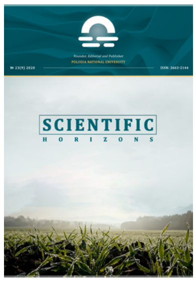
SCIENTIFIC HORIZONS

Journal homepage: https://sciencehorizon.com.ua
Scientific Horizons, 26(3), 124-134



UDC 621.73.011.001.5
DOI: 10.48077/scihor3.2023.124

Determination of porosity functions in the pressure treatment of iron-based powder materials in agricultural engineering

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Article's History:
Received: 18.01.2023
Revised: 18.03.2023
Accepted: 16.04.2023

Suggested Citation:
Sivak, R., Kulykivskyi, V., Savchenko, V., Minenko, S., & Borovskyi V. (2023). Determination of porosity functions in the pressure treatment of iron-based powder materials in agricultural engineering. *Scientific Horizons*, 26(3), 124-134.

Abstract. One of the most effective ways to obtain products with the required performance characteristics is the cold plastic deformation of porous workpieces. The relevance of the subject under study is due to the need to increase the reliability of the stress-strain state assessment during the plastic processing of porous workpieces by clarifying the porosity functions. The purpose of the study is to develop a method for describing the mechanical characteristics of porous bodies by single functions, the nature of which is determined by the properties of the base material and does not depend on the initial porosity. Analytical, numerical, experimental, and computational methods using modern specialised software systems were used to examine the processes of plastic deformation. The study presents a method for describing the mechanical characteristics of porous bodies with single functions. A set of interrelated methods and techniques is based

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on the basic provisions of the mechanics of plastic deformation of porous bodies and allows obtaining reliable porosity functions for this material, by clarifying theoretical dependencies by experimental studies. Therewith, experimental data were obtained in experiments on axisymmetric upsetting of cylindrical samples without friction at the ends. Based on the conducted theoretical studies, porosity functions for iron-based materials are obtained. Samples of five different initial porosities were used for the study. As a result of processing experimental data, final expressions for the porosity functions of the iron-based powder workpiece material are obtained. The study also presents a method for calculating the accumulated deformation of the base material. Flow curves for iron-based powder materials are plotted. The obtained results will allow formulating the practical recommendations for the development of technological processes for the plastic processing of powder materials by pressure to obtain products with specified physical and mechanical properties

Keywords: material; porosity function; pressure treatment; cold plastic deformation; flow curve; stress

INTRODUCTION

Information about the effect of shape change on each material particle in the volume of the workpiece is necessary to ensure the technological quality of finished products in pressure processing. Reliable information about the stress-strain state and the regularity of its change can be obtained by solving the boundary value problem of plasticity theory. Various types of surface plasticity and the associated flow law allow considering the influence of hardening and the type of stress state when solving problems in the theory of plasticity of a porous body. Therefore, the development and improvement of plastic deformation processes of porous materials depend on the accuracy of the stress-strain state assessment since the result substantially affects the idea of rheological properties in real pressure treatment processes.

The damage accumulation model is based on the hypothesis (Grushko *et al.,* 2017) that changes are directed in nature and are described by a second-rank tensor. The components of this tensor are determined by the mechanics of plastic deformation in a particular technological process, and functions that describe the physical and mechanical properties of the material. The accuracy of determining the value of the plasticity resource used depends not only on the criteria that consider the tensor nature of damage accumulation (Titov *et al.,* 2018; Puzyr *et al.,* 2021) but also on the reliability of the stress-strain state assessment. The considered criteria for estimating the value of the plasticity resource used include determining the stress-strain state considering the strain anisotropy. The formulation and solution of problems of the theory of plasticity of a porous material differs substantially from the formulation of a similar problem for a solid body. Not all the main provisions of the theory of irreversible deformation of compressible media are fully established. Model representations are most widely used to solve technological problems of pressure treatment of porous bodies (Shtern *et al.,* 2021), which are based on a continuous approach. A locally inhomogeneous medium is considered as a continuous medium, the state of which can be described using force and kinematic parameters that obey the laws of a continuous medium. When mathematically modelling the process of irreversible shape change of a porous body, it is necessary to determine the rheological properties of porous materials, which is the main condition. Therefore, theories that define plasticity surfaces of various types and use the flow law associated with them have become widespread. This approach allows considering the effect of hardening caused by the amount of shape and volume change and the type of stress state.

K. Gogaev *et al.* (2017), I. Prikhod'ko *et al.* (2016) proposed a model of a nonlinear viscous-elastic medium for examining the extrusion of powder materials through axisymmetric matrices, which allowed for formulating and solving the boundary value problem. Therewith, experimental data obtained on field samples using the coordinate grid method were used to determine the velocity field in the strain cell.

The development of various generalisations of the simplest theory of plasticity of a porous body towards accounting for strain hardening in the solid phase of a porous body was conducted in two ways: using independent experiments on a porous material (Beygelzimer *et al.,* 2021) and using a static model of a porous body and the law of strain hardening of a solid (non-porous) base material (Shtern & Kartuzov, 2016). Modification of the load surface in the form of an ellipsoid of rotation, which is shifted relative to the origin (Aliieva *et al.,* 2020), allows considering the difference in deformation resistance and changes in volume at hydrostatic pressure. Along with considering the macroscopic features of the deformation of porous bodies, attempts are made to analyse the nature of changes in the porous structure. Therewith, visual representations related to the shape and location of the load surface are used. This type includes a model (Shtern *et al.,* 2021), which considers the presence of two-dimensional defects along with pores.

A general constraint (Skorokhod & Shtern, 2019) linking porosity functions follows from the analysis of known models. Expressions for the material constant from the Mises plasticity condition for a porous billet based on iron and copper, depending on the porosity functions, are obtained by V. Rud *et al.* (2020). At the stage of isostatic loading and subsequent unloading, with combined deformation (Abdelmula *et al.*, 2017), an established seal is established. However, it is believed that the reduction in porosity during unloading can be ignored. Therefore, for the final stage of combined deformation under axial load, the initial porosity is greater than at the first stage under isostatic load. The density increase at the last stage under axial load is reduced to solving the Cauchy problem for an ordinary differential equation, in which the porosity function satisfies the expression with respect to the radial stress component and the initial conditions.

Thus, it can be argued that in most cases the solution of pressure treatment problems using the developed mathematical models of various processes and phenomena that occur during plastic deformation of porous bodies is reduced to the formulation of boundary value problems for certain systems of differential equations. The reliability of mathematical methods of the theory of plasticity of porous bodies directly depends on how accurate the porosity functions are. Therefore, the purpose of the study is to increase the reliability and accuracy of determining the real physical and mechanical properties of porous materials by experimentally clarifying the porosity functions, which is considered an important stage in the creation of new technological processes for plastic processing.

MATERIALS AND METHODS

The plasticity equations of a porous body were used to develop the theory of forming powder materials (Shtern *et al.*, 2021). In this case, the basic equations of the theory of porous body flow have the form:

$$\tau_0^2 = \frac{p^2}{f_2(\theta)(1-\theta)} + \frac{\tau^2}{f_1(\theta)(1-\theta)}\,; \quad (1)$$

$$\dot{e}_{ij} - \frac{1}{3}\dot{e}\delta_{ij} = \frac{\dot{\gamma}}{\tau}\left(\sigma_{ij} - p\delta_{ij}\right); \quad (2)$$

$$p f_1(\theta)\dot{\gamma} = \tau f_2(\theta)\dot{e}\,; \quad (3)$$

$$\dot{\gamma}_0^2 = \frac{f_2(\theta)\dot{e}^2}{(1-\theta)} + \frac{f_1(\theta)\dot{\gamma}^2}{(1-\theta)}; \quad (4)$$

$$G_0 = \int_0^t \dot{\gamma}_0 d\tau, \quad (5)$$

where τ_0 – intensity of the stress deviator in the base material; p – average voltage, $p = \frac{1}{3}\sigma_{ij}\delta_{ij}$; θ – porosity; τ – intensity of the stress deviator,

$$\tau = \sqrt{(\sigma_{ij} - p\delta_{ij})(\sigma_{ij} - p\delta_{ij})}\,; \quad (6)$$

$\dot{e}_{ij}$ – components of the strain velocity tensor; $\dot{e}$ – rate of relative volume change, $\dot{e} = \delta_{ij}\dot{e}_{ij}$; $\dot{\gamma}$ – intensity of the strain velocity deviator,

$$\dot{\gamma} = \sqrt{\left(\dot{e}_{ij} - \frac{1}{3}\dot{e}\delta_{ij}\right)\left(\dot{e}_{ij} - \frac{1}{3}\dot{e}\delta_{ij}\right)} = \sqrt{\frac{2}{3}}\dot{\varepsilon}_u; \quad (7)$$

σ_{ij} – components of the stress tensor; $\dot{\gamma}_0$ – intensity of the strain velocity deviator in the base material; G_0 – accumulated deformation of the base material.

The following expressions for porosity functions are obtained from theoretical references (Beygelzimer *et al.*, 2022):

$$f_{10}(\theta) = (1-\theta)^2\,; \quad (8)$$

$$f_{20}(\theta) = \frac{2}{3}\cdot\frac{(1-\theta)^3}{\theta}. \quad (9)$$

The theoretical dependencies (8) and (9) for the porosity functions will be clarified according to the data of M. Shtern *et al.* (2021), via coefficients m and n, which are determined experimentally and are different (Sivak, 1996) for each porous material:

$$f_1(\theta) = (f_{10}(\theta))^{n+1} = (1-\theta)^{2n+2}. \quad (10)$$

According to the studies by I. Sivak (1996), function $\alpha(\theta)$ has the form:

$$\alpha(\theta) = \frac{1}{6}\cdot\frac{f_1(\theta)}{f_2(\theta)}. \quad (11)$$

Theoretical expression for the porosity function:

$$\alpha_0(\theta) = \frac{1}{6}\cdot\frac{f_{10}(\theta)}{f_{20}(\theta)} = \frac{\theta}{4(1-\theta)}. \quad (12)$$

Function $\alpha(\theta)$ considering the parameter m takes this form:

$$\alpha(\theta) = \alpha_0^m(\theta) = \left(\frac{\theta}{4(1-\theta)}\right)^m. \quad (13)$$

Experimental values m and n will be calculated for axisymmetric upsetting of cylindrical samples without friction at the ends in experiments. Since the deformation is axisymmetric, then . The rate of volume change is:

$$\dot{e} = \dot{e}_z + 2\dot{e}_\phi, \quad (14)$$

and the intensity of the strain velocity deviator is described by the expression

$$\dot{\gamma} = \sqrt{\left(\dot{e}_z - \frac{\dot{e}_z+2\dot{e}_\varphi}{3}\right)^2 + 2\left(\dot{e}_\varphi - \frac{\dot{e}_z+2\dot{e}_\varphi}{3}\right)^2} = \sqrt{\frac{2}{3}}(\dot{e}_\varphi - \dot{e}_z). \quad (15)$$

In addition, with the frictionless upsetting σ_r=σ_φ=0, σ_z=-σ, then the relations have the form:

$$p = \frac{\sigma_r+\sigma_\varphi+\sigma_z}{3} = -\frac{\sigma}{3}; \quad (16)$$

$$\dot{\gamma} = \sqrt{\left(\dot{e}_{ij} - \frac{1}{3}\dot{e}\delta_{ij}\right)\left(\dot{e}_{ij} - \frac{1}{3}\dot{e}\delta_{ij}\right)} = \sqrt{\frac{2}{3}}\dot{\varepsilon}_u. \quad (17)$$

The volume deformation rate is equal to:

$$\dot{e} = \frac{1}{V}\cdot\frac{dV}{dt} = -\frac{1}{\rho}\cdot\frac{d\rho}{dt}. \quad (18)$$

The density of the material is:

$$\rho = \rho_0(1-\theta), \quad (19)$$

where ρ – density of the porous body; ρ_0 – density of the base material.

The following is considered time parameter:

$$t = |e_z| = ln\frac{h_0}{h}, \quad (20)$$

where h_0, h – initial and current height of the upsetting sample.

The expression for axial deformation has the form:

$$\frac{de_z}{dt} = -1. \quad (21)$$

Since the accumulated deformation of the base material G_0 depends on the time and porosity, the following can be written:

$$\frac{dG_0}{dt} = \frac{dG_0}{d\theta} \cdot \frac{d\theta}{dt}. \quad (22)$$

Equivalent stress intensity in the base material τ_0 corresponds to the upsetting stress:

$$\sigma = |\sigma_z| = \frac{P}{A}, \quad (23)$$

where P – upsetting force; A – cross-sectional area of the sample.

It is necessary to determine parameter m numerically by solving the differential equation for different initial porosities. Integration of a system of differential equations and calculation of the corresponding values of the stress intensity of the base material at an arbitrary initial porosity and parameter n is performed using methods of approximate or precise solution of applied mathematics problems. The curves of the upsetting stress dependence on time must be approximated by a power dependence. Approximation coefficients are determined by the least squares method. Implicit dependencies of the stress intensity of the base material on the accumulated deformation of the base material are also approximated by power functions using the least squares method. Values m and n which are included in the porosity functions obtained as a result of processing experimental data.

Experiments on axisymmetric upsetting of cylindrical samples with low friction at the ends were conducted to determine porosity functions $f_1(\theta)$, $f_2(\theta)$, and $\alpha(\theta)$ (Ogorodnikov *et al.,* 2018). The studies were conducted on samples made of powder material PZh4M2, height h_0=15.9 mm and diameter d_0=4.32 mm. The samples had five different initial porosity values: 1) θ_0=0.283; 2) θ_0=0.246; 3) θ_0=0.208; 4) θ_0=0.164; 5) θ_0=0.126. The density of the samples was determined by hydro-weighing in distilled water. The sample was weighed in a dry state (weight G_d), and then, to protect against liquid ingress into the pores of the material, it was soaked with paraffin. After wiping, the sample with the filler was weighed (weight G_f). Then a prototype, on a thin (d=0.05 mm) copper wire, was weighed in water (weight G_w). The experimental density value was determined by the formula:

$$\rho = \frac{G_d}{G_f - G_w}\rho_w, \quad (24)$$

where ρ_w – water density at the measurement temperature.

The experimental value of porosity was calculated from the relation:

$$\theta = 1 - \frac{\rho}{\rho_0}, \quad (25)$$

where ρ_0 – density of iron, the base material of a porous body, ρ_0=7.85 g/cm^3.

After hydro-weighing, the paraffin was removed. Complete removal of the mixture was monitored during subsequent weighing. upsetting of 2-4 samples of each initial density was performed in 10-15 stages. At each stage, the upsetting force P was determined, and the height h and diameter d of the sample were measured. In addition, the distance between two labels applied in the middle of the sample in the axial direction was measured. The density of the samples was determined by hydro-weighing in 5-8 stages. Low friction at the ends was ensured by the use of lead gaskets and lubrication with graphite grease lubricant. Compressive stresses were determined from the upsetting results:

$$\sigma = \frac{4P}{\pi d^2}, \quad (26)$$

and deformations

$$e_z = -\ln\frac{h_0}{h}; \quad (27)$$

$$e_r = e_\varphi = \ln\frac{d}{d_0}. \quad (28)$$

In addition to hydro-weighing, the variation in the porosity of the material was determined by the change in the average calculated volume:

$$V = \frac{\pi d^2}{4}h. \quad (29)$$

RESULTS AND DISCUSSION

As a result of the development of mathematical methods of the theory of plasticity of porous bodies, a harmonic theory was obtained (Shtern *et al.,* 2021). It is based on the hypothesis of the existence of a surface plasticity of a porous body:

$$3I_2(D_\sigma) + \alpha I_1^2(T_\sigma) - \beta k^2 = 0, \quad (30)$$

where α, β – porosity functions; $I_2(D_\sigma)$ – the second invariant of the stress deviator; $I_1(D_\sigma)$ – the first invariant of the stress tensor; k – flow stress of the base material.

The plasticity condition proposed by Y. Beygelzimer *et al.* (2022) is slightly different and has the form:

$$3I_2(D_\sigma) - (1 - 2\upsilon)I_2(T_\sigma) + \varphi^2 k^2 = 0, \quad (31)$$

where υ – coefficient of transverse deformation; ϕ – porosity function.

Analysis of known experimental and theoretical results shows that the plasticity conditions (30) and (31) qualitatively describe the mechanism of plastic deformation of porous materials. The main dependencies of the flow theory, in particular, the increase in plastic deformations from stresses, are determined from the plasticity condition and the associated flow law, according to which, the vector of increasing plastic deformations is perpendicular to the load surface at the point that corresponds to the stresses.

A. Kuzmov *et al.* (2020), M. Shtern, and E. Kartuzov (2016) used an approach related to the need to set the properties of a dissipative function when formulating the defining equations. Based on the assumption that the third invariants do not affect the behaviour of a porous body and considering isotropy, the plasticity condition of the compacted material was obtained. This condition is used quite widely (Skorokhod & Shtern, 2019) and is usually written as:

$$\frac{p^2}{f_2(\theta)} + \frac{\tau^2}{f_1(\theta)} = (1-\theta)k^2, \quad (32)$$

where $f_1(\theta)$, $f_2(\theta)$ – porosity functions; $k = \sqrt{\frac{3}{2}}\sigma_f$, σ_f – the flow stress of the solid phase under uniaxial tension.

The plasticity conditions (30), (31), and (32) are symmetric with respect to the plane p=0. However, this requirement is not mandatory. E. Beygelzimer and Y. Beygelzimer (2022) experimentally established that the plasticity condition of porous bodies under the bulk stress state is described by the expression:

$$\frac{(p-p^*)^2}{f_2(\theta)} + \frac{\tau^2}{f_1(\theta)} = (1-\theta)k^2, \quad (33)$$

where p^* – function of internal stresses and porosity.

If p=0, then the flow law associated with this plasticity condition leads to the fact that the volume deformation rate is:

$$\dot{e} = -\frac{f_1(\theta)}{f_2(\theta)} \cdot \frac{\dot{\gamma}}{\tau} p^*. \quad (34)$$

It follows from expression (34) that during the plastic deformation of a porous body, there is a change in volume in the absence of a spherical component of the stress tensor.

The material model is one of the main components in the research on the processes of forming porous materials. This model is formed in the form of defining equations that relate the components of stress tensors and strain rates to the state parameters of the deformable material (Shtern & Kartuzov, 2016). One of these parameters is the current porosity. Therefore, the relationship between stress tensors and strain rates contains internal variables for which conditions are formulated in the form of equations for state parameters. The kinetic equation of conservation of mass for materials the behaviour of which depends on porosity can be used as these relations. The defining relations for such materials are shown as a generalised model, which is a partial case of the equations of sensitivity of the plastic potential to the Nadai-Lode parameter (Skorokhod & Shtern, 2019) or the third invariant of the stress tensor (Sivak, 2017). In most cases, the porosity functions included in these equations are determined from experimental data.

Since the experimental values m and n will be calculated for axisymmetric upsetting of cylindrical samples without friction at the ends in experiments, the relation (3) is reduced to the form:

$$\frac{\dot{\gamma}}{\dot{e}} = \frac{f_2(\theta)}{f_1(\theta)} \cdot \frac{\tau}{p} = \frac{1}{6\alpha(\theta)} \cdot \frac{\tau}{p}. \quad (35)$$

The relations (16) and (17) are substituted in equation (35) and the expression is written:

$$\frac{\dot{\gamma}}{\dot{e}} = \frac{1}{6\alpha(\theta)} \cdot \frac{\sqrt{\frac{2}{3}}\sigma}{-\frac{\sigma}{3}} = -\frac{1}{\sqrt{6}\alpha(\theta)}. \quad (36)$$

After substituting the relations (14) and (15) into equation (36), the following expressions are obtained:

$$\dot{e}_z + 2\dot{e}_\varphi = -\sqrt{\frac{2}{3}}(\dot{e}_\varphi - \dot{e}_z)\sqrt{6}\alpha(\theta); \quad (37)$$

$$\frac{\dot{e}_\varphi}{\dot{e}_z} = -\frac{1-2\alpha}{2(1+\alpha)} = -\frac{1}{2} + \frac{3}{2} \cdot \frac{\alpha}{1+\alpha}. \quad (38)$$

It follows from expression (38) that:

$$\dot{e}_\varphi = -\dot{e}_z \frac{1-2\alpha}{2(1+\alpha)}. \quad (39)$$

Expression (39) is substituted to relation (14) and the following equation is obtained:

$$\dot{e} = \dot{e}_z - \frac{1-2\alpha}{2(1+\alpha)}\dot{e}_z = \frac{3\alpha}{1+\alpha}\dot{e}_z. \quad (40)$$

Using expressions (18) and (19), the volume strain rate is calculated:

$$\dot{e} = \frac{\dot{\theta}}{1-\theta}. \quad (41)$$

It follows from expressions (40) and (41) that:

$$\frac{\dot{\theta}}{1-\theta} = \frac{3\alpha(\theta)}{1+\alpha(\theta)}\dot{e}_z, \quad (42)$$

given function (13), the following relation is obtained

$$\frac{\dot{\theta}}{1-\theta} = \frac{3\alpha_0^m(\theta)}{1+\alpha_0^m(\theta)}\dot{e}_z, \quad (43)$$

or:

$$\frac{d\theta}{dt} = \frac{3(1-\theta)\alpha_0^m(\theta)de_z}{1+\alpha_0^m(\theta)dt}. \quad (44)$$

Expression (21) is substituted into relation (44), and as a result, the following is obtained:

$$\frac{d\theta}{dt} = -\frac{3(1-\theta)\alpha_0^m}{1+\alpha_0^m}, \quad (45)$$

then the differential equation is obtained

$$\frac{dt}{d\theta} = -\frac{1+\alpha_0^m}{3(1-\theta)\alpha_0^m}, \quad (46)$$

solving which, the following is calculated

$$t = -\int_0^\theta \frac{1+\alpha_0^m(\theta_*)}{3(1-\theta_*)\alpha_0^m(\theta_*)} d\theta_*. \quad (47)$$

The solution of equation (47) allows determining the value of the parameter m, at which the calculated curves coincide with the experimental ones.

The intensity of the deformation rates of the base material $\dot{\gamma}_0$ is expressed through the volume deformation rate $\dot{e}$ with a frictionless draft. Expression (4) is reduced to this form to do this:

$$\dot{\gamma}_0^2 = \frac{f_1(\theta)\dot{e}^2}{1-\theta}\left(\frac{f_2(\theta)}{f_1(\theta)} - \frac{\dot{\gamma}^2}{\dot{e}^2}\right). \quad (48)$$

Expressions (11) and (36) are substituted in equation (48) and the following the relation is obtained:

$$\dot{\gamma}_0^2 = \frac{f_1(\theta)\dot{e}^2}{1-\theta}\left(\frac{1}{6\alpha} + \frac{1}{6\alpha^2}\right). \quad (49)$$

Considering expressions (40) and (21), the relation (49) is reduced to the form:

$$\dot{\gamma}_0^2 = \frac{3}{2}\frac{f_1(\theta)}{(1-\theta)(1+\alpha)}, \quad (50)$$

hence

$$\dot{\gamma}_0 = \sqrt{\frac{3}{2}}\sqrt{\frac{f_1(\theta)}{(1-\theta)(1+\alpha)}}. \quad (51)$$

Expressions (46) and (51) are substituted in equation (22), considering that $\dot{\gamma}_0 = dG_0/dt$, the following is calculated:

$$\frac{dG_0}{d\theta} = \frac{dG_0}{dt}\cdot\frac{dt}{d\theta} = -\frac{1}{6}\sqrt{\frac{f_1(\theta)}{(1-\theta)^3}}\cdot\frac{\sqrt{1+\alpha}}{\alpha}. \quad (52)$$

Given that $f_1(\theta) = f_{10}^{1+n}(\theta) = (1-\theta)^{2+2n}$, the following expression is obtained:

$$\frac{dG_0}{d\theta} = -\frac{1}{\sqrt{6}}(1-\theta)^{n-0.5}\frac{\sqrt{1+\alpha_0^m}}{\alpha_0^m}. \quad (53)$$

Equation (1) is transformed considering the relations (11), (16), and (14) and the following is obtained:

$$\tau_0^2 = \frac{1}{f_1(\theta)(1-\theta)}\left(p^2\frac{f_1(\theta)}{f_2(\theta)} + \tau^2\right) = \frac{\frac{2}{3}\sigma^2(1+\alpha)}{f_1(\theta)(1-\theta)}. \quad (54)$$

As a result, the equivalent stress intensity of the base material during frictionless upsetting is determined by the relation:

$$\tau_0 = \sqrt{\frac{2}{3}}\frac{\sqrt{1+\alpha}}{\sqrt{f_1(\theta)(1-\theta)}}\sigma = \sqrt{\frac{2}{3}}\frac{\sqrt{1+\alpha_0^m(\theta)}}{(1-\theta)^{n+1.5}}\sigma. \quad (55)$$

Parameter value *n* can be determined from relation (55) by the iteration method, based on the condition that the dependence $\tau_0=f(G_0)$ must be uniform for any initial porosity of the material.

Therefore, to estimate the stress-strain state in the plastic region, the flow curve of the deformable material is necessary. In terms of plastic deformation of porous bodies, the problem is complicated by the fact that the type of flow curve depends on both the base material and the initial porosity (Sivak, 1996). The presented method allows constructing the dependence of the intensity of the stress deviator in the base material τ_0 from accumulated deformation G_0, which is uniform for this material and does not depend on the initial porosity. It is necessary to know the porosity functions to examine the processes of plastic deformation of porous bodies and construct a single flow curve $\tau_0(G_0)$.

From the storage conditions of the mass:

$$\rho V = \rho_0 V_0; \quad (56)$$

$$\rho_0(1-\theta)V = \rho_0(1-\theta_0)V_0, \quad (57)$$

finding the porosity

$$\theta = 1 - \frac{V_0}{V}(1-\theta) = 1 - \frac{d_0^2 h_0}{d^2 h}(1-\theta_0). \quad (58)$$

Dependency graphs $e_\varphi(t)$, $\sigma_z(t)$, $\theta(t)$ obtained from the equations (26), (28), and (58) are shown in Fig. 1-3. In graphical dependencies $t = ln\frac{h_0}{h} = -e_z$ – time parameter.

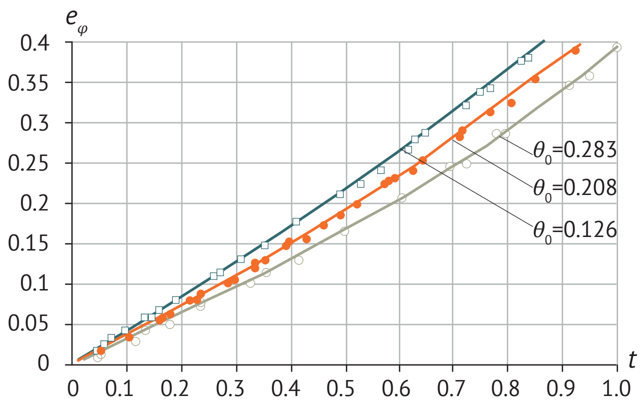

Figure 1. *Strain dependence e_φ depending on the degree of upsetting (iron)*

Source: *compiled by the authors*

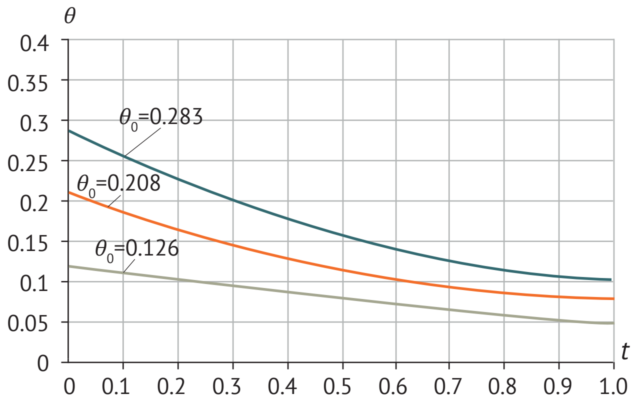

Figure 2. *Porosity dependence θ depending on the degree of upsetting (iron)*

Source: *compiled by the authors*

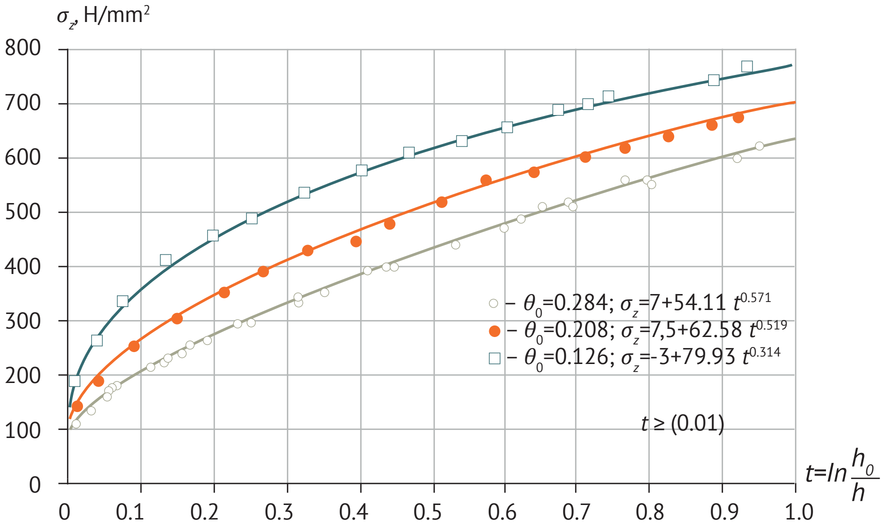

Figure 3. *Axial stress during upsetting of samples with different initial porosity* θ_0

Source: *compiled by the authors*

Parameter m was determined numerically by solving equation (46) for various initial porosities θ_0. Analysing the dependency $t(\theta)$ at different m, the following value was eventually identified (m=0.86), at which the experimental curve was identified to be close to the calculated one at the initial porosity θ_0=0.208. The following calculations showed that when m=0.86, calculated curves $\theta(t)$ coincide with the experimental ones for the remaining initial porosities. Integration of the system of differential equations (46), (53), and calculation of the corresponding values τ_0 according to relation (55), for arbitrary initial porosity and parameter n was performed using numerical methods. Under this condition, the coefficient was entered as the initial data m=0.86, and curves $\sigma_z(t)$ (Fig. 3). Therewith, curves $\sigma_z(t)$ were approximated by power dependence:

$$\sigma_z(t) = a_0 + a_1 t^{a_2}. \qquad (59)$$

The approximation coefficients depending on (59) were determined by the least squares method. As a result, it is established that the accepted expression is a valid approximation for describing the experimental dependence $\sigma_z(t)$. When solving the system of equations (46), (53), and calculating the ratio (55), data for two initial porosities were applied simultaneously θ_0. The obtained implicit dependencies $\tau_0=f(G_0)$ were approximated by power functions:

$$\tau_0 = b_0 + b_1 {G_0}^{b_2}, \qquad (60)$$

using the least squares method.

Curves (60) for both initial porosities were obtained at different values of the parameter n starting from n=0. When n=0.75, curves $\tau_0=f(G_0)$ coincided for both initial porosities. Applying the resulting value n=0.75 in the programme for other initial porosities confirmed the uniqueness of the curve $\tau_0=f(G_0)$. Thus, as a result of processing experimental data, the values m and n were obtained, which are included in the porosity functions of the workpiece based on PZh4M2 powder. Finally, the following dependencies are obtained for the porosity functions:

$$f_1(\theta) = f_{10}^{1+n}(\theta) = ((1-\theta)^2)^{1+1,75} = (1-\theta)^{3,5}; \qquad (61)$$

$$f_2(\theta) = \frac{1}{6\alpha(\theta)} f_1(\theta) = \frac{1}{6\alpha_0^m(\theta)} f_2(\theta) = 0.546\frac{(1-\theta)^{4,36}}{\theta^{0,86}}. \qquad (62)$$

For the flow curve, when $G_0 \geq 0.01$ the following expression was obtained:

$$\tau_0 = -15 + 83{,}73 G_0^{0,186}. \qquad (63)$$

Dependencies (61), (62), and (63) are used to calculate stresses during plastic deformations of porous workpieces in pressure treatment processes. Functions $\alpha(\theta), f_1(\theta), f_2(\theta)$ are shown in Fig. 4, 5, and the flow curve $\tau_0(G_0)$ – in Figure 6.

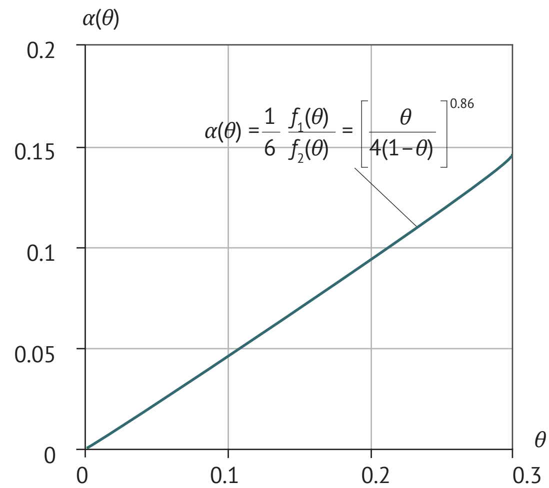

Figure 4. *Porosity function α(θ) for the PZh4M2 iron*

Source: *compiled by the authors*

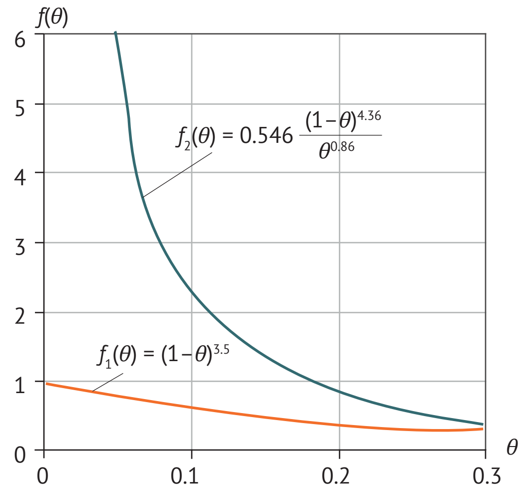

Figure 5. *Porosity functions $f_1(\theta)$, $f_2(\theta)$ for the PZh4M2 iron*

Source: *compiled by the authors*

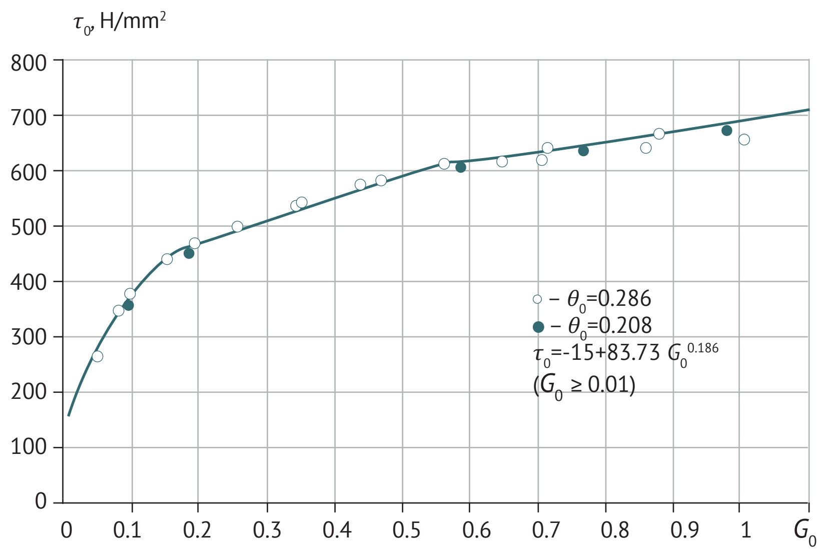

Figure 6. *Flow curve of the porous PZh4M2 iron base material*

Source: *compiled by the authors*

Mathematical models created based on ideas about the concept and methods of continuous environment mechanics are used to solve specific technological problems. These are mainly problems for determining the fields of residual porosity and deformation. Methods of mechanics of deformable powder materials allow explaining quite a lot of regularities at the macro level. However, the task of modelling the processes of obtaining powder materials, which consists in establishing the relationship between the deformation scheme and the properties of the product, cannot be reduced only to calculating residual deformations. It is necessary to consider changes in the parameters that determine the structure and physical properties. During research in the field of physics of dispersed and porous media, a number of important, qualitative results were obtained regarding the influence of porosity on elementary deformation processes and mechanical characteristics of porous bodies. The most common theory of plasticity of porous bodies in the form of condition (30) emerged as a generalisation of the Mises plastic body model (Green, 1973; Kuhn & Downey, 1971). The first stress tensor invariant included in the load function (30) makes the load surface closed, convex, and smooth. The closeness of the load surface reflects the property of volume-deformed bodies, which consists in the occurrence of plastic deformations at any monotonous trajectory of the applied forces. The most common modification of the theory of plasticity of porous bodies is based on the fluidity condition (32). In stress space, condition (32) corresponds to an ellipsoid of rotation relative to the hydrostatic axis. There are at least two ways to determine the half-axes of a given ellipsoid. The first method is based on independent experiments

(Kuhn & Downey, 1971) to obtain the relationship between stress and strain for a porous material under the stress state of simple types.

The second is the way to determine the porosity and flow stress functions under uniaxial compression. It is based on ideas about the mechanics of a micro-inhomogeneous continuum. Assuming a certain type of real microstructure of a porous body, the relative position of the pores, the nature of their interaction, and using known solutions to problems about the behaviour of an isolated pore in deformable space, it is possible to determine the effective characteristics of media that have an infinite number of pores. V. Skorokhod and M. Shtern (2019) calculated various boundaries of the flow stress of a porous body based on the static model of a porous body and solved the simplest macroscopically homogeneous problems of the theory of plasticity of compressible materials. Another area is based on the assumption of regular pore stacking (Green, 1973). Using the solution of two problems on the behaviour of an isolated pore in an infinite perfectly plastic medium, porosity functions such as the semi-axis of an ellipsoid were determined. This path allows considering the features of the porous structure. Consistent softening of the initial assumptions can lead to a change in the porosity functions and lead to a more complete consideration of physical and structural factors in the framework of the described theory of plasticity of porous bodies. In this form, the theory of plasticity of porous bodies is widely recognised and has allowed explaining quite a lot of the regularities of volume changes identified at the macro level in the most common deformation schemes. This method also helped to solve boundary value problems on density distribution and determine the fields of residual deformations. Based on the postulate of exact certainty of the dissipative function (Skorokhod & Shtern, 2019), the measure of accumulated deformation of the solid phase is calculated by formula (5), considering expression (48). A. Kuzmov *et al.* (2020) suggested that the relationship between the flow stress for uniaxial compression and accumulated deformation is identical to the deformation hardening curve of the solid phase.

N. Abdelmoula *et al.* (2017), A. Kuzmov *et al.* (2020) analysed the evolution of porosity for porous workpieces. As a result, by solving the equation and determining the radial stress component, the dependence of porosity on axial strain is obtained for three values of the initial porosity. Analysis of the results shows that the number of cavities increases for materials with lower initial porosity. Therewith, a decrease in the number of cavities is observed in blanks with high initial porosity. The initial porosity is also indicated, which corresponds to the absence of volume changes. It is necessary to consider that at the first stage, under isostatic load, there is a temporary decrease in pressure to coordinate the theoretical and experimental results, so in this case, it is necessary to use a certain law of change in the radial stress component. At the initial stage of deformation, such a replacement ensures that the calculated data correspond to the experimental curves "porosity – axial deformation".

CONCLUSIONS

Using the basic provisions of the mechanics of plastic deformation of porous bodies, a method for describing the mechanical characteristics of porous bodies by single functions was developed, the nature of which is determined by the properties of the material from which the workpiece is made, and does not depend on the initial porosity. The considered approach allows for obtaining reliable porosity functions for this material by clarifying the theoretical dependencies through experimental studies. Porosity functions for iron-based materials were obtained. A method for calculating the accumulated deformation of the base material was developed. A flow curve was plotted for iron-based materials, which, regardless of the initial porosity, describes the dependence of the stress intensity in the base material on the accumulated deformation of the base material as a single curve. Porosity leads not only to quantitative changes in the service and mechanical properties of materials but also makes them more susceptible and quite sensitive to the type of stress-strain state. Porous materials have a number of properties that are not inherent in solid bodies. In particular, the dependence of the flow stress on hydrostatic pressure, dilatancy, multi-resistance during tension and compression, etc. These features are reflected in mechanical models of porous bodies. However, research in the field of mechanics of porous media should also be aimed at further creating theories focused on solving the problems of compaction and shaping of porous materials, determining the boundary states that characterise the structural properties of powder materials.

AKNOWLEDGEMENTS

None.

CONFLICT OF INTEREST

None.

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Визначення функцій пористості при обробці тиском порошкових матеріалів на основі заліза в агроінженерії

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Анотація. Одним із ефективних способів отримання виробів з необхідними експлуатаційними характеристиками є холодна пластична деформація пористих заготовок. Актуальність досліджуваної теми обумовлена необхідністю підвищення достовірності оцінки напружено-деформованого стану при пластичній обробці пористих заготовок шляхом уточнення функцій пористості. Метою дослідження є розробка методики описання механічних характеристик пористих тіл єдиними функціями, характер яких визначається властивостями матеріалу основи і не залежить від початкової пористості. Для дослідження процесів пластичної деформації використані аналітичні, числові, експериментально-розрахункові методи із застосуванням сучасних спеціалізованих програмних систем. В статті представлена методика описання механічних характеристик пористих тіл єдиними функціями. Сукупність взаємопов'язаних способів та прийомів базується на основних положеннях механіки пластичної деформації пористих тіл і дозволяє отримувати достовірні функції пористості для даного матеріалу, шляхом уточнення теоретичних залежностей експериментальними дослідженнями. Водночас експериментальні дані отримували в дослідах на осесиметричну осадку циліндричних зразків без тертя на торцях. Ґрунтуючись на проведених теоретичних дослідженнях отримано функції пористості для матеріалів на основі заліза. Для досліджень використовували зразки п'яти різних початкових пористостей. В результаті обробки експериментальних даних отримані остаточні вирази для функцій пористості матеріалу заготовки з порошку на основі заліза. Також в статті представлена методика розрахунку накопиченої деформації матеріалу основи. Побудовано криві течії для порошкових матеріалів на основі заліза. Отримані результати досліджень дозволять сформулювати практичні рекомендації з розробки технологічних процесів пластичної обробки порошкових матеріалів тиском для одержання виробів із заданими фізико-механічними властивостями

Ключові слова: матеріал; функція пористості; обробка тиском; холодна пластична деформація; крива течії; напруження