into consideration the economic, technical, and environmental factors that are related. To develop technical solutions, it is necessary to take into consideration not only technical limitations on the part of IESU but also the regime restrictions of DEN.

3. The aim and objectives of the study

The aim of this work is to increase the efficiency of the functioning of IESU in distribution networks by optimizing their energy intensity and connection units according to a comprehensive technical and economic criterion. This will make it possible to coordinate the issues of operational safety of electricity transmission systems with issues of improving the reliability and quality of electricity supply to consumers.

To achieve the set aim, the following tasks have been solved:

– to formalize the problem of optimizing energy intensity and IESU connection nodes and justify assumptions that do not contradict the purpose of the study;

– to adapt to solve the optimization problem in the described formal setting by the method of ideal current distribution [17], based on the Hamilton-Ostrogradsky principle;

– to develop a technique to adjust the parameters of the ideal current distribution model to take into consideration the economic factors associated with the installation and operation of IESU;

– to develop an algorithm for complex optimization of the capacity and connection nodes of IESU based on the proposed method;

– to conduct a computational experiment to confirm the effectiveness of the proposed method and algorithm on the example of distribution networks of 110–35 kV.

4. The study materials and methods

The object of our research is the process of integrating IESU into distribution electrical networks. The task of optimizing this process belongs to the problems of nonlinear multifactor optimization under the conditions of partial uncertainty. The solution to such problems is associated with a number of algorithmic and information issues [16]. To eliminate them, during the formalization of the problem, a number of assumptions were adopted.

The process of operation of existing and new IESU in electrical networks was averaged at intervals Δt=30 minutes. The time interval was determined on the basis of the standard frequency of the survey of monitoring systems for distribution network modes. The discreteness of representation of typical load and generation graphs was also taken into consideration [18].

The sets of permissible capacities and rated capacities of IESU were accepted continuous at the stage of their optimization, followed by rounding to standard values. For rounding, a separate algorithm for optimized selection of options is proposed, taking into consideration the complex technical and economic criterion.

The time factor was represented as a discrete set of counts corresponding to the standard intervals Δt.

The rate of charge of IESU batteries was taken into consideration by the maximum increase in the capacity $\Delta E_{i\_\max}$ during the standard interval Δt, which depends on the IESU current capacity $E_{i(t)}$. The effect of voltage levels on IESU buses on the charge and discharge speed was not taken into consideration. The factor of degradation of IESU batteries was not taken into account.

To analyze and solve the tasks of this study, the method of ideal current distribution (at a minimum of electricity losses) obtained on the basis of the Hamilton-Ostrogradsky principle was used. For the formalization of optimization problems and the construction of iterative procedures for their solution, generalizing methods of modeling theory, methods of linear and nonlinear programming are used.

The established DEN modes were modeled and analyzed using the method of nodal voltages in the form of a current balance. To calculate losses, the method of numerical integration and the method of average loads were used. For the distribution of electricity losses between individual transactions caused by the charge/discharge of IESU, the method of loss distribution coefficients [17] was applied. For the development of optimization algorithms, matrix algebra, graph theory, and decomposition were used.

To conduct a computational experiment, a module was used to find optimal solutions from the Microsoft Excel tabular processor, the suite of specialized software "Grafskaner", as well as our own software.

To check the performance of the proposed method and algorithm for optimizing the placement of IESU in distribution networks, a computational experiment was conducted on the example of an electrical network of 110/35 kV at the Joint Stock Company "Vinnytsiaoblenergo" (Ukraine). The initial data were obtained from representatives of the energy supply company and from open sources.

5. Results of studying the process of integration of industrial energy storage units into distribution networks

5. 1. Setting the optimization problem

The problem of optimizing the capacity and connection nodes of IESU was solved in the following statement. For the distribution grid, it is necessary to determine the optimal energy intensity and places of connection of IESU, which will provide a group intraday adjustment of electricity supply schedules. The effectiveness of solutions should be assessed by the return on investment R. It is necessary to take into consideration the restrictions on voltage deviations on the buses of substations, current loads of power lines, and power transformers of DEN in the modes of charge and discharge of IESU. The placement of industrial drives should be determined taking into consideration the technical capabilities of their connection at the substations of distribution networks. The energy intensity of IESU should be determined taking into consideration the efficiency of the charge/discharge cycle.

Taking into consideration our assumptions, the formalized statement of the optimization problem, taking into consideration active restrictions, can be represented as follows:

$$\begin{cases} R(\mathbf{X}) = \dfrac{AP(\mathbf{X}) + ACR(\mathbf{X})}{INV(\mathbf{X})} \to \max, \\ \mathbf{X} = \{P_i, E_i, L_i,\ i \in [1 \ldots n_{esu}]\};\ L_i \in \mathbf{L}; \\ E_{i(t)} = E_{i(0)} - \sum_{j=0}^{t} \left(\eta P^{+}_{ESU\,i(j)|j\in \mathbf{T}_D} - P^{-}_{ESU\,i(j)|j\in \mathbf{T}_C} \right); \\ D_t = \sum_{i=1}^{n_{esu}} P^{+}_{ESU\,i(t)} - \sum_{i=1}^{n_c} P_{\mathrm{C}i(t)} - \Delta P(\mathbf{X}_t); \\ D_t \le D_{\max}, t \in \mathbf{T}_D; \\ C_t = \sum_{i=1}^{n_{esu}} P^{-}_{ESU\,i(t)} + \sum_{i=1}^{n_c} P_{\mathrm{C}i(t)} + \Delta P(\mathbf{X}_t); \\ C_t \le C_{\max}, t \in \mathbf{T}_C; \\ E_{i(t)} \le E_i,\ t \in \mathbf{T};\ \ \mathbf{T} = \mathbf{T}_C \cup \mathbf{T}_D; \\ \left| E_{i(t+1)} - E_{i(t)} \right| \le \Delta E_{i\_\max}\left(E_{i(t)}\right),\ t \in \mathbf{T}, \\ P^{+}_{ESU\,i(t)} \le P_i,\ P^{-}_{ESU\,i(t)} \le P_i,\ i \in [1 \ldots n_{esu}], t \in \mathbf{T}; \\ U_{i\_\min} \le U_{i(t)} \le U_{i\_\max},\ i \in [1 \ldots n_n], \\ t \in \mathbf{T};\ I_{i(t)} \le I_{i\_\max},\ i \in [1 \ldots n_b], t \in \mathbf{T}, \end{cases} \tag{1}$$

where $AP(\mathbf{X})$ is the current annual profit;

$ACR(\mathbf{X})$ is the additional annual depreciation charges for the renovation of DEN equipment;

$INV(\mathbf{X})$ is the investments related to the implementation of ESS;

$\mathbf{X}$ is the set of optimized variables that characterize the placement of IESU, in particular, installed power (P_i), energy intensity (E_i), and the connection point (L_i) of the ith storage unit;

n_c, n_{esu}, n_n, n_b is, accordingly, the number of consumption units with controlled capacities, IESU, controlled voltage units, and electricity-controlled branches;

$\mathbf{L}$ is the set of potential places of installation of IESU;

$E_{i(0)}$, $E_{i(t)}$ is the energy of the storage unit, which can be acquired at the beginning of the billing period and at the time t;

$\Delta E_{i\_\max}$ is the maximum increase in the capacity of the storage unit during a given period Δt, which characterizes the charge/discharge rate;

η is the indicator of the efficiency of the charge/discharge cycle of IESU, which determines the share of accumulated energy that can be used;

$P^{+}_{ESU\,i(t)}$, $P^{-}_{ESU\,i(t)}$ is the average storage unit power, respectively, during the battery discharge and charge;

$P_{\mathrm{C}i(t)}$ is the average power of the ith node of consumption during the period t;

$\mathbf{T}_D$, $\mathbf{T}_\mathrm{C}$ is the set of periods of time t of a given duration corresponding to the discharge and charge of the storage system batteries, respectively;

D_t, C_t is the power imbalances arising in DEN during the period t of the storage system battery discharge or charge, respectively;

$U_{i\_\max}$, $U_{i\_\max}$, $I_{i\_\max}$ is the maximum permissible values of voltages in controlled units and currents in controlled branches of the substitution DEN scheme;

$C_{\max}$, $D_{\max}$ is the limit values of electricity flows at the limit of the balance sheet ownership of DEN in direct and reverse directions, respectively.

A significant simplification of problem (1) can be obtained if one neglects the influence of the operating modes of IESU on the loss of electricity in distribution networks. This statement of the problem is relevant for the operator of accumulation systems since it makes it possible to estimate the amount of investment in the development of accumulation systems, which will ensure the fastest return of funds (maximum cumulative effect):

$$\begin{cases} R(\mathbf{X}) = \dfrac{AP(\mathbf{X}) + ACR(\mathbf{X})}{INV(\mathbf{X})} \to \max,\ \mathbf{X} = \{P_\Sigma, E_\Sigma\}; \\ E_{\Sigma(t)} = E_{\Sigma(0)} - \sum_{j=0}^{t} \left(\eta P^{+}_{ESS\,\Sigma(j)|j\in \mathbf{T}_D} - P^{-}_{ESS\,\Sigma(j)|j\in \mathbf{T}_C} \right); \\ D_t = P^{+}_{ESS\,\Sigma(t)} - \sum_{i=1}^{n_c} P_{\mathrm{C}i(t)} - \Delta P(\mathbf{X});\ D_t \le D_{\max}, t \in \mathbf{T}_D; \\ C_t = P^{-}_{ESS\,\Sigma(t)} + \sum_{i=1}^{n_c} P_{\mathrm{C}i(t)} + \Delta P(\mathbf{X});\ C_t \le C_{\max}, t \in \mathbf{T}_C; \\ \mathbf{T} = \mathbf{T}_C \cup \mathbf{T}_D; \\ E_{\Sigma(t)} \le E_\Sigma,\ t \in \mathbf{T}, \\ E_{\Sigma(t+1)} - E_{\Sigma(t)} \le \Delta E_{\Sigma\_\max}\left(E_{\Sigma(t)}\right),\ t \in \mathbf{T}, \\ P^{+}_{ESS\,\Sigma(t)} \le P_\Sigma,\ P^{-}_{ESS\,\Sigma(t)} \le P_\Sigma, t \in \mathbf{T}; \end{cases} \tag{2}$$

where P_Σ, E_Σ is the total installed capacity and energy intensity of ESS;

$P^{+}_{ESS\,\Sigma(t)}$, $P^{-}_{ESS\,\Sigma(t)}$ is the total average power of storage systems, respectively, during a battery discharge and charge.

The current annual profit in this formulation is determined according to the data of measured or typical plots [18] of the total electricity consumption of DEN and the dynamics of electricity prices in the intraday electricity market. To solve the optimization problem in statement (2), any method of optimizing nonlinear continuous functions, such as the gradient method [16], can be used.

However, statement (2) has a significant drawback due to the fact that during the decomposition of a complex problem, the connection between economic factors relevant to SSO and technical restrictions relevant to DSO is lost. Such a simplification can lead to making erroneous decisions at the stage of issuing technical specifications for the connection of IESU. Their implementation in the future, instead of simplifying the operation of distribution networks, may lead to the need for their technical re-equipment due to the inconsistency of the modes of accumulation systems.

Thus, the optimization problem in statement (2) can be used only to limit the search area for optimal solutions $\mathbf{X}$ by the total power and capacity of ESS.

To refine the optimal solution for the placement and capacity of specific IESU, the issues of energy efficiency of distribution networks should be taken into consideration. The criterion of optimality is the maximum profitability of IESU connection to distribution networks. Independent optimized variables are the capacity and place of connection of individual IESU.

For each characteristic period of charge or discharge of IESU, the task of optimizing the DEN mode by power and voltage can be represented as follows:

$$\begin{cases} R'(\mathbf{X}) = \dfrac{AP'(\mathbf{X}) + ACR'(\mathbf{X})}{INV'(\mathbf{X})} \to \max, \\ \mathbf{X}_t = \left\{E_i, L_i,\ i \in [1...n_{esu}]\right\},\ L_i \in \mathbf{L}; \\ E_{i(t)} = E_{i(t-1)} - \left(\eta P^{+}_{ESU\,i(t)|t\in\mathbf{T}_D} - P^{-}_{ESU\,i(t)|t\in\mathbf{T}_C}\right); \\ \left|E_{i(t)} - E_{i(t-1)}\right| \le \Delta E_{i\_\max}\left(E_{i(t)}\right),\ t \in \mathbf{T}, \\ U_{i\_\min} \le U_{i(t)} \le U_{i\_\max},\ i \in [1...n_n], t \in \mathbf{T}; \\ I_{i(t)} \le I_{i\_\max},\ i \in [1...n_b], t \in \mathbf{T}, \\ E_{i(t)} \le E_i,\ i \in [1...n_{esu}], \\ \sum_{i=1}^{n_{esu}} E_{i(t)} \le E_\Sigma,\ \sum_{i=1}^{n_{esu}} P_{i(t)} \le P_\Sigma,\ t \in \mathbf{T}, \end{cases} \tag{3}$$

where $AP'(\mathbf{X})$ is the current annual profit of DEN, due to a decrease in electricity losses;

$INV'(\mathbf{X})$ is the investments related to the connection of IESU to the power grids;

$ACR'(\mathbf{X})$ is the additional annual depreciation charges due to the connection of IESU.

The problem in such a statement makes it possible to take into consideration the interests of DSO during the formation and issuance of technical specifications for connecting energy storage units to distribution networks.

5. 2. General approach to solving the optimization problem

The problem of optimizing the parameters and nodes of IESU connection (1) belongs to the tasks of optimizing nonlinear continuous functions with restrictions. To obtain maximum computational efficiency in solving problem (1), it is proposed to use its decomposition. At the first stage, we use the statement of problem (2) and determine the optimal total capacity of storage systems for connection to DEN. As a source of income, a reduction in the cost of purchasing electricity is considered to provide for a given set of consumers. The result of calculations is the total capacity of storage systems for DEN, the amount of investment for their installation, and the optimal schedule of charge and discharge of the ESS during the estimation period.

In the second stage, the statement of problem (3) and the method of optimization of nonlinear functions with restrictions are used. For each charge interval or discharge of the energy storage system (for example, Δt=30 minutes), the places of connection of specific IESU and the optimal power consumption or generation are determined. At this stage, the source of income is to reduce the cost of purchasing electricity, taking into consideration the reduction of electricity losses. According to the values of the capacities, the optimal capacity of IESU for installation at a certain substation of DEN is calculated.

To solve such problems with respect to electrical networks and systems, the method of ideal (economic) current distribution showed high efficiency [17]. The method is built on the basis of the principle of the least action in the formulation of Hamilton-Ostrogradsky.

According to the method of ideal current distribution, the optimal capacities of electricity sources and consumers (including IESU) according to the minimum loss of electricity criterion can be determined from the results of modeling the "ideal" power grid modes. To do this, one uses the substitution schemes of networks with active resistances.

Economic factors associated with the installation and operation of new equipment can be taken into consideration by adding appropriate fictitious resistances to the substitution scheme. These resistances are not permanent. They change nonlinearly depending on the mode parameters of the power grid and independent optimized **X** variables.

The use of this approach, unlike classical methods of nonlinear programming, significantly reduces the duration of calculations to optimize the parameters of a separate power grid mode (at a certain time interval). The solution is as close as possible to the global minimum of the objective function if it can be represented in the form of energy consumption.

Computational efficiency and reliability of the approach is ensured by reducing the problem of multifactorial optimization of the placement of IESU to the iterative calculation of "ideal" DEN modes according to the corresponding substitution schemes. The stability of the solution is ensured by the absence of accumulation of errors of calculations between iterations and consecutive time slices.

Due to the reduction of time spent on optimizing the placement of IESU for a separate degree of the load schedule (or mode of operation) of DEN, it is possible to analyze the efficiency of operation of ESS over long periods (a year or more). The source data are typical, measured, or projected load diagrams and DEN generation with standard discreteness Δt.

5. 3. Adjusting the parameters of the ideal current distribution model in distribution networks with energy storage units

The problem of optimizing the flow of active power in electrical networks according to the criterion of minimum losses of electricity can be reduced to the calculation of economic current distribution using the substitution *r*-circuit of power grids [17]. A similar approach was used to optimize the placement and capacity of IESU based on the profitability of their connection. In order to take into consideration economic factors, in particular the costs of joining and operating IESU, additional economic resistances R_e were introduced into the substitution *r*-scheme (Fig. 1).

Expressions for calculating and adjusting economic resistances are obtained through equivalent transformations of the expression to determine the return on investment [19]:

$$R = \frac{AP + ACR}{INV}. \tag{4}$$

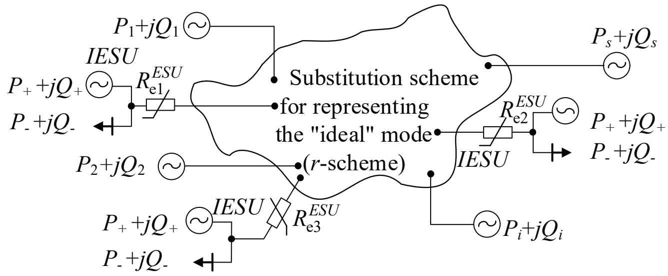


Fig. 1. The substitution scheme of the power grid for calculating the ideal current distribution by the profitability of connecting energy storage units

Current annual profit takes into consideration the balance sheet profit due to DEN's income from reducing electricity losses, except for deductions for servicing credit funds and income tax.

After mathematical expressions for the components of profits and deductions, expression (4) took the following form:

$$R=\sum_t \left(\begin{array}{l} \Delta P^0_{(t)}\Delta t \dfrac{C_{(t)}}{INV} - \Delta P_{(t)}\Delta t\left(1-\alpha_{bp}\right)\dfrac{C_{(t)}}{INV} - \\ -\left(\alpha_{oc}+\alpha_{\Delta W}\sum_{i=1}^{n_{ess}} P_i \Delta t \dfrac{C_{(t)}}{INV}+\alpha_{da}\right)\times \\ \times\left(1-\alpha_{bp}\right)-\alpha_{cr}-\alpha_{da} \end{array} \right), \quad (5)$$

where ΔP^0, ΔP is the loss of power for the characteristic DEN mode before and after the connection of IESU, taking into consideration the specified (on the part of SSO) schedule of their operation;

$C_{(t)}$ is the price of electricity on the intraday market during the t-th period;

α_{oc} is the specific annual operating costs for IESU;

$\alpha_{\Delta W}$ is the specific energy losses in IESU, in particular conversion efficiency and costs for own needs;

α_{da} is the specific annual depreciation and amortization deductions for IESU connection;

α_{bp} is the tax on balance sheet profit;

α_{cr} is the annual cost of servicing credit funds;

K is the investments for the connection of ESS.

Typical technical solutions are used to connect IESU to distribution networks. They may vary depending on the substation scheme to which one plans to connect. However, the total investments for the connection of a given set of IESU can be taken conditionally stable. Based on this, the first component of expression (5) will practically not affect the optimality of decisions on the capacities and places of connection of IESU. Then, problem (5) can be naturally reduced to finding a minimum of operating costs in DEN, brought to the unit of investment:

$$OC_* = \\ =\sum_t \left(\begin{array}{l} \Delta P_{(t)}\Delta t\left(1-\alpha_{bp}\right)\dfrac{C_{(t)}}{INV}+ \\ +\left(\begin{array}{l} \alpha_{oc}+ \\ +\alpha_{\Delta W}\sum\limits_{i=1}^{n_{ess}} \begin{array}{l} P_i\Delta t\times \\ \times\dfrac{C_{(t)}}{INV} \end{array} +\alpha_{da} \end{array} \right)\times \\ \times\left(1-\alpha_{bp}\right)+\alpha_{cr}+\alpha_{da} \end{array} \right). \quad (6)$$

For each time period of the schedule of operation of ESS with a conditional duration Δt, the relative cost of electricity losses [p.u./kWh] is stable:

$$b_{\Delta W(t)*}=\left(1-\alpha_{bp}\right)\frac{C_{(t)}}{INV}.$$

Therefore, the function of total operating costs can be replaced by the function of equivalent losses of electricity, which will practically not affect the solution to the problem:

$$\Delta W_{eq}=\sum_t \left(\begin{array}{l} \Delta P_{(t)}\Delta t+\alpha_{\Delta W}\sum\limits_{i=1}^{n_{ess}} P_i\Delta t+ \\ +\dfrac{INV}{C_{(t)}}\left(\left(\alpha_{oc}+\alpha_{da}\right)+\dfrac{\alpha_{cr}+\alpha_{da}}{\left(1-\alpha_{bp}\right)}\right) \end{array} \right). \quad (7)$$

Thus, the function of electricity losses in DEN was built, equivalent in cost to the costs associated with the operation of ESS, taking into consideration the reduction of electricity losses after their installation. Minimization of equivalent losses ΔW_{eq} in the field of balance restrictions and restrictions on parameters (3) will make it possible to obtain for each of the potential places of connection of IESU the values of P_i capacities that correspond to the solutions to the problem of maximizing profitability (3).

In order to take into consideration economic factors according to (7) when determining the ideal current distribution, it is necessary to introduce active resistances to the substitution scheme of the "ideal" DEN mode, the losses of which will be equivalent to additional losses of electricity.

$$R_{ei}=\frac{U_i^2}{P_i}\sum_t \left(\alpha_{\Delta W}+\frac{INV}{C_{(t)}P_i\Delta t}\left(\begin{array}{l} \left(\alpha_{oc}+\alpha_{da}\right)+ \\ +\dfrac{\alpha_{cr}+\alpha_{da}}{\left(1-\alpha_{bp}\right)} \end{array} \right) \right). \quad (8)$$

The expression for determining the economic resistances R_{ei} contains independent optimized P_i variables, economic indicators $C_{(t)}$, as well as dependent parameters of the U_i mode. Therefore, economic resistances will change when looking for an optimal solution (Fig. 2).

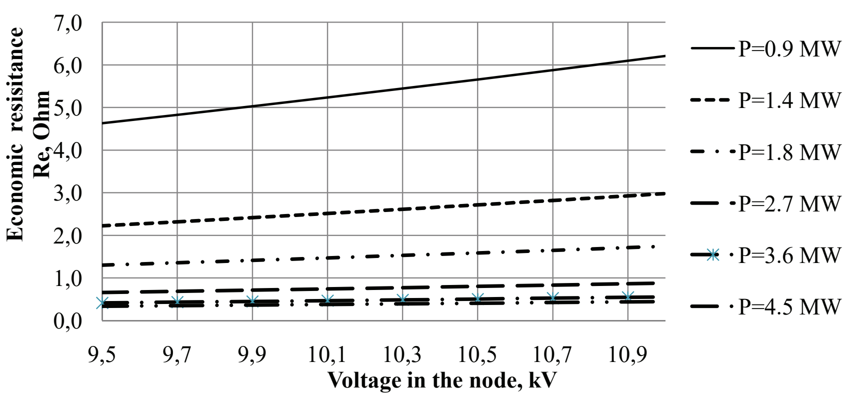


Fig. 2. The value of economic resistances for the specified capacity of discharge of energy storage units

From the plots of the dependence of the economic resistances of IESU on the voltage levels in the connection units (Fig. 2), it is clear that the increase in the maximum power of IESU is accompanied by a decrease in the absolute values of economic resistances. In addition, the sensitivity of economic resistances to voltage changes in the connection nodes decreases. Therefore, if at a given capacity of the accumulation system it was advisable to connect a separate IESU to the i-th substation, then further capacity growth will be accompanied by an increase in the installed capacity of the i-th IESU. This feature of the approach ensures a reasonable reduction in the number of necessary IESU connection nodes and reduction of capital expenditures.

5. 4. Algorithm for optimizing the connection of energy storage units

The source data for optimizing the placement of IESU in distribution networks are:

– the scheme of normal DEN mode;

– typical or measured load diagram of individual substations for the calculation period;

– the range of permissible IESU and their design parameters;

– optimized plots of the charge-discharge of the energy accumulation system (ESS totality), which are obtained based on the results of solving the problem in statement (2).

The optimal nodes for connecting individual IESU are determined in the following sequence. Based on the source data, the established DEN mode is calculated. According to the results of the calculation, the setting currents in the DEN nodes are determined, and the model of the current mode is replaced by a linearized one.

Next, a calculated model of DEN is built to simulate an ideal current distribution. Electrical networks are represented by the substitution *r*-scheme. Economic resistances are added to potential IESU connection nodes. IESU connection nodes are assigned an attribute of balancing in active power, which makes it possible to determine the capacities of IESU by calculated means.

According to the linearized model of the established DEN mode, the Gaussian method calculates the current distribution, which corresponds to a minimum of power losses in the substitution *r*-scheme. Taking into consideration the presence of economic resistances in the substitution scheme of DEN, the calculated current distribution will correspond to the maximum profitability of IESU connection.

To determine the charge/discharge capacities of individual IESU, the calculated currents in the branches of the substitution circuit with the economic resistances of IESU are recalculated to power.

Next, check the voltage restrictions in the DEN nodes. If the restrictions are not met, the values of the adjusting devices at the substations are corrected. In the case of inefficiency of existing regulatory devices, the estimated capacities of IESU are adjusted.

The given sequence is repeated until the specified accuracy of determining the economic resistances at each stage of the charge/discharge diagram of the energy storage system is achieved.

According to the results of calculations, charge/discharge diagrams of individual IESU are formed during the settlement period. According to these plots, using (1), we calculate the necessary capacities and other design parameters of IESU for connection in a given DEN node.

At the final stage, the estimated capacities, as well as the capacities of IESU, are rounded to standardized values. Mainly, the capacity of IESU is rounded to a higher standard value to take into consideration the evolution of electricity consumption.

5. 5. Results of computational experiments

The process of optimizing the parameters and placing the electrical installations of energy storage units in DEN consisted of two stages, in accordance with the one described above. To solve the problem of optimizing the total capacity of the ESS, which would ensure the maximum return on investment in energy storage system for SSO (the first stage of optimization), a gradient method implemented in the Microsoft Excel tabular processor was used. The optimization problem was represented in statement (2).

Using the plots of the flow of electricity to the experimental electrical network of 110/35 kV (Fig. 3), as well as diagrams of price changes in the intraday market (Fig. 4), the trajectories of the charge/discharge of the energy storage system of a given capacity were calculated. An example of the optimal diagram of the ESS with a capacity of 40 MWh is shown in Fig. 5, *a*. To obtain maximum income, the system performs two full charge/discharge cycles during the day in accordance with the dynamics of prices in the intraday market (Fig. 4).

Similar plots were obtained for 365 days of 2021, which made it possible to estimate the likely income from the connection of IESU of various capacities to the power grids of the power supply company (Table 1). The effect of the introduction of the energy storage system was estimated for the annual period, as well as for the autumn-winter and spring-summer periods. They are characterized by different dynamics of prices in the intraday market, which was manifested in the values of return on investment.

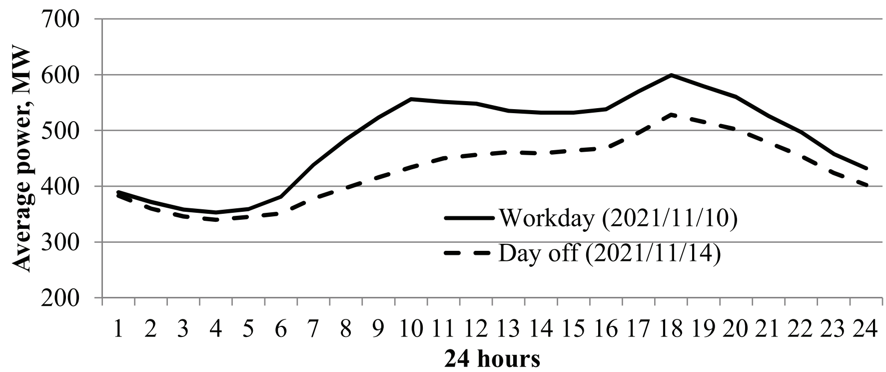


Fig. 3. An example of plots of electricity flow to the experimental grid

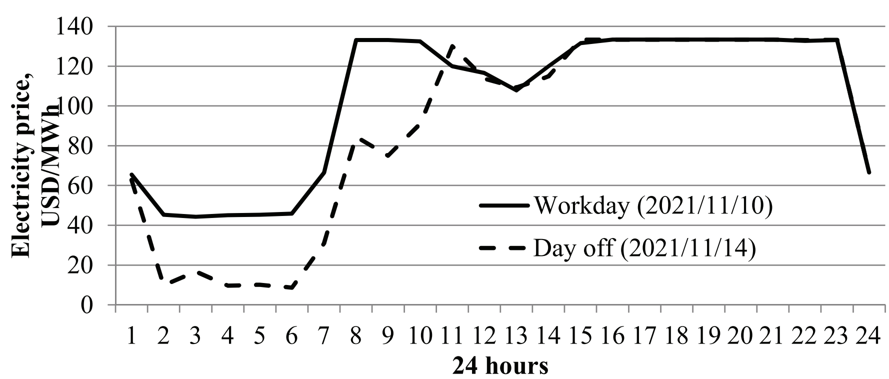


Fig. 4. Dynamics of electricity prices in the intraday market

From the results of optimizing the capacity of the energy storage system, it is clear that investments in the development of IESU have almost the same profitability, regardless of their capacity. At the same time, the value of profitability is influenced by the peculiarities of pricing in the intraday market. For example, the introduction of restrictions on the maximum price in the autumn-winter period reduced daily price volatility and, as a result, reduced the expected income from the introduction of energy storage systems (Table 1). At the same time, in the spring-summer period, due to the growth of renewable energy sources productivity, as well as an increase in the required reserve of sources of guaranteed generation, price variability was higher, which contributed to reducing the payback period of ESS to six years.

All variants of ESS capacity were economically justified for the operator of the accumulation system. Based on this, for each of the options, the connection of IESU to the substations of the energy supply company was optimized. The results are given in Table 2. For implementation, container-type IESU with a rated capacity of 4 MWh was adopted.

Table 1

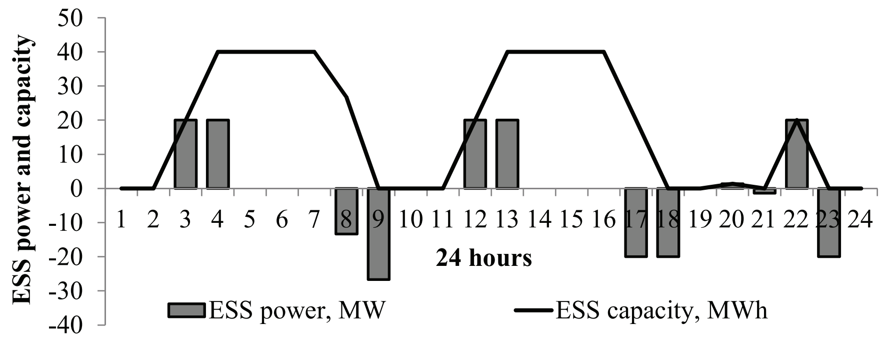
Results of evaluation of profitability of ESS implementation in distribution networks of 110/35 kV at Joint Stock Company "Vinnytsiaoblenergo"

Indicator	Measurement unit	Storage system capacity, MWh						
		4	8	12	16	24	32	40
Annual price dynamics								
Reducing the cost of electricity	%	0.03	0.07	0.10	0.14	0.21	0.28	0.34
Income from reducing the cost of electricity	USD thousand/year	78.0	156.8	235.2	313.6	470.4	627.1	782.5
Capital investment in ESS	USD thousand	771.4	1,542.9	2,314.3	3,085.7	4,628.6	6,171.4	7,714.3
Expenses for ESS maintenance and connection	USD thousand/year	61.7	123.4	185.1	246.9	370.3	493.7	617.1
Balance sheet profit	USD thousand/year	16.3	33.4	50.0	66.7	100.1	133.4	165.3
Depreciation deductions	USD thousand/year	77.1	154.3	231.4	308.6	462.9	617.1	771.4
Net cash flow	USD thousand/year	93.5	187.6	281.5	375.3	562.9	750.6	936.8
Profitability	p.u.	0.12	0.12	0.12	0.12	0.12	0.12	0.12
Payback period	year	8.25	8.22	8.22	8.22	8.22	8.22	8.23
Dynamics of prices in the spring and summer								
Reducing the cost of electricity	%	0.05	0.10	0.16	0.21	0.31	0.42	0.52
Income from reducing the cost of electricity	USD thousand/year	104.0	208.1	312.1	416.1	624.2	832.2	1,034.6
Net cash flow	USD thousand/year	119.5	238.9	358.4	477.8	716.7	955.6	1,188.9
Profitability	p.u.	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Payback period	year	6.46	6.46	6.46	6.46	6.46	6.46	6.49
Dynamics of prices in the autumn-winter period								
Reducing the cost of electricity	%	0.03	0.06	0.09	0.12	0.18	0.23	0.29
Income from reducing the cost of electricity	USD thousand/year	67.9	135.8	203.7	271.6	407.3	543.1	678.4
Net cash flow	USD thousand/year	83.3	166.6	249.9	333.3	499.9	666.5	832.7
Profitability	p.u.	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Payback period	year	9.26	9.26	9.26	9.26	9.26	9.26	9.26
Taking into account the reduction of electricity losses in the power grid								
Reducing the cost of electricity	%	0.03	0.07	0.10	0.14	0.21	0.28	0.35
Income from reducing the cost of electricity	USD thousand/year	78.0	156.8	235.2	313.6	470.4	627.1	782.5
Income from the reduction of the cost of losses	USD thousand/year	20.3	34.0	46.0	60.0	85.0	110.0	136.4
Total income	USD thousand/year	98.4	190.79	281.2	373.6	555.4	737.1	918.9
Capital investments in ESS	USD thousand	771.4	1,542.86	2,314.3	3,085.7	4,628.6	6,171.4	7,714.3
Expenses for ESS maintenance and connection	USD thousand/year	61.7	123.43	185.1	246.9	370.3	493.7	617.1
Balance sheet profit	USD thousand/year	36.7	67.36	96.0	126.7	185.1	243.4	301.7
Depreciation deductions	USD thousand/year	77.1	154.29	231.4	308.6	462.9	617.1	771.4
Net cash flow	USD thousand/year	113.8	221.64	327.5	435.3	647.9	860.6	1,073.2
Profitability	p.u.	0.15	0.14	0.14	0.14	0.14	0.14	0.14
Payback period	year	6.78	6.96	7.07	7.09	7.14	7.17	7.19

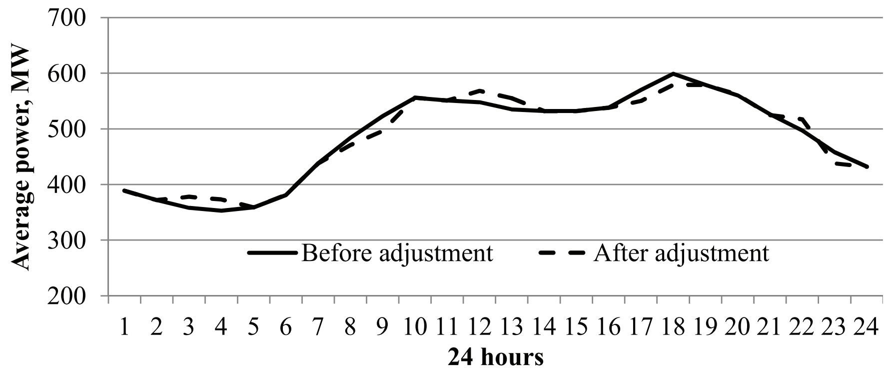
Table 2

Results of optimization of ESS placement in the distribution networks of Joint Stock Company "Vinnytsiaoblenergo"

Name of the distribution network node	Rated power, MWA	Active load, MW	Power loss sensitivity, MW/MW	Potential loss reduction, MW	Storage system capacity, MWh						
					4	8	12	16	24	32	40
Substation 35 kV Central (bus 1)	16	9.98	0.040	0.399	4	8	8	8	8	12	12
Substation 110 kV Southern (bus 1)	25	13.76	0.025	0.337	–	–	4	8	12	12	12
Substation 110 kV Southern (bus 2)	25	12.19	0.023	0.284	–	–	–	–	4	8	8
Substation 110 kV Chechelnyk	16	5.16	0.048	0.246	–	–	–	–	–	–	8
Substation 110 kV Yampil	10	5.22	0.044	0.231	–	–	–	–	–	–	–



a



b

Fig. 5. The results of adjusting the flow of electricity to the power grids by the energy storage system with a capacity of 40 MWh: *a* — dynamics in the energy storage system parameters; *b* — adjustment of the schedule of electricity supply

From the results of our calculation (Table 1), it is clear that due to the effective placement of IESU, a steady increase in the return on investment was achieved due to the additional income from reducing electricity losses in the distribution networks of 110/35 kV.

6. Discussion of results of studying the process of integrating energy storage systems into distribution networks

For distribution networks, the introduction of energy storage systems is logical and justified in view of improving operational safety and energy efficiency of network operation. Given the complexity of integrated optimization of IESU connection, the rated capacity and energy intensity are usually considered as independent technical indicators, which are determined by the investment capabilities of SSO. The dynamics of prices in the electricity market are taken into consideration as consolidated or approximate.

Based on our studies, a method and algorithm for optimizing the capacity and connection nodes of IESU, which have certain advantages, were proposed. The initial statement of problem (1) implied determining the parameters for the energy storage unit to achieve a combined effect of reducing energy flow volatility and reducing energy losses in DEN. Due to the complexity of the problem, it was proposed to decompose it into two subtasks that complement each other and can be reliably solved by known methods.

The general energy intensity of the ESS is proposed to be determined taking into consideration only the dynamics of prices in the intraday electricity market and the investment opportunities of SSO. To determine the optimal capacity of the charge/discharge of IESU and the places of their connection according to the criterion of maximum profitability, modeling of "ideal" EM modes according to the substitution schemes with active resistances was proposed. The effectiveness of the approach was confirmed by practical calculations.

According to the results of determining the capacity of ESS to optimize the flow of electricity to DEN, it was established that the payback period of investments in energy storage systems for Ukraine is about eight years. This indicates the economic feasibility of the development of this sector of the market. The return on investment practically does not depend on the capacity of the accumulation system because with the growth of investments, income from the reduction of costs for the purchase of electricity increases proportionally (Table 1).

In the case of the introduction of energy storage systems in DEN, they significantly affect the voltage regime and electricity loss. Therefore, optimal IESU connection schemes provide the effect of reducing losses, which, in value terms, is commensurate with the effect of reducing the cost of purchasing electricity (Table 1). In the optimization algorithm, each approximation is determined by the results of calculating the established DEN mode with control of restrictions on the parameters of the mode. Thus, the defined connection schemes and modes of operation of ESS do not worsen the quality of electricity but mainly contribute to its increase.

The disadvantages of the study's results include the following. The proposed optimization algorithm does not sufficiently formalize the decision-making process for connecting IESU to a certain substation based on a significant number of calculation results. For this purpose, weights were used in the form of the specific cost of electricity losses. However, this solution is not ideal and needs to be improved. In addition, to analyze the effectiveness of the implementation of ESS, data on the dynamics of electricity prices for only one year were used. The intensity of the formation of the balance of electricity during that year made it possible to draw interesting conclusions. However, in order to obtain generalizations, it is advisable to conduct additional studies with a greater depth of retrospective data.

7. Conclusions

1. The required capacity of ESS is mainly determined by the analysis of uneven schedules of electricity supply at different levels of the power system. The effect of their implementation is estimated as a profit from the provision of auxiliary services, or as a reduction in penalties for unforeseen power deviations. However, the increase in the capacity of power hubs may entail additional restrictions for the transportation of electricity. Taking into consideration the influence of ESS on the regime parameters of distribution networks is associated with objective complications. To overcome them, it was proposed to use the natural decomposition of the problem of optimizing the parameters and places of connection of IESU into tasks that are relevant for individual subjects of the energy system.

2. The process of connecting IESU should be preceded by determining the generalized parameters of the energy storage system. After that, it is necessary to optimize the connection of a certain set of IESU, taking into consideration the regime restrictions of distribution networks. To obtain a stable effect of reducing electricity losses and improving the quality of voltage to solve this problem, the method of ideal current distribution was used. This approach has made it possible to obtain a new solution to the problem of multifactorial optimization of the ESS according to a comprehensive criterion of optimality, taking into consideration restrictions on dependent and independent parameters.

3. The method of ideal current distribution of DEN provides a quick search for the optimal distribution of IESU according to the criterion of minimum electricity losses. To take into consideration the costs of their connection and subsequent operation to the substitution *r*-scheme, it is necessary to introduce fictitious resistances. Their values are calculated so that the cost of electricity losses in them corresponds to the annual operating costs of the connections of individual IESU.

4. According to the results of our research, an optimization algorithm has been developed that provides the possibility of distributing computing processes, reducing the number of computing operations, and improving the reliability of obtaining an optimal solution. This makes it possible to take into consideration a wider range of influential factors, in particular the dynamics of pricing, consumption and electricity generation processes over long periods.

5. For electrical networks of 110–35 kV, the places of connection of individual installations of the ESS were determined. It is established that without taking into consideration the impact of IESU on the loss of electricity, the capacity of storage systems in the range from 4 to 40 MWh provides a payback period of investments of about eight years. If the factor of reducing electricity losses is taken into consideration, the payback period is reduced to seven years and becomes sensitive to the distribution and capacity of ESS.

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