1.Vasyl V. KUKHARCHUK[1], 2. Sergii V. PAVLOV[1], 3. Samoil Sh. KATSYV[1], 4. Andrii M. KOVAL[1], .5. Volodymyr S. HOLODIUK[1], 6. Mykhailo V. LYSYI[1], 7. Andrzej KOTYRA[2], 8. Orken MAMYRBAEV[3], 9. Aidana KALABAYEVA[4]

Vinnytsia National Technical University, Ukraine (1), Lublin University of Technology, Poland (2), Institute of Information and Computational Technologies CS MES RK, Almaty, Kazakhstan (3); Kazakh Academy of Transport & Communication (4)
ORCID: 1. 0000-0001-9920-2726, 2. 0000-0002-0051-5560, 3. 0000-0003-1375-5229, 4. 0000-0003-4985-9433, 5. 0000-0002-2449-5488, 6. 0000-0002-2329-1619, 7. 0000-0002-9442-6090, 8. 0000-0001-8318-3794; 9. 0000-0003-4398-3759

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Transient analysis in 1st order electrical circuits in violation of commutation laws

***Abstract**. The paper considers the usage of non-standard analysis mathematical apparatus to solve some non-trivial problems of electrical engineering theory. The axiomatics of non-standard analysis makes it possible to simplify the transient analysis in the 1st order electrical circuits in violation of the commutation laws. Examples of solving such problems are given.*

***Streszczenie.** W artykule rozważono zastosowanie aparatu matematycznego analizy niestandardowej do rozwiązywania niektórych nietrywialnych zadań z teorii elektrotechniki. Aksjomatyka analizy niestandardowej pozwala na uproszczenie analizy stanów nieustalonych w obwodach elektrycznych I rzędu z naruszeniem praw komutacji. Podane są przykłady rozwiązywania takich przypadków. (**Analiza stanów przejściowych w obwodach elektrycznych 1 rzędu z naruszeniem praw komutacji**).*

Keywords: infinitesimal number, infinitely large number, hyperreal number, non-standard number, real number, inductive electric circuit.
Słowa kluczowe: liczba nieskończenie mała, liczba nieskończenie duża, liczba hiperrzeczywista, liczba niestandardowa, liczba rzeczywista, indukcyjny obwód elektryczny.

Introduction

In [1, 2] the problems of calculating DC circuits with ideal reactive elements using the mathematical apparatus of non-standard analysis were considered. This mathematical approach made it possible to apply unified calculation methods using complex numbers to analyze such circuits. However, methods of non-standard analysis can also be useful for other electric circuits theory problems, for the transient analysis in circuits in violation of commutation laws.

Before proceeding to these problems, the axiomatics of non-standard analysis can be recalled. The number α will be called an infinitesimal number, then the number β = 1/α – infinitely large number. All algebraic operations (addition, subtraction, multiplication, division, squaring, etc.) and theorems (commutativity, associativity, etc.) can be applied to infinitesimal and large numbers. There are infinitesimal and large numbers of different order viz. $\alpha > \alpha^k$ – infinitesimal numbers of the first and k-th order; $\beta < \beta^k$ – infinitely large numbers of the first and kth order. Together with real numbers$r \in R$ infinitesimal and large numbers form an ordered set of hyperreal numbers. $*R$. Real numbers $r \in R$ calledstandard or Archimedes as opposed to non-standard (non-Archimedes) numbers $*r \in * R$. Two non-standard numbers $*a$ and $*b$ called equivalent (or infinitely close to each other) if and only if $*a - *b \approx \alpha$. The notation $\approx$ will mean the equivalence of two non-standard numbers.

In [1, 2] various relations which are necessary for carrying out algebraic transformations within the limits of the non-standard analysis were resulted. Most of these ratios will be used in this paper. Before proceeding to the usage of the above expressions to solve various applications, it should be noted that there are no general rules for selecting a parameter that should be equated to an infinitesimal (or infinitely large) number. This choice is made by the researcher depending on the context of a particular task. It should be borne in mind that in the case of the need to replace infinitesimal numbers of several heterogeneous parameters of one problem, determining ratios of the orders of these numbers is a difficult problem and sometimes requires additional research. More details about this mathematical apparatus can be found in [3, 4, 5].

Transient Analysis Of Inductive Circuits In Violation Of The Commutation Laws

Example 1. Consider this example of a circuit, which is shown in Fig.1, *a*. Scheme's parameters: $U = 600$ V, $r_1 = r_2 = 20\,\Omega$ ohm, $L_1 = 0.2$ H, $L_2 = 0.3$ H. Determine the transient currents $i_1(t)$, $i_2(t)$.

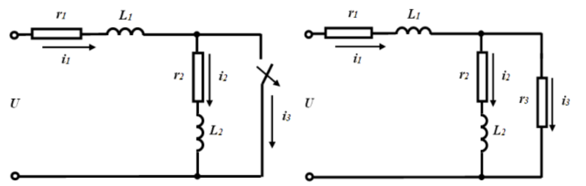


Fig. 1. Example of inductive circuits

Typically, in such circuit, the analysis of the transient process is performed using a generalized commutation law, which applies to flux couplings. However, the usual commutation law can be used, if the methods of non-standard analysis are applied.

Since the branch with the switching device was shorted before switching, It is assumed that it had some resistance $r_3 = \alpha$. After switching, replace the rupture of this branch with infinitely large resistance $r_3 = \beta$ (Fig. 1, *b*), thus removing the inductive cross section, which makes it possible to apply the standard commutation laws [6,7].

Then the initial conditions are:

(1) $i_1(0_+) = i_1(0_-) = \frac{U}{r_1 + \frac{r_2 r_3}{r_2 + r_3}} = \frac{U}{r_1 + \frac{r_2 \alpha}{r_2 + \alpha}} \approx \frac{U}{r_1} = 30$A.

(2) $i_2(0_+) = i_2(0_-) = i_1(0_-)\frac{r_3}{r_2 + r_3} = i_1(0_-)\frac{\alpha}{r_2 + \alpha} \approx 0$,

The forced current is defined as:

(3) $i_{1пр} = i_{2пр} = \frac{U}{r_1 + \frac{r_2\beta}{r_2+\beta}}\frac{\beta}{r_2+\beta} \approx \frac{U}{r_1 + \frac{r_2\beta}{\beta}}\frac{\beta}{\beta} = \frac{U}{r_1 + r_2} = 15\ \text{A}$ A.

By the method of input resistance

(4) $$Z_{вх}(p) = r_1 + pL_1 + \frac{(r_2+pL_2)\beta}{r_2+pL_2+\beta} = \frac{r_1r_2+pr_1L_2+r_1\beta+pr_2L_1+p\beta L_1+p^2L_1L_2+r_2\beta+p\beta L_2}{r_2+pL_2+\beta}$$

forming a characteristic equation:

(5) $$p^2L_1L_2 + p(r_2L_1 + r_1L_2 + \beta L_1 + \beta L_2) + r_1r_2 + \beta(r_1 + r_2) = 0.$$

This quadratic equation has two roots. The first of these can be determined by performing equivalent transformations of the equation (5):

(6) $$p^2L_1L_2 + p(r_2L_1 + r_1L_2 + \beta L_1 + \beta L_2) + r_1r_2 + \beta(r_1 + r_2) \approx \beta[p(L_1 + L_2) + (r_1 + r_2)] = 0$$

Hence,

(7) $$p_1 = -\frac{r_1+r_2}{L_1+L_2} = -80\text{s}^{-1}.$$

The second root is found by Vieta's theorem (for the quadratic equation $ap^2 + bp + c = 0$ the valid formu(16)is $p_1p_2 = \frac{c}{a}$, or $p_2 = \frac{c}{ap_1}$).

From the characteristic equation (5) it follows that $c = r_1r_2 + \beta(r_1 + r_2)$, a $a = L_1L_2$, then:

(8) $$p_2 = \frac{r_1r_2+\beta(r_1+r_2)}{L_1L_2\left(-\frac{r_1+r_2}{L_1+L_2}\right)} \approx -\frac{\beta(r_1+r_2)(L_1+L_2)}{L_1L_2(r_1+r_2)} = -\frac{\beta(L_1+L_2)}{L_1L_2} = -8.333\beta \text{ s}^{-1}.$$

Then

(9) $$i_2(t) = i_{2пр} + A_1e^{p_1t} + A_2e^{p_2t} = 15 + A_1e^{-80t} + A_2e^{-8.333\beta t},$$

and

(10) $$i_1(t) = i_2(t) + \frac{i_2(t)r_2 + L_2\frac{di_2(t)}{dt}}{r_3} = \left(i_{2пр} + A_1e^{p_1t} + A_2e^{p_2t}\right)\left(1 + \frac{r_2}{\beta}\right) + \frac{L_2}{\beta}\left(p_1A_1e^{p_1t} + p_2A_2e^{p_2t}\right) \approx 15 + A_1e^{-80t} - 1.5A_2e^{-8.333\beta t}.$$

Current $i_3(t)$ determined as:

(11) $$i_3(t) = i_1(t) - i_2(t) = -2.5A_2e^{-8.333\beta t}.$$

Determining the integration constants in expressions (9) and (10) by substituting the variable *t*, the value of the initial time $t = 0_+ \approx \alpha_1$ (the initial time is indicated by a symbol α_1, because by its physical nature it differs from resistance $r_3 = \beta = \frac{1}{\alpha}$). This raises the uncertainty $e^{-8.333\frac{\alpha_1}{\alpha}}$.

From the ratio of infinitesimal numbers α and α_1 it is impossible to establish purely mathematically, because they belong to heterogeneous parameters. Analyzing them from a physical point of view and recalling that α_1 – is initial time, and α – this is the active conduction of the circuit breaker, which were specifically introduced to fulfill the standard commutation laws. Since these values are independent of each other, it is always possible to choose them so as to provide a condition $\alpha_1 \approx \alpha^2$. This way it can be written as

(12) $$e^{-8.333\frac{\alpha_1}{\alpha}} = e^{-8.333\frac{\alpha^2}{\alpha}} = e^{-8.333\alpha} \approx 1,$$

Taking into account (9), (10), (11) and (12), a system of equations for determining the integration constant can be obtained as follows

(13) $$15 + A_1 - 1.5A_2 = 30,\ 15 + A_1 + A_2 = 0.$$

Hence $A_1 = -3$, $A_2 = -12$. Therefore

(14) $$i_1(t) = 15 - 3e^{-80t} + 18e^{-8.333\beta t}\text{A},$$
$$i_2(t) = 15 - 3e^{-80t} - 12e^{-8.333\beta t}\text{A},$$
$$i_3(t) = 30e^{-8.333\beta t}\text{A}.$$

Since $e^{-\beta t} = \alpha$, it is possible to write

(15) $$\forall(t > 0 \wedge t \neq 0_+)\begin{cases} i_1(t) = 15 - 3e^{-80t} + 18e^{-8.333\beta t} \\ \approx 15 - 3e^{-80t} + 18\alpha \approx 15 - 3e^{-80t}, \\ i_2(t) = 15 - 3e^{-80t} - 12e^{-8.333\beta t} \\ \approx 15 - 3e^{-80t} - 12\alpha \approx 15 - 3e^{-80t}, \\ i_3(t) = 30e^{-8.333\beta t} \approx 30\alpha \approx 0. \end{cases}$$

Recalling that infinitely large resistance$r_3 = \beta$ was artificially introduced to ensure compliance with standard commutation laws. Considering the values of the currents flows in the branches at the moments $t = 0$ i $t = 0_+$ in the real circuit, with $r_3 = \infty \approx \beta^\beta$.

As already determined, before switching at $t < 0$ (in particular at $t = 0_-$) $i_1 = i_3 = 30\text{A}$, $i_2 = 0\text{A}$.

At the time $t = 0_+ \approx \alpha_1$ expression (14) considering (12) will take the form

$$i_1(0_+) = 15 - 3e^{-80\alpha_1} + 18e^{-8.333\infty\cdot\alpha_1} \approx 15 - 3 + 18\alpha \approx 12\text{A}$$
$$i_2(0_+) = 15 - 3e^{-80\alpha_1} - 12e^{-8.333\infty\cdot\alpha_1} \approx 15 - 3 - 12\alpha \approx 12\text{A}$$
$$i_3(t) = 30e^{-8.333\infty\cdot\alpha_1} \approx 30\alpha \approx 0\text{A}.$$

At time $t = 0 \approx {\alpha_1}^\beta$ expression $e^{-8.333\infty\cdot 0}$ becomes uncertain, since time and resistance are heterogeneous parameters, the exact value of currents at this time cannot be determined. Only the intervals of their possible values are known as [8]:

$$30 \geq i_1(0) \geq 12,$$
$$0 \leq i_2(0) \leq 12,$$
$$30 \geq i_3(0) \geq 0.$$

Finally let us consider the energy ratios in the circuit. Before switching ($t < 0$) the energy was stored only in the magnetic field of the first coil and was equal to

(17) $$W(0_-) = \frac{L_1i_1^2(0_-)}{2} = \frac{0.2\cdot 30^2}{2} = 90\text{ J}.$$

In the first moment of time after switching ($t = 0_+$) energy is already present in the field of both coils

(18) $$W(0_+) = \frac{L_1i_1^2(0_+)}{2} + \frac{L_2i_2^2(0_+)}{2} = 36\text{J}$$

Thus, the energy deficit is as follows:

(19) $$\Delta W = 90 - 36 = 54\text{J}.$$

At the time of switching currents in all branches change their value. The time of these changes is an infinitesimal number, so in the first and second branches, where the active resistance have finite values, there is no loss of active energy on these resistors during this time. The resistance of the third branch$r_3 = \beta$, so it consumes so much energy

(20) $$\Delta W = \int_0^\infty i_3^2(t)r_3dt = \int_0^\infty \left(30e^{-8.333\beta t}\right)^2\beta dt = \left.\frac{900e^{-16.666\beta t}\beta}{-16.666\beta}\right|_0^\infty = -54e^{-16.666\beta\cdot\infty} + 54e^{-16.666\beta\cdot 0} \approx 54\text{ J}.$$

Example 2. Consider now a circuit where is a magnetic connection between the inductors (Fig. 2). Scheme parameters:$U = 60$ V V, $r_1 = 5\ \Omega$, $r_2 = r_3 = 10\ \Omega$, $L_1 = 0.1$H, $L_2 = 0.2$ H, $M = 0.05$ H. Let us determine the transient current $i_1(t)$.

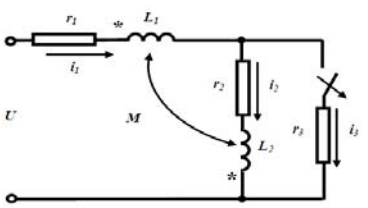


Fig.2. Circuit with is a magnetic coupling between the inductors

Assuming that the third branch after switching will have resistance $r_3 = \beta$. Then the initial conditions are:

(21) $i_1(0_+) = i_1(0_-) = \frac{U}{r_1 + \frac{r_2 r_3}{r_2 + r_3}} = 6$ A,

$i_2(0_+) =_2 (0_-) = i_1(0_-)\frac{r_3}{r_2+r_3} = 3$A.

The forced component is defined as:

(22) $i_{1пр} = \frac{U}{r_1 + \frac{r_2\beta}{r_2+\beta}} \approx \frac{U}{r_1 + \frac{r_2\beta}{\beta}} = \frac{U}{r_1 + r_2} = 4$A.

To get a characteristic equation, we compose a system of equations according to Kirchhoff's laws

(23) $i_1 = i_2 + i_3,\quad L_1\frac{di_1}{dt} - M\frac{di_2}{dt} + r_1 i_1 + r_3 i_3 = U,$

$$L_2\frac{di_2}{dt} - M\frac{di_1}{dt} + r_2 i_2 - r_3 i_3 = 0.$$

Substituting $i_3 = i_1 - i_2$.

(24) $(L_1 p + r_1 + r_3)i_1 - (Mp + r_3)i_2 = U,$

$-(Mp + r_3)i_1 + (L_2 p + r_2 + r_3)i_2 = 0.$

Equating the determinant of the system (24) to zero after minor transformations, the characteristic equation in the next form is obtained

$$(L_1L_2 - M^2)p^2 + [L_1r_2 + L_2r_1 + (L_1 + L_2 - 2M)r_3]p + r_1r_2 + (r_1 + r_2)r_3 = 0,$$

Substituting initial data

(25) $0.018p^2 + (2 + 0.2\beta)p + 50 + 15\beta = 0.$

This quadratic equation has two roots, first of them is determined by performing the necessary transformations

(26) $0.018p^2 + (2 + 0.2\beta)p + 50 + 15\beta \approx \beta(0.2p + 15) = 0$, from $p_1 = -\frac{15}{0.2} = -75\text{s}^{-1}$.

The second root is found by Viet theorem:

(27) $p_2 = \frac{c}{p_1 a} = \frac{50+15\beta}{(-75)0.018} \approx -\frac{15\beta}{(-75)0.018} = -11.43\beta\text{s}^{-1}$.

Then $i_1(t) = i_{1пр} + A_1e^{p_1t} + A_2e^{p_2t}$ and the first equation for determining the integration constants as follows

(28) $i_1(0_+) = i_{1пр} + A_1 + A_2 = 4 + A_1 + A_2 = 6.$

Finding the first derivative of the current $i_1(t)$:

(29) $\frac{di_1(t)}{dt} = p_1A_1e^{p_1t} + p_2A_2e^{p_2t}.$

The initial condition for the first derivative of the current is determined by the system of equations (23) for the moment $t = (0_+)$.

(30)
$$i_1(0_+) = i_2(0_+) + i_3(0_+),$$
$$L_1\frac{di_1}{dt}\Big|_{t=0_+} - M\frac{di_2}{dt}\Big|_{t=0_+} + r_1i_1(0_+) + r_3i_3(0_+) = U,$$
$$L_2\frac{di_2}{dt}\Big|_{t=0_+} - M\frac{di_1}{dt}\Big|_{t=0_+} + r_2i_2(0_+) - r_3i_3(0_+) = 0.$$

Taking into account that $i_3(0_+) = i_1(0_+) - i_2(0_+) = 6 - 3 = 3$A, and substituting the numerical values, the system (30) in the next form is written as

(31) $0.1\frac{di_1}{dt}|t = 0_+ - 0.05\frac{di_2}{dt}|t = 0_+ + 30 + 3\beta = 60,$

$$0.2\frac{di_2}{dt}|t = 0_+ - 0.05\frac{di_1}{dt}|t = 0_+ + 30 - 3\beta = 0.$$

Hence $\frac{di_1}{dt}|t = 0_+ = 428.57 - 25.7\beta = p_1A_1 + p_2A_2$ and the system of equations for determining the integration constants will take the final form as

(32) $A_1 + A_2 = 2,$

$-75A_1 - 11.43\beta A_2 = 428.57 - 25.7\beta.$

After the transformations of the second equation of the system obtained as $-11.43\beta A_2 = -25.7\beta$, hence

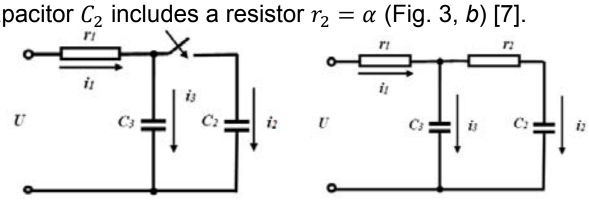
(33) $A_2 = \frac{25.7\beta}{11.43\beta} = 2.25$, and $A_1 = 2 - A_2 = -0.25$.

Therefore

(34) $i_1(t) = 4 - 0.25e^{-75t} + 2.25e^{-11{,}45\beta t} \approx 4 - 0.25e^{-75t}$A.

Transient Analysis of Capacitive Circuits In Violation Of The Commutation Laws

Example 3. Determine the transient voltages on the capacitors in the circuit, which is shown in Fig. 3, a. Scheme parameters: $U = 100$ V, $r_1 = 10\Omega$, $C_2 = 100\mu\text{F}$, $C_3 = 150\mu\text{F}$. To be able to use the second law of commutation will assume that the capacity of branch with capacitor C_2 includes a resistor $r_2 = \alpha$ (Fig. 3, *b*) [7].



a) b)

Fig. 3. Example of a capacitive circuit

The initial conditions are

(35) $u_{C_3}(0_+) = u_{C_3}(0_-) = U = 100\text{V},$

$u_{C_2}(0_+) = u_{C_2}(0_-) = 0\text{V}.$

The forced component we define as

(36) $u_{C_2пр} = u_{C_3пр} = U = 100\text{V}.$

By the method of input resistance

(37) $Z_{вх}(p) = r_1 + \frac{\left(r_2 + \frac{1}{pC_2}\right)\frac{1}{pC_3}}{r_2 + \frac{1}{pC_2} + \frac{1}{pC_3}} = r_1 + \frac{\left(\alpha + \frac{1}{pC_2}\right)\frac{1}{pC_3}}{\alpha + \frac{1}{pC_2} + \frac{1}{pC_3}} = \frac{r_1(\alpha C_2C_3p^2 + (C_2+C_3)p) + \alpha C_2 p + 1}{\alpha C_2C_3p^2 + (C_2+C_3)p}$

form a characteristic equation:

(38) $\alpha r_1C_2C_3p^2 + [r_1(C_2 + C_3) + \alpha C_2]p + 1 = 0.$

This quadratic equation has two roots, first of them is determined by performing the appropriate transformations

(39) $\alpha r_1C_2C_3p^2 + [r_1(C_2 + C_3) + \alpha C_2]p + 1 \approx r_1(C_2 + C_3)p + 1 = 0,$

hence $p_1 = -\frac{1}{r_1(C_2+C_3)} = -400\text{s}^{-1}$.

The second root is found by Viet theorem

(40) $p_2 = \frac{c}{p_1 a} = -\frac{1}{\alpha r_1C_2C_3\frac{1}{r_1(C_2+C_3)}} = -\frac{C_2+C_3}{\alpha C_2C_3} = -\frac{16667}{\alpha}\text{s}^{-1}$.

Then

(41) $u_{C_2}(t) = U + A_1e^{p_1t} + A_2e^{p_2t} = 100 + A_1e^{-400t} + A_2e^{-\frac{16667}{\alpha}t},$

(42) $u_{C_3}(t) = u_{C_2}(t) + \alpha C_2\frac{du_{C_2}(t)}{dt} = U + A_1e^{p_1t} + A_2e^{p_2t} + \alpha C_2(A_1p_1e^{p_1t} + A_2p_2e^{p_2t})$

$\approx 100 + A_1e^{-400t} - 0.667A_2e^{-\frac{16667}{\alpha}t}.$

To determine the integration constants, it is necessary to substitute the initial values of the first moment of time instead of the variable *t* in expressions (41) and (42). $t = 0_+ \approx \alpha_1$ (the initial moment of time is denoted by a symbol α_1, because its physical nature differs from the resistance $r_2 = \alpha$). This raises uncertainty $e^{-16667\frac{\alpha_1}{\alpha}}$, which is solved similarly to expression (30), ie

(43) $e^{-16667\frac{\alpha_1}{\alpha}} = e^{-16667\frac{\alpha^2}{\alpha}} = e^{-16667\alpha} \approx 1.$

Taking into account (41), (42) and (43), a system of equations for determining the integration constants is obtained as follows

(44) $u_{C_2}(0_+) = 100 + A_1 + A_2 = 0,$

$u_{C_3}(0+) = 100 + A_1 - 0.667A_2 = 100.$

Hence $A_1 = -40$, a $A_2 = -60$.

Therefore

(45) $u_{C_2}(t) = 100 - 40e^{-400t} - 60e^{-\frac{16667}{\alpha}t}$ V,

(46) $u_{C_3}(t) = 100 - 40e^{-400t} + 40e^{-\frac{16667}{\alpha}t}$ A.

Since $e^{-\frac{16667}{\alpha}t} = \alpha$, it is possible to write

(47) $$\forall(t > 0 \wedge t \neq 0_+)\begin{cases} u_{C_2}(t) = 100 - 40e^{-400t} - 60e^{-\frac{16667}{\alpha}t} \\ \approx 100 - 40e^{-400t} - 60\alpha \approx 100 - 40e^{-400t}, \\ u_{C_3}(t) = 100 - 40e^{-400t} + 40e^{-\frac{16667}{\alpha}t} \approx \\ 100 - 40e^{-400t} + 40\alpha \approx 100 - 40e^{-400t}. \end{cases}$$

Recalling that the infinitesimal resistance $r_2 = \alpha$ was artificially introduced to enforce standard commutation laws. Considering the values of the voltages on the capacitors take at moments $t = 0$ and $t = 0_+$ in the real circuit, with $r_2 = 0 \approx \alpha^{\beta}$.

As already determined, before switching at $t < 0$ (especially at $t = 0_-$) $u_{C_3} = 100$ V, $u_{C_2} = 0$ V.

Now $t = 0_+ \approx \alpha_1$ expressions (32) and (38) considering (30) will take the form

(48) $u_{C_2}(0_+) = 100 - 40e^{-400\alpha_1} - 60e^{-\frac{16667}{0}\alpha_1} \approx 100 - 40e^{-400\alpha_1} - 60\alpha \approx 60$V,

$u_{C_3}(0_+) = 100 - 40e^{-400\alpha_1} + 40e^{-\frac{16667}{0}\alpha_1} \approx 100 - 40e^{-400\alpha_1} + 40\alpha \approx 60$V.

At the moment $t = 0 \approx \alpha_1{}^{\beta}$ expression $e^{-16667\frac{0}{0}}$ becomes uncertain, since time and resistance are heterogeneous parameters, the exact value of voltages at this time cannot be determined. Only the intervals of their possible values known as

$$0 \le u_{C_2}(0) \le 60,$$
$$60 \le u_{C_3}(0) \le 100.$$

Finally, considering the energy ratios in the circuit and determining the current in the capacitor, which was switched

(49) $i_2(t) = C_2 \frac{du_{C_2}(t)}{dt} = 10^{-4} \frac{d\left(100 - 40e^{-400t} - 60e^{-\frac{16667}{\alpha}t}\right)}{dt} = 1.6e^{-400t} + \frac{100}{\alpha}e^{-\frac{16667}{\alpha}t}$ A.

Before switching ($t < 0$), the energy of the electric field was stored only in the first capacitor and was equal to

(50) $W(0_-) = \frac{C_3 u_{C_3}^2(0_-)}{2} = \frac{150 \cdot 10^{-6} \cdot 100^2}{2} = 0.75$ J.

At the first moment of time after switching ($t = 0_+$) the energy is already stored in both capacitors and is equal to

(51) $W(0_+) = \frac{(C_3 + C_2) u_{C_3}^2(0_+)}{2} = \frac{250 \cdot 10^{-6} \cdot 60^2}{2} = 0.45$ J.

Thus the energy deficit is

(52) $\Delta W = 0.75 - 0.45 = 0.3$ J.

In traditional electrical engineering books, the presence of this deficit is explained by the loss of energy when the capacitor is charging, but there are not provided any mathematical evidence. It is proven within the limits of the non-standard analysis.

(53) $$\Delta W = \int_0^{\infty} i_2^2(t) r_2 dt = \int_0^{\infty}\left(1.6e^{-400t} + \frac{100}{\alpha}e^{-\frac{16667}{\alpha}t}\right)^2 \alpha dt =$$
$$= \int_0^{\infty}\left(2.56\alpha e^{-800t} + 320e^{-\left(400+\frac{16667}{\alpha}\right)t} + \frac{10000}{\alpha}e^{-\frac{33333}{\alpha}t}\right)dt =$$
$$= \left(-3.210^{-3}\alpha e^{-800t}\frac{320}{400 + \frac{16667}{\alpha}}e^{-\left(400+\frac{16667}{\alpha}\right)t} - 0.3e^{-\frac{33333}{\alpha}t}\right)\Bigg|_0^{\infty} \approx$$
$$\approx -0.3e^{-\frac{33333}{\alpha}\infty} + 0.3e^{-\frac{33333}{\alpha}0} \approx 0.3e^{-33333\cdot 0} \approx 0.3 \text{ J.}$$

The law of conservation of energy is fulfilled.

Conclusions

1) The application of ideas and methods of non-standard analysis in the field of electrical engineering makes it possible to use the traditional classical method of transient analysis in circuits in violation of the commutation laws.

2) Only using the methods of non-standard analysis can strictly prove the implementation of the law of conservation of energy in such circuits.

3) In order to expand the scope of non-standard methods of analysis one should identify similar problems in different areas of science and technology, which use differential calculus and boundary crossings and solution by standard approaches is limited or impossible.

Authors: *Vasyl V. Kukharchuk, Vinnytsia National Technical University, Khmelnytsky highway, 95, 21000, Vinnytsia, Ukraine, Sergii V. Pavlov, Vinnytsia National Technical University, Khmelnytsky highway, 95, 21000, Samoil Sh. Katsyv, Vinnytsia National Technical University, Khmelnytsky highway, 95, 21000, Vinnytsia, Ukraine, Andrii M. Koval, Vinnytsia National Technical University, Khmelnytsky highway, 95, 21000, Vinnica, Ukraine, Volodymyr S.Holodiuk, Vinnytsia National Technical University, Khmelnytsky highway, 95, 21000, Vinnica, Ukraine, e-mail: vgolodyk@gmail.com, Mykhailo V. Lysyi, Vinnytsia National Technical University, 93 Khmelnitsky Road, Vinnitsa, Ukraine, e-mail: m.lysyi@bigmir.net Andrzej Kotyra, Lublin University of Technology, Poland, e-mail:a.kotyra@pollub.pl; Orken Mamyrbaev, Institute of Information and Computational Technologies CS MES RK, Almaty, Kazakhstan, e-mail: morkenj@mail.ru; Aidana Kalabayeva, Kazakh Academy of Transport & Communication, , e-mail: a.kalabaeva@list.ru*

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