Journal of Physics: Conference Series

PAPER • OPEN ACCESS

The formation of visual thinking of students in technical universities in the context of higher mathematics education

To cite this article: V I Klochko *et al* 2024 *J. Phys.: Conf. Ser.* **2871** 012029

The formation of visual thinking of students in technical universities in the context of higher mathematics education

V I Klochko, Z V Bondarenko, S A Kyrylashchuk, O P Prozor and T H Kyrylashchuk

Vinnytsia National Technical University, 95 Khmelnytskyi Hw., Vinnytsia, 21021, Ukraine

E-mail: vi.klochko.7@gmail.com, bondarenko@vntu.edu.ua, kyrylashchuk@vntu.edu.ua, prozor@vntu.edu.ua, tan099838@vntu.edu.ua

Abstract. The visualisation of teaching higher mathematics as a development of information and communication technologies determines the transition of the educational process to a new level of its development. As a result, it should focus on the visual orientation and modern practice of its use. The question arises as to the formation of students' visual thinking as a necessary component of the training of a modern specialist. In this context, a study has been conducted on the problem of using visual information and its visual perception in teaching higher mathematics on the example of individual sections, as well as techniques of justification based on visual information. The examples offered in the article form the skills of working with graphic information. Developing visual thinking, they fix attention during the learning process, implicitly and indirectly contributing to the conversion of mental content into a visual image, ensuring the formation of a more complete picture of an image or concept. The article analyses the results of using visualisation in the educational process of students of technical universities. The influence of visualisation on the level of students' knowledge is analysed. In particular, the ability of a certain contingent of students to perceive and use visual images, their ability to think and express thoughts in images, perceive the content of logic, emotions contained in visual information, as well as the ability to operate with visual images in communication, establish cause and effect relationships and relations between concepts, as well as memorise and reproduce ideas about objects is noted.

1. Introduction

In the context of the increasing amount of information and the active "immersion" of youth in the consumption of visual media, the issues of visualizing the thinking of students at technical universities are becoming increasingly relevant [1–3]. Often, the effectiveness of learning mathematics for students is limited precisely by the difficulty of perceiving mathematical abstractions [4, 5]. The need for visualization of educational information is determined by the peculiarities of modern students' thinking, namely: the ability to quickly switch attention and to process information quickly; a preference for perceiving graphical information, and at the same time, an unadaptability to the perception of linear and homogeneous information, especially large book texts [6]. According to the authors, the main obstacle to a deep understanding of mathematics by students of technical universities is the insufficient development of abstract and logical thinking. This deficiency can be somewhat remedied by forming visualizing some basic concepts in mathematics. Overcoming this barrier in some topics will undoubtedly make it easier to perceive the material of other sections, where visualization is either impossible or limited. Free operation of visual objects is an important component of the professional training of future specialists in technical fields and their professional competence. The problems of effective methods of modern presentation of educational material, based on the psychological principles of cognitive visualization, are discussed by Babych and Semenikhina O. [7], Bezuglyi [8], Drushlyak [9], Reznik [10], Bilousova and Zhytienova [11], Gurzhii et al. [12], Gritchenko et al. [13], Zhytienova [14], Lipchevska [15], Rudenko [16], Nind and Lewthwaite [17].

The aim of the article is to theoretically substantiate and practically test the effective methods of modern presentation of educational material in higher mathematics, based on the principles of cognitive visualization.

The hypothesis of the research is as follows: formation of visual thinking as a means of systematizing concepts in the course of higher mathematics helps to improve understanding of mathematical concepts.

2. Materials and methods

To achieve the goal, the following scientific tasks were defined:

1. To analyze the current trends in the field of higher mathematics teaching and the use of visualization in the educational process.
2. To develop a theoretical concept that explains how visualization can influence the systematization of mathematical concepts, concepts understanding and student motivation.
3. To develop educational materials that use techniques and principles of cognitive visualization for presenting higher mathematics material.
4. To conduct a practical appraisal of the developed educational materials in an academic environment to collect data on their effectiveness and interaction with students.
5. To perform an analysis and evaluation of the obtained results, determining how successful and effective the developed techniques and educational materials were to understand the concepts of the higher mathematics course.

The following methods were used to solve the tasks set in the work: critical analysis and synthesis, induction, logical generalization, comparison, graphical and tabular analysis, etc.

3. Theoretical background

At the stage of reforming the education system, adapting to the demands of the society and the development of science and searching for new teaching methods, one of the issues that requires review and clarification is the use of visual aids as one of the main principles of didactics [18]. According to this principle, the perception of an object is possible visually, audially, kinetically and verbally. In this situation, the main task of educational activity is to maximally include visual process tools or visualistics in the educational process.

In pedagogy visuality is understood as one of the main principles of didactics, according to which teaching is based on concrete images that are directly perceived by the learners. The concept of visualization varies among authors. Some believe that visualization is the presentation of numerical and textual information in the form of graphs, diagrams, structural schemes, etc. Others view visualization as the process of presenting data through depiction [7, 8, 11].

The definitions encountered in scientific publications regarding the term "visualization" differ in their generic concept. Some authors perceive visualization as a ready presentation of numerical and textual information in the form of graphs, diagrams, structural schemes, tables, maps, etc. Others believe that visualization is the process of representing data through images for the purpose of maximizing their comprehensibility; giving a visible form to an object, subject or process. The terms “visibility” and “visualization” are typically equated. However, a detailed analysis of scientific and pedagogical research dedicated to the theoretical foundations of visualizing educational material, types, and means of visibility, as well as the theoretical foundations of visual learning support, visual thinking, practical issues of creating visualization tools, etc., shows that these concepts differ from each other. Among the types of visibility, scientists highlight visual clarity so the visualization of educational material can be included in the generic concept of “visibility” as a type. The interpretation of the term “visualization” implies the process of creating a visual image whereas the term “visibility” is associated with an already formed image of the educational object. This allows us to assert that the concept of visualization of educational material goes beyond the scope defined by the term “visibility” [7].

In our view, visualization is a specific category of didactics including the mechanisms of imagination, establishing and consolidating associative links between visual images and the properties of fundamental concepts, and the process of visualizing educational material that involves not only reproducing a visual image, but also the process of its creation as a means of reflecting the facts of reality.

The assertion of visuality as one of the most important factors of modernity is a logical result of the “information boom”. It has “overloaded” people with information, the processing of which exceeds their cognitive, mental, temporal and social resources. As a result, the way of perceiving as well as the way of analyzing reality are changing.

Visual types of communication transform education on a global scale where there is a transition from printed means of information relay to visual forms. This circumstance naturally entails an expansion of the influence of visualization on the educational process and education in general.

Modern forms of visualization of educational material include: reference notes, frames, logical-symbolic models, block diagrams, graph schemes, dynamic models, mental maps, interactive timelines, internet memes, tag clouds.

Visual images are not just an illustration of the author’ s thoughts, but the final expression of thinking itself. Unlike the usual use of visual aids, the work of visual thinking is an activity of the mind in a special environment which makes it possible to transition from one presentation of information to another, to comprehend the connections and relationships between its objects. In that we see the mechanism of activating thinking through visibility. As stated in the work [8], the main purpose of visualization in teaching is to support logical operations at all stages of educational activity and this is the most important issue when performing analytical actions (analysis, synthesis, comparison, search for connections and relationships, systematization, conclusion, etc.).

Among the functions of visualization are also the development among the students the skills to establish cause-and-effect relationships and relations between concepts, as well as skills in memorizing and reproducing images of objects, developing imagination, concentrating attention, etc.

The content of education is the core that connects all levels of the education system, defining their sequence and continuity. In shaping the content, it is important to establish a balance between the fundamentality and professional orientation of training, realizing the principle of visualization.

The main principles that guided us during the development of the content of educational information visualization are defined as follows:

- the development of visualization of educational information in a technical university must be based on a model that includes the goals of using visual means as a component;
- the combination of the visualization means with the ways of organizing students’ independent educational and cognitive activities;

– the methodological approach to organizing the teaching process requires determining the optimal structure of information based on didactic principles;
– the enlargement and visualization of educational information can be realized by various methods and corresponding symbolic models of knowledge representation;
– structuring the content of educational information is based on identified elements of knowledge;
– the application of visual educational information in the teaching process implies the systematic use of visual models of a certain type or their combination in the educational process.

Accompanying the study of mathematical concepts through prior analysis of their visual images (where possible) gives a new quality to higher mathematics classes. Visual thinking is an element of figurative thinking. Where possible, logical thinking should be developed with the inclusion of figurative thinking. In this case, mathematical abstractions find concrete embodiment and the assimilation of mathematical concepts occurs more easily, quickly and becomes more solid. Visual images directly affect the senses, their perception and understanding are more accessible than the perception of analytical information. Visual interpretation of many mathematical statements helps make them more transparent for assimilation.

Scientists propose various ways to solve the problem of forming a system of concepts – writing a genealogy of the concept, classification of concepts. For concepts of mathematical analysis, reliance on graphical, visual representations is relevant. Therefore, it is natural to assume that this approach will work in forming a system of concepts. What is the specificity of using visualization of concepts of mathematical analysis in forming their system?

First of all it is necessary to consider visual (graphical) representations of concepts and only then to move on to its analytical representation. Such an approach will allow for a linear study of mathematical analysis, and for forming a system of concepts, it is necessary to include tasks in the learning process that carry the sense of expanding the application of the theory of one section of analysis to other sections, i.e., transferring the concept from one section to another based on their visual images.

Thus, the visualization of concepts and tasks of applying the concepts of mathematical analysis of one section in the tasks of another section are identified as necessary conditions for systematizing the concepts of mathematical analysis. The high degree of abstraction in presenting information about concepts and their properties in the process of teaching mathematics to students of technical universities necessitates such an organization of mathematics teaching where the representations that arise in the minds of students reflect the main and essential aspects of mathematical objects and laws, in particular through the visual presentation of mathematical content.

It is necessary to achieve that the recall of a concept is primarily associated with the corresponding visual factor. The transition from a visual representation to a formal definition is the next step in forming a stable representation of a particular concept. Since this representation in such an approach is based on a visual image, students should more consciously, confidently, and with fewer errors define the various properties of the functions being studied, apply the learned concepts to solving tasks.

From the point of view of cognitive psychology, the visual context, firstly, gives students an external reference that reinforces and supports their expectations, namely, the substantive representation, prediction of performing corresponding actions; and secondly, forms an essential stimulus for perceiving new information. A varied form of stimulus, presented at a high frequency and reinforced by a visual context, allows not only to quickly classify or recognize perceived information, but also to effectively assimilate it at an actively operative level, i.e., to translate it from the algorithmic sphere of activity into the productive one.

As for the lecture-visualization, it can be most effectively used during the consideration of generalizing and abstract topics that are difficult to perceive in traditional lectures, as well as in initial lectures with the aim of increasing students' interest in the content of the lecture. The possibilities of lecture-visualization are offered to be used at the stage of starting a new section, for example, vector analysis (field theory).

In figure 1, a vector field, vector lines, an element of a field flow tube, a vector gradient, the physical formation of circulation and the rotor of a vector field are depicted. Such a visual acquaintance with mathematical concepts, the visual perception of their properties, connections and relationships between them in lectures and practical classes allows to quickly and visually unfold before students individual fragments of the theory, to form and spread a generalized algorithm of operations, to apply the acquired knowledge and skills to the cognition of the content of other fields of knowledge.

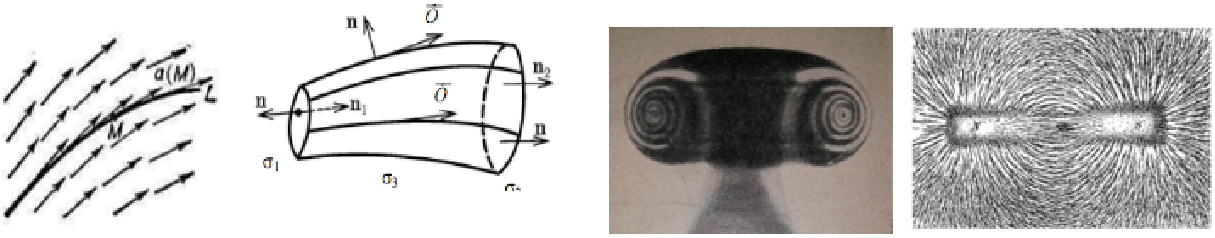

Figure 1. The visualization of basic concepts of the field theory.

However, lecture-visualization also has its drawbacks, particularly the rapid pace of information delivery can create difficulties during the note-taking of educational material. For better absorption of new knowledge it is advisable for students to be provided with educational material for review or the main text of the lecture for preliminary familiarization and a set of visual aids without text and connections, so that during the lecture students only need to take notes on explanations and not have to draw diagrams and figures.

In this case, it is appropriate to adhere to didactic requirements:

– not to overload the lecture with visual objects as this reduces the educational and cognitive activity of students;
– a visual object should not contain superfluous information to avoid the emergence of side analogies and associations.

Unlike the traditional use of visual aids, the work of thinking during the perception of a visual object is an activity of the mind in a special environment which makes it possible to transition from one form of information presentation to another, to understand the connections and relationships between objects.

Visualization plays an important role in systematizing and understanding the concepts of higher mathematics. It can help students transform abstract mathematical concepts into concrete visual images, easing their perception and memorization. Here are some ways to use visualization for systematizing concepts in higher mathematics:

Graphs and visual representations of functions:

- The construction of graphs of mathematical functions, the analysis of their shape and properties.
- Studying the changes in graphs with changes in the parameters of functions.

Vector algebra and geometry: Work with vectors, matrices, and operations on them, which can be visualized using vectors in space.

- Applying geometric concepts to solving problems using visual models.

Differential and integral equations:

- Studying the change in functions using concepts of derivatives and integrals through their graphs.
- Applying visual representations to solve problems in kinetics, motion.

Trigonometry and geometric figures:

- Studying trigonometric functions and their graphs to understand the relationship between triangles and angles.
- Applying geometric properties to analyze functions and their graphs.

Complex numbers and geometry on the complex plane:

- Studying complex numbers and their representation on the complex plane.
- Using complex numbers for the visual representation of geometric transformations.

Three-dimensional models:

- Using three-dimensional models to illustrate vectors, spatial relationships, and geometric constructions, visualizing curves in three-dimensional space.

Let's consider an example where, thanks to visualization, the solution of a Cauchy problem (1) allows the student to perform elements of qualitative analysis of integral curves.

$$y^{'} + ycos(x) = exp\left(-\sin(x)\right),\ y(0) = 2. \tag{1}$$

A student conducts an analysis of the corresponding process based on the construction of a fragment of the relevant integral curve (figure 2) and attempts to predict what the other integral curves will be like. It' s important to analyze the content of the general and particular solutions. The content of the general solution (usually, within the set of constructed graphs) allows one to determine the features of the dynamics of the process and to identify its behavior near certain points. The analysis of such situations enables students to realize the significance of mathematical modeling. For example, the graph of a particular solution (figure 2) indicates the presence of a significant number of extrema, and at the same time, the graphs of the general solution (figure 3) characterize the process differently near certain points (the max point changes to a min point). This visual approach to learning mathematics forms the ability to interpret information presented in a mathematical way, to use mathematics accurately for conveying information and solving problems [19].

Visual representation of mathematical concepts, visual perception of their properties, connections and relationships between them allow to quickly and visually unfold individual fragments of the theory before students focusing attention on the key moments of the problem-solving process. Consider this in the example of a qualitative description of dynamic systems. It usually comes down to constructing phase portraits. For visualizing the phase space, both general-purpose programs like Mathematica, MathCAD and specialized programs are used. It' s necessary to build a phase portrait of the system of differential equations and ensure that the non-closed integral curves on the phase plane correspond to the non-periodic solution of the system. For this purpose, construct a three-dimensional model of the system (2).

$$\begin{cases} \dfrac{dx}{dt} = a_{11}x + a_{12}y; \\ \dfrac{dx}{dt} = a_{11}x + a_{12}y. \end{cases} \tag{2}$$

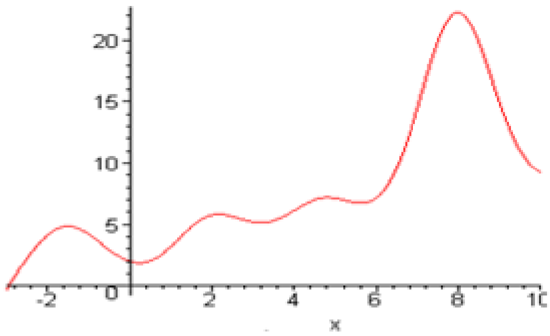

Figure 2. The graph of the particular solution, $y = (x+2) \cdot e^{-\sin x}$.

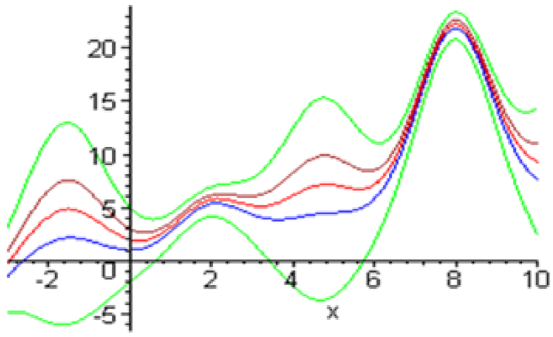

Figure 3. The graph of the elements of the general solution, $y = (x+C) \cdot e^{-\sin x}$.

In figure 4, the phase portrait of the system is presented, and in figure 5, a three-dimensional model of the system is shown (for specific values of coefficients and initial conditions). Figure 4 depicts the orthogonal projection of the system' s phase portrait onto the XY coordinate plane (similarly, orthogonal projections onto the XZ and YZ coordinate planes can also be considered).

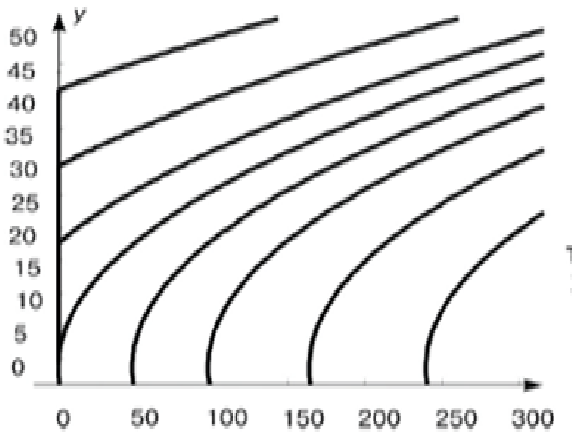

Figure 4. The phase portrait of the system.

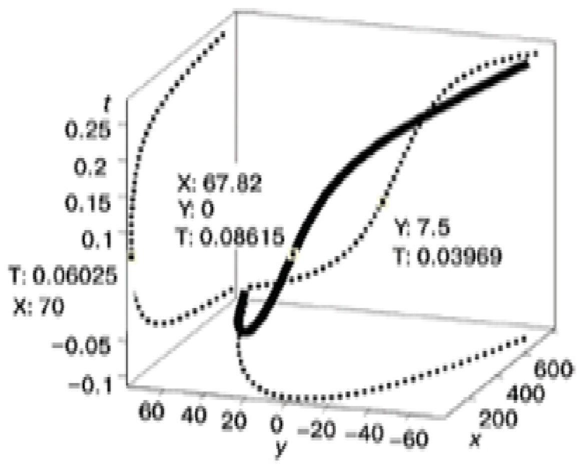

Figure 5. A three-dimensional model of the of differential equations system of the solution.

The visualization mentioned indicates that the solution of the system is not periodic (as shown in figure 5), hence the integral curves on the phase plane are not closed.

This is a crucial insight in the study of nonlinear dynamic systems. Apart from constructing and analyzing the phase space using various numerical algorithms, computer visualization enables students to comprehend important concepts of synergetics, such as limit cycles and strange attractors.

Experience with first-year students shows that they do not always clearly understand what it means to find the root of an equation with a given accuracy. By definition, x_i is an approximation of the root of the equation $f(x) = 0$ with accuracy ε if the condition $|x_i - \xi| < \varepsilon$ is met, where ξ is the exact root of the equation.

The problem arises because the exact root ξ is unknown (and mostly will not be found), so the actual difference $|x_i - \xi| < \varepsilon$ cannot be calculated. Students who are comfortable with numbers often ask questions like "What should be compared?", "When should the process of finding the root be stopped?", "Why are the conditions $|x_i - x_{i-1}| < \varepsilon$ or $|f(x_i)| < \varepsilon$ fost often used to ensure the specified accuracy?". The best way to address these issues is visually, by showing examples of intervals $[\xi,\ x_i]$, $[x_{i-1},\ x_i]$, $|f(x_i)|$.

Visualization helps clarify confusing areas and, in addition, enables choosing the criterion for stopping the process depending on the behavior of the function $f(x)$ (for example, the condition $|f(x)| < \varepsilon$ is not suitable for rapidly growing functions).

The visualization of "machine epsilon" is particularly interesting for students. When constructing graphs, "machine epsilon" can be used to visualize small differences between numbers that, although very close, still differ due to the limited accuracy of floating-point numbers. It's important for students to understand that this is crucial in areas where accuracy is key, such as numerical computations or signal processing. The impact of "machine epsilon" on graphical representation can arise, in particular, in cases involving interaction with graphs or data where the precision of numerical values matters. Here are a few situations where this may become noticeable:

- *Graphs of functions:* If the graph of a function has values very close to 1, then "machine epsilon" might cause the expression to be rounded to 1 when displayed on the graph. This can lead to underrepresentation or loss of some details in the visual representation (figure 6).
- *Comparison of values:* When comparing numbers on a graph (e.g., determining the points of intersection of curves), "machine epsilon" might cause values with minor differences from 1 to be perceived as equal, affecting the correctness of graph analysis.
- *Processing of graphical data:* In working with graphical data, such as images, "machine epsilon" can influence calculations or comparisons of pixel values, which can also affect image processing or graph analysis.

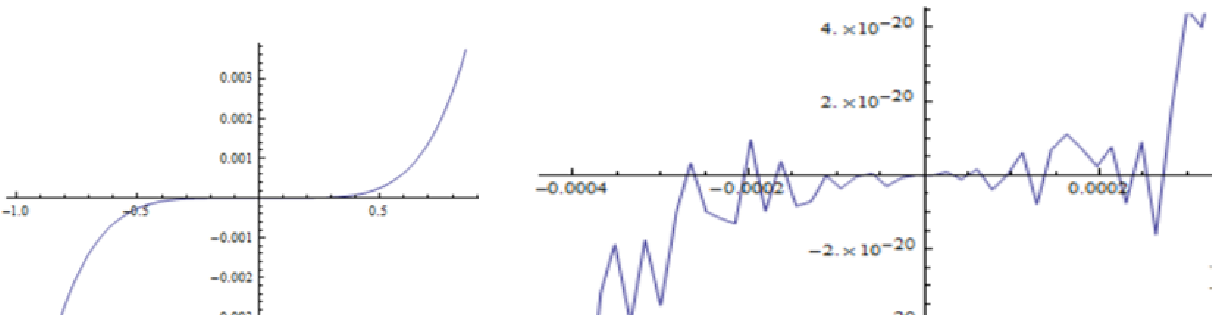

Figure 6. The structure of the graph connected with "machine epsilon".

Thanks to appropriate visualizations, students not only hear such information but also understand that when developing very accurate algorithms or in tasks where high numerical precision is critical, attention should be paid to the impact of "machine epsilon".

The process of perceiving, understanding, and memorizing knowledge quickly is based on visual cognitive images. Cognitive visualization has deep genetic forms and is used in various activities, of which the following are directly related to the sphere of higher mathematics education: aggregating significant didactic units of information (elements of knowledge) and their reproduction through various schematic-symbolic means; demonstrating theoretical formalized knowledge in a visual form necessary for easing perception; multidimensional visualization of problem areas in intellectual information technologies, such as the structural-logical scheme of studying a higher mathematics course.

According to the above mentioned methods of cognitive visualization, which are directly related to the field of higher mathematics education, the following levels of visualization can be identified in didactic means, based on the degree of adequacy to the objects:

- Reference signals, tables for aggregating information, the method of reference notes, workbooks, etc;
- Structural-logical schemes, graphs, and methods of schematizing knowledge;
- Modeling (logical-content models).

At each of these levels, key modeling principles are implemented: structuring information, linking elements of the structure, and aggregating information.

It should be noted that visualization of educational material reduces its verbal explanation, thereby aggregating information. For example, presenting material in the form of a presentation is a vivid example of such condensation.

Visualization of educational material at the first level allows its presentation as a guide for action in a form convenient for memorization and reproduction based on assimilated associations.

The effect of schematizing educational knowledge through structural-logical schemes (the second level) lies in the visual representation of the hierarchy of diverse elements of knowledge and concepts, indicating internal logical connections between them using directed lines that connect them. Structural-logical schemes help visualize, organize, summarize, visually build knowledge and concepts, freely navigate in the material, and correctly systematize it.

The third level of visualization (modeling) represents a new frontier in the theory and practice of didactic clarity, occurring at the level of designing new aesthetic, technological didactic means of the modeling type. The modeling form of knowledge representation is characterized by compactness and is a necessary form for performing, supporting assimilated information at all stages of activity – perception, processing, storage, reproduction, etc. It should be noted that there is a gradual transition from partially intuitive compilation of visual didactic means to their design using various compact elements of image models (conceptual, pictogram, sign-symbolic, etc.), as well as visually convenient methods of their logical organization.

The level of development of visual thinking can be assessed using various criteria that evaluate the ability of students to analyze, interpret, and use visual representations in the context of higher mathematics and engineering tasks. We offer the following criteria:

1. *Graphic skills*:
   - Ability to create quality graphs of mathematical functions and analyze their shape and properties.
   - Ability to use graphs to solve mathematical problems.
2. *Spatial thinking*:
   - Ability to identify and understand spatial relationships, especially in the context of vectors, matrices, geometric objects.
   - Use of visual representations to solve problems involving three-dimensional space.
3. *Working with computer programs*:
   - Ability to effectively use computer-aided design (CAD) programs and other engineering tools.
   - Application of visual capabilities of programs to solve engineering problems.
4. *Use of visual representations in analysis and problem solving*:
   - Ability to use visual models to solve complex engineering problems.
   - Study and analysis of visual data to solve mathematical tasks.
5. *Visual communication*:

- Ability to clearly and effectively present engineering ideas through visual means.
- Ability to prepare visual presentations and documentation.

These criteria were used to develop tasks and assessment tools that allow evaluating the level of development of visual thinking in students of technical specialties.

4. Results and discussion

In accordance with the set goals and hypotheses of the study, a pedagogical experiment was conducted in the first term with first-year students of specialty 121 – Software Engineering.

The purpose of the pedagogical experiment was to determine the effectiveness of modern methods of presenting educational material in higher mathematics, based on the psychological principles of cognitive visualization.

During the experiment, the hypothesis was tested: the use of innovative methods of presenting mathematical material using elements of visualization would contribute to the formation of visual thinking in students of technical universities, improving the process of assimilation and understanding of mathematical concepts.

In the initial stage of the experiment, 150 first-year students were tested using a preliminary written test to determine the initial level of development of visual thinking as far as the formation of mathematical concepts is concerned.

The majority of students, as shown by the testing, had a low level of development of visual thinking, as evidenced by the results presented in table 1.

Table 1. The results of the students' testing for identifying the initial level of visual thinking formation as for the formation of mathematical concepts.

The initial level of visual thinking formation	low	medium	high
The amount of students (150 people)	102 (68%)	39 (26%)	9 (6%)

A high level of development of visual thinking and heuristic skills was mainly shown by students who were talented in mathematics and who once again confirmed their results.

Additionally, at the beginning of the study using the experimental methodology, students were divided into control (C) and experimental (E) groups. Based on the results of the initial control work at the beginning of the study, the level of mathematics preparation of the students in these groups was determined (residual school knowledge and skills in mathematics were tested).

To test the statistical significance of the difference between the samples of marks for residual school knowledge of mathematics of students in the experimental and control groups, the Pearson criterion χ^2 was used. The null hypothesis in this case was formulated as follows: there is no statistically significant difference between the two empirical distributions. It was shown that this difference is not statistically significant, meaning there is no difference in residual school knowledge of mathematics between groups E and C. The statistical data obtained as a result of the pedagogical experiment meet the conditions for applying the Pearson criterion χ^2: the samples are random and independent; the sample size is more than 20; the sum of observations across all intervals equals the total number of observations.

To calculate χ^2, the formula was used: $\chi^2_{emp} = \frac{1}{n_1 \cdot n_2} \sum_{i=1}^{n} \left(\frac{n_1 Q_{2i} - n_2 Q_{1i}}{Q_{1i} + Q_{2i}} \right)^2$ where n_1 is the sum of all grades of the first sample, n_2 is the sum of all grades of the second sample, Q_{1i}, Q_{2i} is the number of students in the experimental (control) group belonging to category i (i = 1, 2, 3, 4) according to the state of the property being studied. In our case n_1 is the sum of all grades of students in the experimental group, n_2 is the sum of all grades of students in the control group [20].

The number of degrees of freedom is found using the formula $v = (k-1)(c-1)$, where k is the number of samples being studied, c is the number of gradations in the samples being studied. If the inequality $\chi^2_{emp} > \chi^2_{cr}$ is met, then the null hypothesis is rejected at the significance level α and the alternative hypothesis is accepted, but if $\chi^2_{emp} < \chi^2_{cr}$, then the null hypothesis is accepted, i.e., there is no sufficient basis to consider the state of the property being studied different in both samples.

For example, let's describe the statistical processing of results of the E group (72 students) and the C group (78 students). The total number of grades received by the students is presented in table 2.

Table 2. Total grades of residual school knowledge in Mathematics.

Groups of initial research	Grades				$\sum$
	1-3	4-6	7-9	10-12	
Experimental	$Q_{11} = 15$	$Q_{12} = 42$	$Q_{13} = 12$	$Q_{14} = 3$	72
Control	$Q_{21} = 20$	$Q_{22} = 45$	$Q_{23} = 11$	$Q_{24} = 2$	78
$\sum$	$Q_{11} + Q_{21} = 35$	$Q_{12} + Q_{22} = 87$	$Q_{13} + Q_{23} = 23$	$Q_{14} + Q_{24} = 5$	150

As a result of the calculations based on table 2, we obtained: $\chi^2_{emp} = 0.67$.

According to the table of critical values for the Pearson χ^2 criterion, we find that $\chi^2_{cr} = 11.345$ ($\alpha \leq 0.01$; $v = 3$). We obtained that $\chi^2_{emp} < \chi^2_{cr}$. Therefore, the null hypothesis is accepted. The difference in the total distribution of grades for residual school knowledge in mathematics in groups E and C is statistically not significant, therefore, there is no difference between the residual school knowledge in mathematics in the experimental and control groups.

The identification of the levels of formation of visual thinking in students was carried out using criteria (graphic skills, spatial thinking, work with computer programs, use of visual representations in analyzing and solving problems, visual communication) that allowed assessing the level of correctness and completeness of solving standard tasks and tasks using visualization.

The results of the experiment are presented in table 3.

Table 3. Total grades of residual school knowledge in Mathematics.

Groups	Number of students	Tasks			
		Standard task		Task with the use of visualization	
		Correct solution	Complete and correct solution	Correct solution	Complete and correct solution
Experimental	72	67	62	48	43
Control	78	65	56	30	23

Present the data of the table in the form of a diagram (figure 7).

Taking into account the obtained results, we will evaluate the probability of the E group's superiority compared to the C group in terms of correctness and completeness of task solutions, using the Wilcoxon-Mann-Whitney criterion.

 doi:10.1088/1742-6596/2871/1/012029

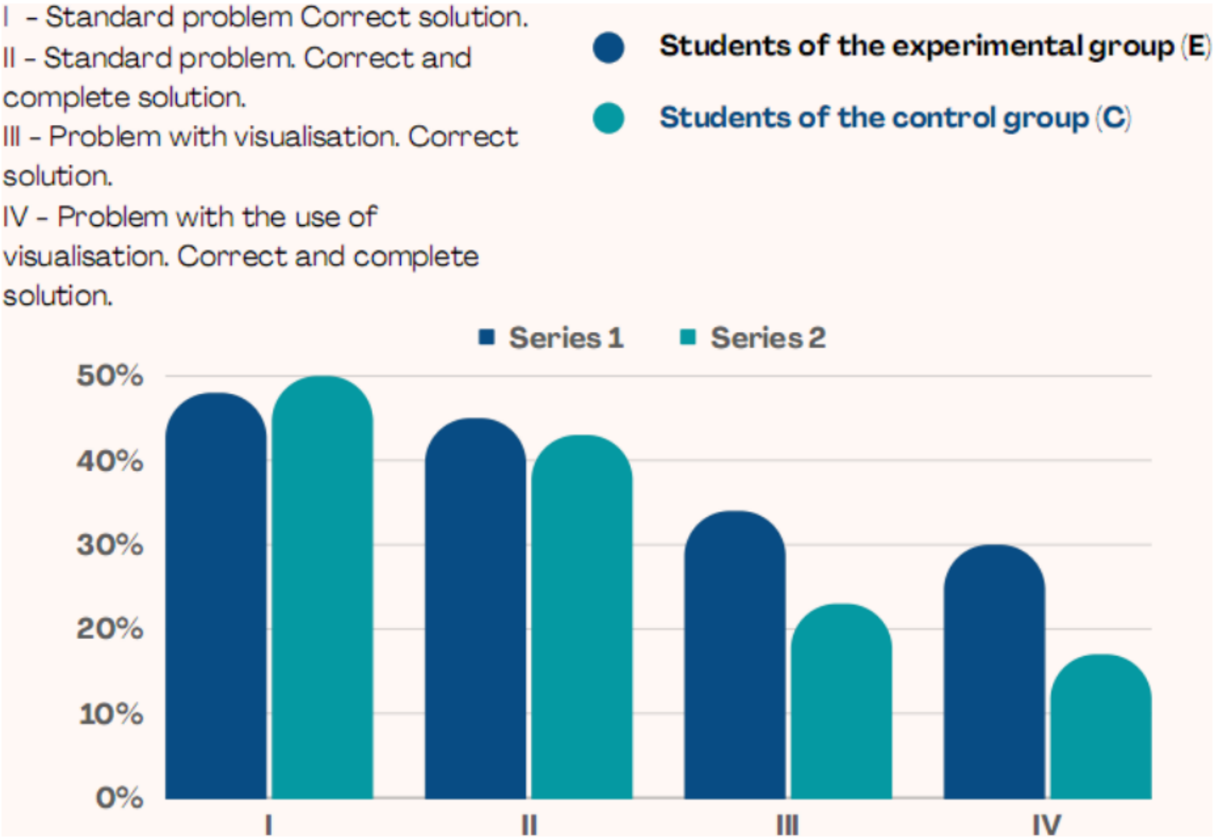

Figure 7. Experimental study of the formation of visual thinking.

The Wilcoxon-Mann-Whitney criterion is used in pedagogical esearch, as it allows testing several assumptions about the nature of distribution differences of a certain property in two populations (differences in medians, average values of both populations' distributions, as well as the presence of a tendency for objects of the first population to be on average larger or, conversely, smaller than members of the second population) based on ordinal measurements of the state of this property. Therefore, to test the statistical significance of the divergence between the samples of grades obtained by students in a control work consisting of standard tasks and tasks using visualization, and taking into account the results of evaluating the probability of the E group's advantage over the C group in terms of completeness and correctness of solving tasks, we used precisely this criterion [20].

The hypothesis being tested is H_0: $P(X < Y) = \frac{1}{2}$ at the significance level $\alpha = 0.01$ ($x_{\alpha/2} = 1.64$) with the alternative $P(X < Y) < \frac{1}{2}$. The correctness and completeness of the solution were assessed on a dichotomous scale. That is, the hypothesis H_0 assumes that the completeness and correctness of solving standard and problems using the visualization of the control work of the students of the experimental group (variable X), with the same probability equal to 0.5, is statistically more or less than the completeness and correctness of solving the problems by the students of the control group (variable Y), that is, students of both groups will solve problems at the same level. At the same time, the statistical data obtained as a result of conducting a pedagogical experiment meet the conditions for applying the Wilcoxon-Mann-Whitney criterion: the samples are random and independent, the members of each sample are also independent of each other; the sample size is more than 20; the property of objects under study is continuously distributed in both samples. According to the rule of using the criterion, the members of the two samples are combined into one, which has a size of $N = n_1 + n_2$, are written in a row in ascending order and ranked, i.e., each member of the row is assigned a rank, which numerically equals the place number this member occupies in the row. If several consecutive members of the row have the same value, each is assigned the same rank, equal to the arithmetic mean of the place numbers these members occupy. To test hypotheses using the Wilcoxon-Mann-Whitney, first calculate the sum of ranks S: $S = \sum_{i=1}^{n} R(x_i)$, where $R(x_i)$ is the rang assigned to the i-th object of the sample, n_1, n_2 are the sample sizes, and n equals the smaller of the values n_1, n_2. The value of the criterion statistic T is calculated using the formula (3):

$$T = S - \frac{n(n+1)}{2}. \tag{3}$$

If at least one of the values n_1, n_2 is greater than 20, then the critical value of the T-statistic is calculated using the formula (4):

$$W_{\alpha/2} = \frac{n_1 n_2}{2} + x_{\alpha/2}\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}, \tag{4}$$

where $x_{\alpha/2}$ is the quantile of the normal distribution for α=0.01, $x_{\alpha/2} = 1.64$.

The null hypothesis H_0: $P(X < Y) = \frac{1}{2}$ is rejected at the significance level α if one of the inequalities $T_{obs} < W_{\alpha/2}$ or $T_{obs} > n_1 n_2 - W_{\alpha/2}$ is true.

For an example of calculation, let's assess the correctness of solving a standard problem.

Let's compare the results of groups E and C. Out of 72 individuals in group E, 67 students succeeded in solving the problem. From 78 individuals in group C, 65 correctly solved the problem. From the entire sample of 150 students, there were 132 acts of student interaction with the problem that received a score of 1, and these are assigned a rank ($\frac{18+\ldots+150}{132} = 84$). There were 17 acts of interaction (incorrectly solved problems) that have a rank ($\frac{1+\ldots+17}{17} = 9$). Thus:

$$S = 84 \cdot 67 + 5 \cdot 9 = 5673;$$

$$T_{obs} = 5673 - \frac{72 \cdot 73}{2} = 3045;$$

$$W_{\alpha/2} = \frac{72 \cdot 78}{2} + 1.64\sqrt{\frac{72 \cdot 78 \cdot 151}{12}} = 3244;$$

$$T_{obs} < W_{\alpha/2}.$$

The accuracy and completeness of solving a standard problem were similarly assessed. For this problem $T_{obs} < W_{\alpha/2}$ as well. Therefore, in all cases of the standard problem, we have $T_{obs} < W_{\alpha/2}$. In all cases of problems with visualization, $T_{obs} > n_1 n_2 - W_{\alpha/2}$. Thus, we can conclude that the alternative hypothesis H_1: $P(X < Y) \neq \frac{1}{2}$ is accepted when rejecting the null hypothesis H_0: $P(X < Y) = \frac{1}{2}$. Acceptance of this hypothesis means that the analysis of experimental data allows us to conclude: the distribution laws of each group are different, indicating significantly better results for group E compared to group C. In other words, the experimental methodology improves the quality of performing standard tasks and also develops the ability to solve problems using visualization.

Considering the obtained results, the statistical significance of the differences between group E and group C was also evaluated using the Pearson χ^2 criterion (the difference in the distribution of residual knowledge and skills of groups E and C at the end of the experiment was determined). The null hypothesis was adopted: there is no difference between the two empirical distributions.

It was found that $\chi^2_{emp} = 71.8$ and $\chi^2_{cr} = 9.21$ ($\alpha = 0.01$, $v = 2$). Since $\chi^2_{emp} > \chi^2_{cr}$, the null hypothesis is not accepted. Therefore, the difference in the total distribution of grades for residual knowledge in higher mathematics in groups E and C is statistically significant, indicating differences between the residual knowledge in higher mathematics in the experimental and control groups. Thus, the level of formation of visual thinking in group E has significantly increased compared to group C, and these changes are statistically significant.

Thus, through mathematical methods of analyzing the results of an educational experiment, it was confirmed that there is an increase in the level of visual thinking formation and the level of mathematical preparation of students in experimental groups compared to students in the control group.

5. Conclusions

Visualization technology of educational information is a system that includes: a complex of educational knowledge; visual methods of their representation; visual-technical means of transmitting information; psychological techniques for using and developing visual thinking in the learning process. Visualization of educational material, conditioned by the development of information and communication technologies, provides the possibility of its compression, presentation as an orienting basis of action in a compact form, convenient for memorization and reproduction based on the learned associations and analogies. Visualization is associated with the formation of stable visual images and mastering various mental operations on them, similar to such general processes as abstraction, distinguishing the main from the secondary, structuring, logical reasoning, association, analogy, etc. Active and purposeful use of the reserves of visual thinking in the learning process is based on the selection of stable images in the educational material with an emphasis on the “primacy” of the image, on immediate and possibly more accurate visual association with an abstract concept that precedes verbal description. The essence of learning lies in shifting the priority from the illustrative function to the visual cognitive function, thereby ensuring the shift of emphasis from the educational function to the developmental one.

In this context, research was conducted on the problem of forming visual information and its visual perception in teaching higher mathematics, using examples of individual sections, as well as techniques of substantiation based on visual information. An analysis of the results of using visualization in the educational process of students at technical universities was conducted. The impact of visualization on the level of student knowledge in higher mathematics was analyzed.

In particular, it is noted the ability of a certain contingent of students to perceive and use visual images, their ability to think and express thoughts in images, to perceive the content of logic, emotions contained in visual information, as well as the ability to operate with visual images in communication, establish cause-and-effect relationships and relations between mathematical concepts, as well as memorize and reproduce representations of objects.

ORCID iDs

V I Klochko `https://orcid.org/0000-0002-9415-4451`
Z V Bondarenko `https://orcid.org/0000-0003-3339-0570`
S A Kyrylashchuk `https://orcid.org/0000-0002-8972-3541`
O P Prozor `https://orcid.org/0000-0003-1454-8352`

References

[1] Vlasenko K, Volkov S, Sitak I, Lovianova I and Bobyliev D 2020 Usability analysis of on-line educational courses on the platform “Higher school mathematics teacher” *E3S Web of Conferences* **166** 10012
[2] Vlasenko K, Chumak O, Lovianova I, Kovalenko D and Volkova N 2020 Methodical requirements for training materials of on-line courses on the platform “Higher school mathematics teacher” *E3S Web of Conferences* **166** 10011 URL `https://doi.org/10.1051/e3sconf/202016610011`

[3] Vlasenko K, Chumak O, Achkan V, Lovianova I and Kondratyeva O 2020 Personal e-learning environment of a mathematics teacher *Universal Journal of Educational Research* **8**(8) 3527–3535 URL https://doi.org/10.13189/ujer.2020.080828

[4] Bobyliev D Y and Vihrova E V 2021 Problems and prospects of distance learning in teaching fundamental subjects to future Mathematics teachers *Journal of Physics: Conference Series* **1840**(1) 012002 URL https://doi.org/10.1088/1742-6596/1840/1/012002

[5] Vlasenko K, Chumak O, Sitak I, Lovianova I and Kondratyeva O 2019 Training of mathematical disciplines teachers for higher educational institutions as a contemporary problem *Universal Journal of Educational Research* **7**(9) 1892–1900 URL https://doi.org/10.13189/ujer.2019.070907

[6] Kremen V H and Ilyin V V 2020 Presentation of visual literacy in the educational process and its explication in the culture of thinking *Information Technologies and Learning Tools* **75**(1) 1–12 URL https://doi.org/10.33407/itlt.v75i1.3660

[7] Babych O and Semenikhina O 2014 To questions about interpretation of clarity and visualization *Physical and Mathematical Education* (2) 47–53 URL https://fmo-journal.fizmatsspu.sumy.ua/publ/4-1-0-21

[8] Bezuglyi D 2014 Visualization as a strategy in modern teaching *Physical and Mathematical Education* (1) 5–11 URL https://fmo-journal.fizmatsspu.sumy.ua/journals/2014-v1-2/2014_1-2-Bezuglyi_Scientific_journal_FMO.pdf

[9] Drushlyak M G 2018 Glossary of visual education: Visibility, visualization, visual thinking *Physical and Mathematical Education* (1) 78–83 URL https://fmo-journal.fizmatsspu.sumy.ua/publ/4-1-0-499

[10] Reznik N 2000 Technology of visual thinking *School technologies* **4** 127–141

[11] Bilousova L I and Zhytienova N V 2017 Cloud services as an effective visualization tool *CTE Workshop Proceedings* **4** 25–30 URL https://doi.org/10.55056/cte.320

[12] Gurzhii A, Kotsur B, Volynskyi V and Samsonov V 2003 *Visual and audiovisual means of teaching* (Kyiv: Academy of Pedagogical Sciences of Ukraine, Institute of Pedagogy)

[13] Gritchenko A G, Martyniuk M T and Shut M I 2020 Modern technologies of educational information visualization in future teachers' professional training *Collection of Scientific papers Kamianets-Podilskyi National Ivan Ohiienko University. Pedagogical series* **26** 92–101 URL http://ped-series.kpnu.edu.ua/article/view/226214

[14] Zhyteniova N 2019 *Visual didactic tools: creation and use in educational practice* (Kharkiv: Kharkiv National Pedagogical University named after H.S. Skovoroda)

[15] Lipchevska I 2021 Visualisation in education: a modern approach to the use of visuals *World of didactics: didactics in the modern world* vol 1 pp 196–197 URL http://lib.iitta.gov.ua/728087/

[16] Rudenko Y 2019 Lecture-visualization as the form of the psychological and pedagogical supply of formation of professional and methodic integrity of future learners of preschool education institutions *Scientific Visnyk V.O. Sukhomlynskyi Mykolaiv National University. Pedagogical Sciences* **65**(2) 242–247 URL https://doi.org/10.33310/2518-7813-2019-65-2-242-247

[17] Nind M and Lewthwaite S 2018 Methods that teach: developing pedagogic research methods, developing pedagogy *International Journal of Research & Method in Education* **41**(4) 398–410 URL https://doi.org/10.1080/1743727X.2018.1427057

[18] Kholoshyn I, Nazarenko T, Bondarenko O, Hanchuk O and Varfolomyeyeva I 2021 The application of geographic information systems in schools around the world: A retrospective analysis *Journal of Physics: Conference Series* **1840**(1) 012017 URL https://doi.org/10.1088/1742-6596/1840/1/012017

[19] Klochko V, Bondarenko Z, Kyrylashchuk S and Volkov Y 2022 *Higher mathematics. Ordinary differential equations* (Vinnytsia: Vinnytsia National Technical University)

[20] Rotanova N and Diachenko O 2020 Organization of a pedagogical experiment: stages and mathematical methods of analyzing results *Physical and Mathematical Education* **25**(3) URL https://doi.org/10.31110/2413-1571-2020-025-3-013