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AN ULTIMATE CASE OF THE PROJECTOR OPTIMAL BEHAVIORS IN MODELING ANTAGONISTICALLY THE BUILDING RESOURCES DISTRIBUTION WITH INCORRECTLY PRE-EVALUATED ONE LEFT AND ONE RIGHT END POINTS WITH IN SEGMENT UNCERTAINTIES

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There has been exampled an antagonistic model of the building resources distribution for the four-pillar mount construction. In the model there are ultimate cases of the corresponding antagonistic game kernel relationships, arisen from the incorrectly pre-evaluated one left and one right endpoints within segment uncertainties, giving essentially antagonism. There is being proved the theorem on an ultimate case for making the optimal decision for the projector. A subcase with continuum of the projector optimal behaviors has been marked and resolved for the single optimal behavior.

Наведено приклад однієї антагоністичної моделі розподілу будівельних ресурсів для чотириколонної опорної конструкції. У цій моделі існують граничні випадки співвідношень у ядрі відповідної антагоністичної гри, що виникають внаслідок некоректно попередньо оцінених одного лівого й одного правого кінців у сегментних невизначеностях, котрі породжують, власне, антагонізм. Доводиться теорема за одним граничним випадком для прийняття проектувальником оптимального рішення. Для вибору єдиної оптимальної поведінки відмічено і розв'язано один підвипадок з континуумом оптимальних поведінок проектувальника.

Приведён пример одной антагонистической модели распределения строительных ресурсов для четырёхколонной опорной конструкции. В этой модели существуют предельные случаи соотношений в ядре соответствующей антагонистической игры, возникающие вследствие некорректно предварительно оцененных одного левого и одного правого концов в сегментных неопределённостях, порождающих, собственно, антагонизм. Доказывается теорема по одному предельному случаю для принятия проектировщиком оптимального решения. Для выбора единственного оптимального поведения отмечен и разрешён один подслучай с континуумом оптимальных поведений проектировщика.

Problem general description

There are many conflict processes in the contemporary world, and one of them is the scanty resources distribution. Particularly, there stands the problem of the building resources distribution for projecting mount design constructions [1, 2], might be modeled antagonistically as a lot of uncertain factors in building cause engineering-scientific antagonisms [3, 4]. Having the four-pillar mount construction with uncertain unit-normed cross-section squares $\left\{y_i\right\}_{i=1}^{4}$ (UNCSS) and unit-normed loads $\left\{x_i\right\}_{i=1}^{4}$ (UNL) as

$$x_d \in \left[a_d;\, b_d\right] \subset (0;\, 1),\ y_d \in \left[a_d;\, b_d\right] \subset (0;\, 1),\ a_d < b_d\ \forall\ d = \overline{1,\, 3} \tag{1}$$

but

$$\sum_{i=1}^{4} x_i = 1,\ \sum_{i=1}^{4} y_i = 1, \tag{2}$$

there is a known convex antagonistic game (AG) with the kernel [5, 6]

$$T(\mathbf{X},\, \mathbf{Y}) = T(x_1,\, x_2,\, x_3;\, y_1,\, y_2,\, y_3) = \max\left(\left\{r_i(x_i,\, y_i)\right\}_{i=1}^{4}\right) =$$

$$= \max\left\{x_1 y_1^{-2},\, x_2 y_2^{-2},\, x_3 y_3^{-2},\, \frac{1 - x_1 - x_2 - x_3}{\left(1 - y_1 - y_2 - y_3\right)^2}\right\} \tag{3}$$

on the product

$$\boldsymbol{X} \times \boldsymbol{Y} = \prod_{p=1}^{2} [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] =$$

$$= \prod_{p=1}^{2} \left(\prod_{d=1}^{3} [a_d;\, b_d] \right) \subset \prod_{l=1}^{6} (0;\, 1) \subset \prod_{l=1}^{6} [0;\, 1] \subset \mathbb{R}^6 \qquad (4)$$

of the parallelepiped

$$\boldsymbol{X} = [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \prod_{d=1}^{3} [a_d;\, b_d] \subset \prod_{d=1}^{3} (0;\, 1) \subset \prod_{d=1}^{3} [0;\, 1] \subset \mathbb{R}^3 \qquad (5)$$

of pure strategies

$$\mathbf{X} = [x_1 \quad x_2 \quad x_3] \in [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \boldsymbol{X} \qquad (6)$$

of the first player and of the parallelepiped

$$\boldsymbol{Y} = [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \prod_{d=1}^{3} [a_d;\, b_d] \subset \prod_{d=1}^{3} (0;\, 1) \subset \prod_{d=1}^{3} [0;\, 1] \subset \mathbb{R}^3 \qquad (7)$$

of pure strategies

$$\mathbf{Y} = [y_1 \quad y_2 \quad y_3] \in [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \boldsymbol{Y} \qquad (8)$$

of the second player (projector), where variables x_4 and y_4 due to (2) are excluded. This AG helps removing those segment uncertainties in UNCSS and determining projector optimal behavior

$$\mathbf{Y}_* = \left[y_1^* \quad y_2^* \quad y_3^* \right] \in [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \boldsymbol{Y}, \qquad (9)$$

applying simply the known minimax procedure [7]. Then

$$y_d^* = \frac{\sqrt{b_d}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} \quad \forall\, d = \overline{1,\, 3}. \qquad (10)$$

Nevertheless, the components (10) in (9) are true only if

$$\frac{\sqrt{b_d}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} \in [a_d;\, b_d] \quad \forall\, d = \overline{1,\, 3}, \qquad (11)$$

what means that the endpoints $\{a_d\}_{d=1}^{3}$ and $\{b_d\}_{d=1}^{3}$ had been pre-evaluated correctly. And if

$$\exists\, j \in \left\{\overline{1,\, 3}\right\} \text{ that } \frac{\sqrt{b_j}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} \notin [a_j;\, b_j], \qquad (12)$$

then the common minimax procedure needs revision.

Survey in gorigins on the unsolved point

Actually, the components (10) in (9) are found from the four-parted equality

$$v_* = b_1 \left(y_1^*\right)^{-2} = b_2 \left(y_2^*\right)^{-2} = b_3 \left(y_3^*\right)^{-2} = \frac{1 - a_1 - a_2 - a_3}{\left(1 - y_1^* - y_2^* - y_3^*\right)^2}, \qquad (13)$$

giving the game value v_*. At that this equality should be true within the parallelepiped (7), unless there appears the condition (12). If (12) appeared then the equality (13) within the parallelepiped (7) is violated, and for equalizing the violated equality (13) there should be revised the components $\{y_d^*\}_{d=1}^{3}$, starting necessarily to that y_j^* with (12). Whatever, this all does not infringe upon the known theorem on the second player pure strategies in the convex AG [7], but only is an evidence of the incorrect pre-evaluation of the endpoints $\{a_d\}_{d=1}^{3}$ and $\{b_d\}_{d=1}^{3}$ [2, 8]. The cases with the single (12), where one of the six endpoints $\{a_d,\, b_d\}_{d=1}^{3}$ had been pre-evaluated incorrectly, are pretty trivial and they may be reexamined from the works [6, 8]. More interesting case is that when there are two incorrectly pre-evaluated endpoints, assuming to be more perplexed in being solved.

Papergoal

Will solve the AG with kernel (3) on the product (4) for the projector in the supposition that

$$\frac{\sqrt{b_p}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} < a_p \text{ by } p \in \left\{\overline{1,\,3}\right\}, \tag{14}$$

$$\frac{\sqrt{b_q}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} > b_q \text{ by } q \in \left\{\overline{1,\,3}\right\}, \tag{15}$$

$$\frac{\sqrt{b_k}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} \in [a_k;\, b_k] \text{ by } k \in \left\{\overline{1,\,3}\right\} \setminus \{p,\, q\}. \tag{16}$$

For accomplishing it there should be disclosed the violated equality (13) under incorrectly pre-evaluated endpoints a_p and b_q by $p \in \left\{\overline{1,\,3}\right\}$ and $q \in \left\{\overline{1,\,3}\right\}$ for $p \neq q$.

A theorem on an ultimate case of the projector optimal behaviors in antagonistic game with kernel (3) on the product (4)

It is easy to see clearly that with (14)-(16) the component y_p^* is greater than expected before checking (13), and the component y_q^* is lesser than expected before checking (13). Then there stands the strict inequality

$$\frac{1}{b_q} > \frac{b_k}{y_k^2} > \frac{b_p}{a_p^2} \quad \text{at} \quad k \in \left\{\overline{1,\,3}\right\} \setminus \{p,\, q\} \tag{17}$$

for the k-th component (16). Further more, including the part $\dfrac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2}$ with $k \in \left\{\overline{1,\,3}\right\} \setminus \{p,\, q\}$ and

$$y_k = \frac{\sqrt{b_k}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} \tag{18}$$

here is true one of the following inequalities:

$$\frac{1}{b_q} > \frac{b_k}{y_k^2} > \frac{b_p}{a_p^2} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2}, \tag{19}$$

$$\frac{1}{b_q} > \frac{b_k}{y_k^2} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2} > \frac{b_p}{a_p^2}, \tag{20}$$

$$\frac{1}{b_q} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2} > \frac{b_k}{y_k^2} > \frac{b_p}{a_p^2}, \tag{21}$$

$$\frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2} > \frac{1}{b_q} > \frac{b_k}{y_k^2} > \frac{b_p}{a_p^2}. \tag{22}$$

The ultimate case (22) seems to be rather nontrivial and nonobvious. This urges to prove a theorem on it for the components $\left\{y_d^*\right\}_{d=1}^{3}$ of projectoroptimalbehavior (9) to have or control UNCSS.

Theorem. In AG with kernel (3) on the product (4) by the conditions (14-16) and (22) with $k \in \left\{\overline{1,\,3}\right\} \setminus \{p,\, q\}$ and (18) the projector, selecting its optimal behavior, should decrease the q-th or the k-th components from the values b_q and (16) down correspondingly, and it is guided by the following. If for the decreased k-th component within $[a_k;\, b_k]$ there is the inequality

$$\frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2} > \frac{1}{b_q} = \frac{b_k}{y_k^2} \tag{23}$$

then the game value v_* is determined by the three-parted equality

$$\frac{1-a_1-a_2-a_3}{\left(1-y_q-a_p-y_k\right)^2}=\frac{b_q}{y_q^2}=\frac{b_k}{y_k^2}, \tag{24}$$

wherein the roots $y_q \in \left[a_q;\, b_q\right]$ and $y_k \in \left[a_k;\, b_k\right]$ of (24) give the projector optimal behavior components

$$y_q^*=\frac{\left(1-a_p\right)\sqrt{b_q}}{\sqrt{b_q}+\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}}, \tag{25}$$

$$y_p^*=a_p, \tag{26}$$

$$y_k^*=\frac{\left(1-a_p\right)\sqrt{b_k}}{\sqrt{b_q}+\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}}; \tag{27}$$

the roots $y_q < a_q$ and $y_k \in \left[a_k;\, b_k\right]$ of (24) give the component

$$y_q^*=a_q, \tag{28}$$

the component (26), the k -th component

$$y_k^*=\frac{\left(1-a_q-a_p\right)\sqrt{b_k}}{\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}} \tag{29}$$

by the condition

$$\frac{\left(1-a_q-a_p\right)\sqrt{b_k}}{\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}} \in \left[a_k;\, b_k\right] \tag{30}$$

and the k -th component

$$y_k^*=a_k \tag{31}$$

by the condition

$$\frac{\left(1-a_q-a_p\right)\sqrt{b_k}}{\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}} < a_k; \tag{32}$$

the roots $y_q \in \left[a_q;\, b_q\right]$ and $y_k < a_k$ of (24) give the q -th component

$$y_q^*=\frac{\left(1-a_p-a_k\right)\sqrt{b_q}}{\sqrt{b_q}+\sqrt{1-a_1-a_2-a_3}} \tag{33}$$

by the condition

$$\frac{\left(1-a_p-a_k\right)\sqrt{b_q}}{\sqrt{b_q}+\sqrt{1-a_1-a_2-a_3}} \in \left[a_q;\, b_q\right] \tag{34}$$

and the q -th component (28) by the condition

$$\frac{\left(1-a_p-a_k\right)\sqrt{b_q}}{\sqrt{b_q}+\sqrt{1-a_1-a_2-a_3}} < a_q, \tag{35}$$

the component (26) and the component (31); the roots $y_q < a_q$ and $y_k < a_k$ of (24) give the components (28), (26), (31). If for the decreased k -th component within $\left[a_k;\, b_k\right]$ there is the inequality

$$\frac{1-a_1-a_2-a_3}{\left(1-b_q-a_p-y_k\right)^2}=\frac{1}{b_q}>\frac{b_k}{y_k^2} \tag{36}$$

then the game value $v_*=\frac{1}{b_q}$ and the projector optimal behavior components are

$$y_q^*=b_q, \tag{37}$$

$$y_p^* \in \left[a_p ;\, y_p^{\langle \max \rangle} \right], \tag{38}$$

$$y_k^* \in \left[\frac{1}{2}\left(\sqrt{b_q b_k} + a_k + \left(\sqrt{b_q b_k} - a_k \right) \mathrm{sign}\left(\sqrt{b_q b_k} - a_k \right) \right);\, y_k^{\langle \max \rangle} \right] \tag{39}$$

by

$$y_p^{\langle \max \rangle} + y_k^{\langle \max \rangle} = 1 - b_q - \sqrt{b_q \left(1 - a_1 - a_2 - a_3 \right)}, \tag{40}$$

$$y_p^{\langle \max \rangle} \in \left[a_p ;\, b_p \right], \tag{41}$$

$$y_k^{\langle \max \rangle} \in \left[\frac{1}{2}\left(\sqrt{b_q b_k} + a_k + \left(\sqrt{b_q b_k} - a_k \right) \mathrm{sign}\left(\sqrt{b_q b_k} - a_k \right) \right);\, b_k \right]. \tag{42}$$

If for the decreased k-th component within $\left[a_k ;\, b_k \right]$ there is the inequality

$$\frac{1 - a_1 - a_2 - a_3}{\left(1 - b_q - a_p - a_k \right)^2} > \frac{1}{b_q} > \frac{b_k}{a_k^2} \tag{43}$$

then the game value v_* is determined by the equation

$$\frac{1 - a_1 - a_2 - a_3}{\left(1 - y_q - a_p - a_k \right)^2} = \frac{b_q}{y_q^2}, \tag{44}$$

wherein the root $y_q \geqslant a_q$ of (44) gives the projector optimal behavior components (33), (26), (31); the root $y_q < a_q$ of (44) gives the projector optimal behavior components (28), (26), (31). Locally, the case with

$$\frac{1 - a_1 - a_2 - a_3}{\left(1 - b_q - a_p - y_k \right)^2} = \frac{1}{b_q} = \frac{b_k}{y_k^2} \tag{45}$$

gives the projector optimal behavior components (26), (37), and

$$y_k^* = \sqrt{b_k b_q}\,. \tag{46}$$

Proof. Having faced the conditions (14-16) and (22), the projector, selecting its optimal behavior, should decrease the first player payoff, and it is realizable only with decreasing the q-th or the k-th components from the values b_q and (16) down correspondingly, though controlling them to lie within $\left[a_q ;\, b_q \right]$ and $\left[a_k ;\, b_k \right]$. If for the decreased k-th component within $\left[a_k ;\, b_k \right]$ there is the inequality (23), then its equalization leads to solving the three-parted equality (24), whence

$$y_q = \sqrt{\frac{b_q}{b_k}} y_k\,, \tag{47}$$

$$y_k \sqrt{1 - a_1 - a_2 - a_3} = \sqrt{b_k}\left(1 - y_q - a_p - y_k \right) = \sqrt{b_k}\left(1 - \sqrt{\frac{b_q}{b_k}} y_k - a_p - y_k \right) =$$

$$= \sqrt{b_k} - y_k \sqrt{b_q} - a_p \sqrt{b_k} - y_k \sqrt{b_k} = \sqrt{b_k}\left(1 - a_p \right) - y_k \left(\sqrt{b_q} + \sqrt{b_k} \right), \tag{48}$$

what gives (25) and (27) by the conditions

$$\frac{\left(1 - a_p \right)\sqrt{b_q}}{\sqrt{b_q} + \sqrt{b_k} + \sqrt{1 - a_1 - a_2 - a_3}} \in \left[a_q ;\, b_q \right], \tag{49}$$

$$\frac{\left(1 - a_p \right)\sqrt{b_k}}{\sqrt{b_q} + \sqrt{b_k} + \sqrt{1 - a_1 - a_2 - a_3}} \in \left[a_k ;\, b_k \right]. \tag{50}$$

And whatever it is with the inequality (23), on this case the p-th component must be the least as changing it increases the first player payoff, so (26) is true. Hereinto, by

$$\frac{\left(1-a_p\right)\sqrt{b_q}}{\sqrt{b_q}+\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}}<a_q \tag{51}$$

and (50) the game value v_* is determined by the equation in the relationship

$$\frac{1-a_1-a_2-a_3}{\left(1-a_q-a_p-y_k\right)^2}=\frac{b_k}{y_k^2}>\frac{b_q}{a_q^2}, \tag{52}$$

substantiating the component (28), whence

$$y_k\sqrt{1-a_1-a_2-a_3}=\sqrt{b_k}\left(1-a_q-a_p-y_k\right), \tag{53}$$

what gives (29) by (30) and (31) by (32). Symmetrically, by (49) and

$$\frac{\left(1-a_p\right)\sqrt{b_k}}{\sqrt{b_q}+\sqrt{b_k}+\sqrt{1-a_1-a_2-a_3}}<a_k \tag{54}$$

the game value v_* is determined by the equation in the relationship

$$\frac{1-a_1-a_2-a_3}{\left(1-y_q-a_p-a_k\right)^2}=\frac{b_q}{y_q^2}>\frac{b_k}{a_k^2}, \tag{55}$$

whence

$$y_q\sqrt{1-a_1-a_2-a_3}=\sqrt{b_q}\left(1-y_q-a_p-a_k\right), \tag{56}$$

what gives (33) by (34) and (28) by (35). Surely, if (51) and (54) both are true then the projector cannot decrease the first player payoff anymore, so the components (28), (26), (31) constitute the single projector optimal behavior. If for the decreased k-th component within $\left[a_k;\, b_k\right]$ there is the inequality (36), then the projector cannot decrease the first player payoff down from $\frac{1}{b_q}$, though it may hold it with satisfying the following inequalities:

$$\frac{1}{b_q}\geqslant\frac{b_k}{y_k^2}, \tag{57}$$

$$\frac{1}{b_q}\geqslant\frac{b_p}{y_p^2}, \tag{58}$$

$$\frac{1}{b_q}\geqslant\frac{1-a_1-a_2-a_3}{\left(1-b_q-y_p-y_k\right)^2}. \tag{59}$$

The inequality (57) is equivalent to the inequality $y_k\geqslant\sqrt{b_qb_k}$ by $y_k\geqslant a_k$, what gives the condition

$$y_k\geqslant\frac{1}{2}\left(\sqrt{b_qb_k}+a_k+\left(\sqrt{b_qb_k}-a_k\right)\operatorname{sign}\left(\sqrt{b_qb_k}-a_k\right)\right). \tag{60}$$

The inequality (58) is true $\forall\ y_p\in\left[a_p;\, b_p\right]$ due to the initial condition (22). And from (59) it follows that

$$1-b_q-y_p-y_k\geqslant\sqrt{b_q\left(1-a_1-a_2-a_3\right)},\ y_p+y_k\leqslant 1-b_q-\sqrt{b_q\left(1-a_1-a_2-a_3\right)}, \tag{61}$$

so the maximized sum y_p+y_k must equal to the right side of (40) by (41), (42). All this by (60) allows writing the p-th and the k-th components as (38), (39), but for reaching the game value $v_*=\frac{1}{b_q}$ the projector must hang on (37). Finally, if for the decreased k-th component within $\left[a_k;\, b_k\right]$ there is the inequality (43), then its equalization leads to solving the equation (44), whence

$$y_q\sqrt{1-a_1-a_2-a_3}=\sqrt{b_q}\left(1-y_q-a_p-a_k\right)=\sqrt{b_q}\left(1-a_p-a_k\right)-y_q\sqrt{b_q}, \tag{62}$$

giving (33) by (34), and giving (28) otherwise. Increasing the other components leads to increasing the first player payoff, so here the projector hangs on (26), (31). By the local and pretty rare case (45) the projector clearly hangs on only (26), (37), and (46) as the root of the equality $\frac{1}{b_q} = \frac{b_k}{y_k^2}$. The theorem has been proved.

Conclusion and outlook for further investigation

Within the considered and proved ultimate case the projector controls its optimal behavior (9), using the stated theorem conditions. It ought to be outlined that the subcase with (36) by $y_k > a_k$ generates the continuum of optimal behaviors in the p-th and k-th UNCSS. So there is another problem of selecting the single optimal p-th and k-th UNCSS for that subcase, which, though, is resolved at once: the projector should apply such UNCSS that they would minimize the maximal unbalance of UNL and UNCSS ratio over all four pillars, but simultaneously they would be valued minimally for economizing the building resources. Clearly, for the subcase with (36) the optimal behavior from the continuum of optimal behaviors of the p-th and k-th components is

$$\mathbf{Y}_* = \left[b_q \quad a_p \quad \frac{1}{2}\left(\sqrt{b_q b_k} + a_k + \left(\sqrt{b_q b_k} - a_k \right) \operatorname{sign}\left(\sqrt{b_q b_k} - a_k \right) \right) \right]. \qquad (63)$$

It is obligatory to underline that the papered claims and conclusion are valid for any batched resources distribution, where just the ratio of abstract UNL and square-powered UNCSS is acceptable [4, 7]. For further investigation there still stay questions of the projector optimal behavior with incorrectly pre-evaluated three end points within segment uncertainties, where both left and right end points are.

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